CALCULUS I Exam 3 (Chapter 4 - ANTON) Dr. RAHLJE (UCF)

Non graphics Calculators, Gold sheet are allowed. [SHOW ALL WORK!!]

1. Find the Maximum value M and the minimum value m of $f(x) = 2x^5 - 5x^4 + 7$
   a) on $[-1, 3]$ b) on $(-1, 3)$.

2. Given $y = x\sqrt{4-x^2}$, $y' = \frac{4-2x^2}{(4-x^2)^{1/2}}$, and $y'' = \frac{2x(x^2-6)}{(4-x^2)^{3/2}}$
   a) Find all critical points (x coordinates only)
   b) Find all stationary points (x coord.)
   c) Find all points of inflection (x coord).
   } (Justify!)
   d) Explain why $x=2$, $x=-2$ is or is not a point of inflection.

3a) Graph: $f(x) = \frac{x^2-27}{x-6}$, $f'(x) = \frac{x^2-12x+27}{(x-6)^2}$ $f''(x) = \frac{18}{(x-6)^3}$

b) $\lim_{x\to-\infty} f(x) =$ ____; $\lim_{x\to+\infty} f(x) =$ ____; $\lim_{x\to 6^-} =$ ____; $\lim_{x\to 6^+} =$ ____.

c) Explain the difference between a vertical asymptote and a vertical tangent.

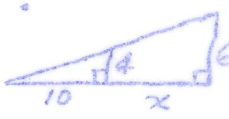
4. Determine if the theorem of the mean applies. If it does, then find all values of c for which the theorem is satisfied:
   a) $f(x) = \frac{x+1}{x-1}$ on $[0, 3]$ b) $f(x) = \frac{x+1}{x-1}$ on $[2, 3]$

5. If $y = \frac{7x^2-3}{(x^2-1)^{1/3}}$, find y' in simplified, factored form.

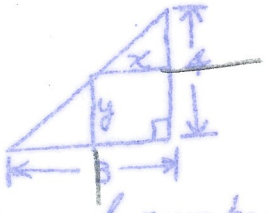
no calculus
6. Find the area and perimeter in terms of

r, h, and π.

no calculus
7.a) Solve for x:



b) Give an equation that shows the relationship between x and y:



8. The sum of one number and 3 times a second number is 60. Find the two numbers so the product of the numbers is a maximum.

9. Romeo is at the top of a 15' ladder, when Juliet's father at the base of the ladder begins to pull it away from the wall at 4 ft/sec. How fast is Romeo dropping when the bottom of the ladder is 9 ft from the wall?

10. Use Newton's method to find an x intercept for $f(x) = x^5 + x^4 - 5$.
    [$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$] Begin with $x=1$ and show each step to obtain 4 place accuracy.

CALCULUS I EXAM 3 Solutions Dr. RAPALJE

1. $f(x) = 2x^5 - 5x^4 + 7$

$f'(x) = 10x^4 - 20x^3$

$= 10x^3(x-2) = 0$

$x = 0, \; x = 2$

$f(0) = 7 \quad f(-1) = 0$

$f(2) = -9 \quad f(3) = 88$

a) Max is 88, Min is -9

b) No Max, Min is -9

2. $y = x\sqrt{4-x^2}$ (domain: $-2 \le x \le 2$)

$y' = \frac{4-2x^2}{(4-x^2)^{1/2}}$

$y'' = \frac{2x(x^2-6)}{(4-x^2)^{3/2}}$

a) Critical pts: $x = \pm\sqrt{2}, \pm 2$

b) Stationary pts: $x = \pm\sqrt{2}$

c) y'': $\overset{-2}{\infty}$ $-$ $\overset{-\sqrt{2}}{0}$ $+$ $\overset{0}{+}$ $+$ $\overset{\sqrt{2}}{0}$ $-$ $\overset{2}{\infty}$

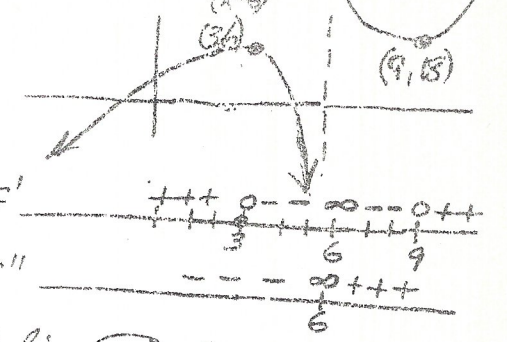
Point of inflection: $x = 0$

d) No – concavity doesn't change. NOT because f'' DNE.

