MAC 3311 CALCULUS I FINAL EXAM UCF Dr. RAPALJE

Show all work as necessary on separate paper.
Non graphics calculators are allowed.

1. a) $\lim_{x\to 6} \frac{x-6}{x^2-36}$ b) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$ (Show algebraically!)

2. Find all points of discontinuity for $f(x)=\frac{x^2-4}{x^2-4x-12}$.
Identify whether removable or non removable.

3 a) $\lim_{\theta\to 0} \frac{\sin 5\theta}{2\theta}$ b) Prove that $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta}=0$.

4. If $y=(3x+2)^4\sqrt{2x-5}$, find $\frac{dy}{dx}$ and give factored form.

5. If $s=\frac{t^2}{t^3+7}$, find $\frac{ds}{dt}$ and simplify.

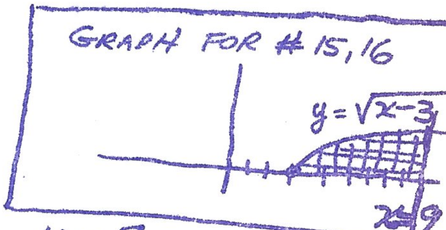
6. If $f(x)=\cos^3(\sin 3x)$, find $f'(x)$.

7. Use the limit definition of the derivative to prove that if $f(x)=\cos x$, then $f'(x)=-\sin x$. $\left[f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right]$

8. A 13-ft ladder is leaning against a wall. If the top of the ladder slides down the wall at 6 ft/sec, how fast is the bottom moving away from the wall when the top is 5 feet above the ground?

9. If $y=\frac{1-x}{x^2}$, $y'=\frac{x-2}{x^3}$, $y''=\frac{2(3-x)}{x^4}$, sketch the graph indicating concavity. Give relative extrema, points of inflection, asymptotes.

10. A cylindrical can is to have a volume of 1 liter (or 1000 cm^3). with
Determine the radius and height in order to minimize the surface area (that is, top, bottom, and side area).



11. $\int \frac{3x\,dx}{\sqrt{x^2+5}}$

12. $\int_{\pi/8}^{\pi/4} \sin^3 2x \cos 2x\,dx$

14. Find the area bounded by the graphs $y=x^3$, $y=-x$, $y=8$.

15-16. Find the volume if the region shown above is rotated
a) about the x axis b) the y axis, c) about $x=10$ d) about $y=-2$

17. A rocket is fired upward with $V_0=49$ m/sec from a tower 150 m. high. If $a=-9.8$,
a) how long will it take to reach max height b) how high does it go, c) how long to hit the ground. d) Find velocity at impact.

18. Find the arclength of $y=\frac{1}{3}x^3+\frac{1}{4x}$, $1\le x\le 2$.

MAC 3311 Calculus I Final Exam Solations Rapalje

1a) $\lim\limits_{x\to 6} \dfrac{\cancel{x-6}}{(\cancel{x-6})(x+6)} = \dfrac{1}{12}$

b) $\lim\limits_{x\to 0} \dfrac{(\sqrt{x+3}-\sqrt{3})}{x} \dfrac{(\sqrt{x+3}+\sqrt{3})}{(\sqrt{x+3}+\sqrt{3})}$

$= \lim\limits_{x\to 0} \dfrac{x+\cancel{3}-\cancel{3}}{x(\sqrt{x+3}+\sqrt{3})} = \dfrac{1}{2\sqrt{3}}$ or $\dfrac{\sqrt{3}}{6}$

2. $f(x) = \dfrac{x^2-4}{x^2-4x-12} = \dfrac{(x-2)\cancel{(x+2)}}{(x-6)\cancel{(x+2)}}$

Discontinuous at $x=6,\ x=-2$

Removable discontinuity at $x=-2$
(Hole in graph)

Non removable discontin. at $x=6$
(Asymptote)

3a) $\lim\limits_{\theta\to 0} \dfrac{5\cdot \sin 5\theta}{2\cdot 5\theta} = \dfrac{5}{2}$

b) $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta}\dfrac{(1+\cos\theta)}{(1+\cos\theta)} = \dfrac{1-\cos^2\theta}{\theta(1+\cos\theta)}$

$= \lim\limits_{\theta\to 0} \dfrac{\sin^2\theta}{\theta(1+\cos\theta)} = \lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}\cdot\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{1+\cos\theta}$

$= 1\cdot\dfrac{0}{1+1} = 0$

4. $y = (3x+2)^4\sqrt{2x-5}$

$\dfrac{dy}{dx} = (3x+2)^4\cdot\dfrac{1}{\cancel{2}}(2x-5)^{-1/2}\cdot\cancel{2} + (2x-5)^{1/2}(4)(3x+2)^3\cdot 3$

