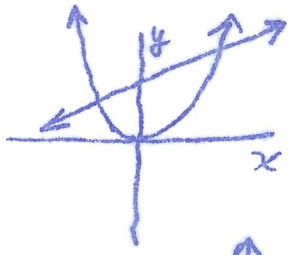
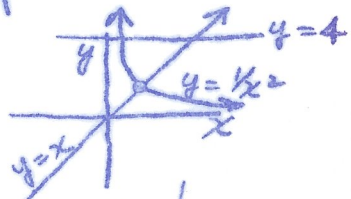
CALCULUS I EXAM 5 Chapter 6 (Anton) Dr. Rapaje UCF

Show all work as necessary on separate paper. Gold sheet, non-graphics calculators are allowed. You may keep this test.

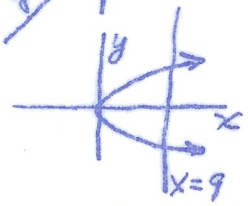
1. Set-up only to find the area enclosed by $y=x^2$ and $-x+y=2$ by integrating
   a) with respect to x
   b) with respect to y.



2. Find the area bounded by $y=\frac{1}{x^2}$, $y=x$, $y=4$.



3. The region bounded by $y^2=x$, $y=0$, and $x=9$ is rotated about the x axis. Find the volume. Set up two ways -- disk and shell method.



4. Set up only to find the volume if the region in #3 is rotated about the y axis:
   a) by disk method
   b) by shell method.

5. Set up only to find the volume if the region in #3 is rotated about the line $x=9$.

6. Find the arclength of $y=\frac{2}{3}x^{3/2}$ from $x=0$ to $x=1$.

7. If $y=\frac{x^4}{16}+\frac{1}{2x^3}$, find $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ and simplify.

8. The acceleration due to gravity on a certain planet is $-40\ ft/sec^2$. Find formulas for velocity and position on this planet in terms of initial velocity v_0, initial position s_0, and t.

9. Find the equations for velocity and position if $a(t)=4\sin 2t$ and $v(0)=4$, $s(0)=100$

10. A projectile is thrown upward from the top of a 64 ft building with an initial velocity of 48 ft/sec. a) How long before the stone hits the ground? b) Find the velocity at impact. c) How high did it go?

11. A cylindrical tank of radius 10 feet and 20 feet deep is filled with water 8 feet deep. Set up only to find the work required to pump all the water over the rim. (Water weighs 62.4 lb/ft^3)

CALCULUS I EXAM 5 (Anton) Solutions UCF Dr. RAPAPORT

1. $y = x^2 - x + y = 2$

$y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$y = 4 \quad y = 1$

a) $\int_{-1}^{2} [(x+2) - x^2]\,dx$

b) $\int_0^1 \sqrt{y} - (-\sqrt{y})\,dy + \int_1^4 [\sqrt{y} - (y-2)]\,dy$

$= 2\int_0^1 \sqrt{y}\,dy + \int_1^4 (\sqrt{y} - y + 2)\,dy$

2a) $y = \frac{1}{x^2}$ $y = x$, $y = 4$ $\quad x = \frac{1}{\sqrt{y}}$

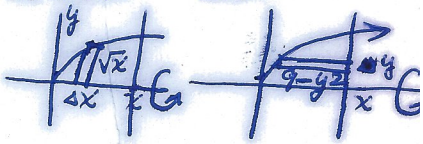
$\int_1^4 \left(y - \frac{1}{\sqrt{y}}\right) dy$ or $\int_{1/2}^{1} \left(4 - \frac{1}{x^2}\right) dx + \int_1^4 (4 - x)\,dx$

$= \frac{y^2}{2} - \frac{2y^{1/2}}{1}\Big|_1^4$

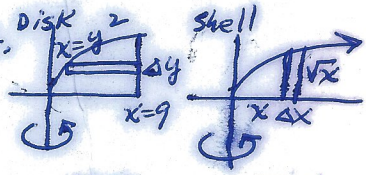
$= (8-4) - \left(\frac{1}{2} - 2\right)$

$= 4 + \frac{3}{2} = \frac{11}{2}$

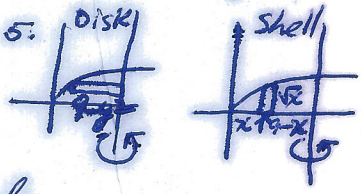
3. Disk: Shell:



4. Disk $x = y^2$ Shell



5. Disk Shell



3. $y^2 = x$, $y = 0$, $x = 9$, x axis.