3. $f(x) = \frac{x^2-27}{x-6}$, $f'(x) = \frac{(x-9)(x-3)}{(x-6)^2}$

$f''(x) = \frac{18}{(x-6)^3}$

a)



b) $\lim_{x\to-\infty} = -\infty$ $\lim_{x\to+\infty} = +\infty$

$\lim_{x\to 6^-} = -\infty$ $\lim_{x\to 6^+} = +\infty$

4. Thm of Mean:

$f'(c) = \frac{f(b)-f(a)}{b-a}$

a) $f(x) = \frac{x+1}{x-1}$ on $[0,3]$

Discontinuous. Thm does not apply

b) $f(x) = \frac{x+1}{x-1}$ on $[2,3]$

$\frac{f(3)-f(2)}{3-2} = \frac{\frac{4}{2} - \frac{3}{1}}{1} = -1$

$f'(x) = \frac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2}$

$= \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$f'(c) = \frac{-2}{(c-1)^2} = -1$

$-2 = -(c-1)^2$

$(c-1)^2 = 2$

$(c-1) = \pm\sqrt{2}$

$c = 1 \pm \sqrt{2}$

$c = 1+\sqrt{2}$ in $[2,3]$

$c = 1-\sqrt{2}$ not in $[2,3]$

5. $y = \frac{7x^2-3}{(x^2-1)^{1/3}}$

$y' = \frac{(x^2-1)^{1/3}14x - (7x^2-3)\frac{1}{3}(x^2-1)^{-2/3}2x}{(x^2-1)^{2/3}}$

$= \frac{2x(x^2-1)^{-2/3}\left[7(x^2-1) - \frac{1}{3}(7x^2-3)\right]}{(x^2-1)^{2/3}}$

$= \frac{2x}{(x^2-1)^{2/3}}\left[7x^2 - 7 - \frac{7}{3}x^2 + 1\right]$

$= \frac{2x}{(x^2-1)^{2/3}}\left[\frac{14x^2}{3} - \frac{18}{3}\right]$

$= \frac{4x(7x^2-9)}{3(x^2-1)^{4/3}}$

c) Vertical asymptote – f, f', f'' are all undefined.

Vertical tangent – f' and f'' are undefined, but f exists.

6.

$A = 2rh + \frac{1}{2}\pi r^2$

$P = 2h + 2r + \pi r$

7a)

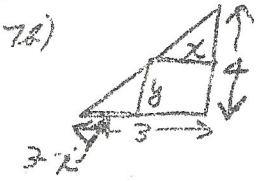
$\frac{4}{10} = \frac{6}{10+x}$

$60 = 40 + 4x$

$4x = 20$

$x = 5$

7b)



$\frac{4}{3} = \frac{y}{3-x}$

$3y = 12 - 4x$

$4x + 3y = 12$

8. Let x, y = 1st, 2nd nos.

$x + 3y = 60 \qquad x = 60 - 3y$

$P = xy = (60-3y)y$

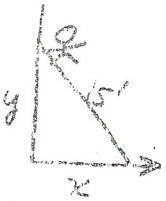
$= 60y - 3y^2$

$\frac{dP}{dy} = 60 - 6y = 0$

$y = 10$

$x = 30$

9.



$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(9)(4) + 2(12)\frac{dy}{dt} = 0$

$24\frac{dy}{dt} = -72$

$\frac{dy}{dt} = -3$ ft/sec

$\frac{dx}{dt} = 4$ ft/sec.

when $x = 9$,

$81 + y^2 = 225$

$y^2 = 144$

$y = 12$

10. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= \frac{4x_n^5 + 3x_n^4 + 5}{5x_n^4 + 4x_n^3}$

$x_1 = 1.3333$

$x_2 = 1.2394$

$x_3 = 1.2248$

$x_4 = 1.2244$

$x_5 = 1.2244$.