Disk – dx $\quad V = \pi \int_0^9 [f(x)]^2\,dx$

$= \pi \int_0^9 x\,dx = \pi \frac{x^2}{2}\Big|_0^9$

$= \frac{81\pi}{2}$

Shell – dy $\quad V = 2\pi \int_0^3 (9 - y^2)\,y\,dy$

$= 2\pi \int_0^3 (9y - y^3)\,dy$

$= 2\pi \left[\frac{9y^2}{2} - \frac{y^4}{4}\right]_0^3$

$= 2\pi \left[\frac{81}{2} - \frac{81}{4}\right] = 2\pi \cdot \frac{81}{4} = \frac{81\pi}{2}$

4a) Disk: $V = \pi \int_0^3 \left(\frac{2}{3} - y^4\right) dy$ $\quad$ yaxis

b) Shell $\quad V = 2\pi \int_0^9 x \cdot \sqrt{x}\,dx$

5. about $x = 9$.

Disk: $V = \pi \int_0^3 (9 - y^2)^2\,dy$

Shell $\quad V = 2\pi \int_0^9 \sqrt{x}(9 - x)\,dx$ E.C.

6. $L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ $\quad y = \frac{2}{3}x^{3/2}$

$y' = x^{1/2}$

$= \int_0^1 \sqrt{1 + x}\,dx$

$= \frac{2}{3}(1+x)^{3/2}\Big|_0^1 = \frac{2}{3}\left(2^{3/2} - 1\right)$ or $\frac{2}{3}(2\sqrt{2} - 1)$

8. $a = -40$

$V = -40t + V_0$

$s = -20t^2 + V_0 t + s_0$

9. $a(t) = 4\sin 2t$

$V(t) = \int 4\sin 2t\,dt \quad V(0) = 4$

$= -2\cos 2t + C_1$

$4 = -2 + C_1 \quad C_1 = 6$

$V = -2\cos 2t + 6$

$s = \int (-2\cos 2t + 6)\,dt \quad s(0) = 100$

$= -\sin 2t + 6t + C_2$

$100 = 0 + 0 + C_2$

$s = -\sin 2t + 6t + 100$

10. $s = -\frac{1}{2}gt^2 + V_0 t + s_0$

$= -16t^2 + 48t + 64$

a) $s = 0 = 16t^2 + 48t + 64$

$-16(t^2 - 3t - 4) = 0$

$(t-4)(t+1) = 0$

$t = 4$ sec $\quad t = -1$ Reject.

b) $V = -32t + 48$

$V(4) = -32(4) + 48$

$= -128 + 40 = -80$ ft/sec

c) $V = -32t + 48 = 0$

$32t = 48$

$t = 3/2$ sec.

$s(3/2) = -16\left(\frac{9}{4}\right) + 48\left(\frac{3}{2}\right) + 64$

$= -36 + 72 + 64 = 100$ ft

7. $y = \frac{x^4}{16} + \frac{1}{2}x^{-2}$

$\frac{dy}{dx} = \frac{x^3}{4} - \frac{3}{2}x^{-3}$

$= \frac{x^3}{4} - \frac{1 \cdot -2}{x^3 \cdot 4 \cdot 2}$

$= \frac{x^6 + 4}{4x^3}$

$\left(\frac{dy}{dx}\right)^2 = \frac{x^{12} - 8x^6 + 16}{16x^6}$

$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{x^{12} - 8x^6 + 16 + 16x^6}{16x^6}$

$= \frac{x^{12} + 8x^6 + 16}{16x^6} = \frac{(x^6+4)^2}{16x^6}$

$\sqrt{\left(\frac{dy}{dx}\right)^2 + 1} = \frac{x^6 + 4}{4x^3}$

11. 

$F = 62.4\pi r^2 \Delta x$

$d = 20 - x$

$\int_0^8 62.4\pi \cdot 10^2 (20 - x)\,dx$

$= 6240\pi \int_0^8 (20 - x)\,dx$