$= (3x+2)^3(2x-5)^{-1/2}\left[(3x+2)+12(2x-5)\right]$

$= \dfrac{(3x+2)^3}{\sqrt{2x-5}}\left[27x-58\right]$

5. $s = \dfrac{t^2}{t^3+7}$

$\dfrac{ds}{dt} = \dfrac{(t^3+7)2t - t^2\cdot 3t^2}{(t^3+7)^2}$

$= \dfrac{2t^4+14t-3t^4}{(t^3+7)^2}$

$= \dfrac{14t-t^4}{(t^3+7)^2}$

6. $f(x) = \cos^3(\sin 3x)$

$f'(x) = 3\left[\cos^2(\sin 3x)\right]\left[-\sin(\sin 3x)\right]\cos 3x\cdot 3$

$= -9\cos^2(\sin 3x)(\sin(\sin 3x))\cos 3x$

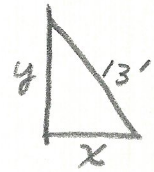
7. Let $f(x)=\cos x$, then $f'(x) = \lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h}$

$f'(x) = \lim\limits_{h\to 0}\dfrac{\cos x\cos h - \sin x\sin h - \cos x}{h}$

$= \lim\limits_{h\to 0}\dfrac{\cos x(\cos h - 1)}{h} - \lim\limits_{h\to 0}\dfrac{\sin x\sin h}{h}$

$= -\cos x\lim\limits_{h\to 0}\left(\dfrac{1-\cos h}{h}\right) - \sin x\lim\limits_{h\to 0}\dfrac{\sin h}{h}$

$= -\cos x\cdot 0 - \sin x\cdot 1 = -\sin x$

8. 

$\dfrac{dy}{dt} = -6$ ft/sec

$\dfrac{dx}{dt} = ??$

$x^2+y^2 = 13$

$\cancel{2}x\dfrac{dx}{dt} + \cancel{2}y\dfrac{dy}{dt} = 0$

when $y=5,\ x=12$.

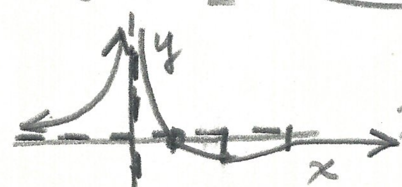
$12\dfrac{dx}{dt} - 30 = 0$

$\dfrac{dx}{dt} = \dfrac{30}{12} = \dfrac{5}{2}$ ft/sec

9. $y = \dfrac{1-x}{x^2}$

$y' = \dfrac{x-2}{x^3}$ $\quad x=0,\ x=2$

$y'' = \dfrac{2(3-x)}{x^4}$ $\quad x=3,\ x=0$



y' + + + ∞ − − 0 + + +
0 2

y'' + + + ∞ + + + 0 − − −

$\lim\limits_{x\to\pm\infty} = 0$

x	0	2	3
y	—	$-\frac{1}{4}$	$-\frac{2}{9}$

Asymptotes: $x=0,\ y=0$.

Rel min at $x=2$

Point of inflection at $x=3$.

Concave up: $(-\infty,0)\cup(0,3)$; Down $(3,\infty)$

10. $V = 1000 = \pi r^2 h$

Surface area

$S = 2\pi r^2 + 2\pi r h \qquad h = \frac{1000}{\pi r^2}$

$S = 2\pi r^2 + 2\cancel{\pi}\cancel{r} \cdot \frac{1000}{\cancel{\pi} r^{\cancel{2}}}$

$= 2\pi r^2 + 2000 r^{-1}$

$\frac{dS}{dr} = 4\pi r - 2000 r^{-2} = 0$

$4r^{-2}(\pi r^3 - 500) = 0$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}}$

$h = \frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}} = \frac{2 \cdot 500}{\pi \frac{500^{2/3}}{\pi^{2/3}}}$

$= 2 \frac{500^{1/3}}{\pi^{1/3}} = 2\sqrt[3]{\frac{500}{\pi}}$

$= 2r$

11. $\int \frac{3x\,dx}{\sqrt{x^2+5}}$ let $u = x^2+5$, $du = 2x\,dx$

$= \int \frac{3\frac{du}{2}}{u^{1/2}} = \frac{3}{2}\int u^{-1/2}\,du$

$= \frac{3}{2}\cdot\frac{2}{1}u^{1/2} = 3\sqrt{x^2+5} + C$

12. $\int_{\pi/8}^{\pi/4} \sin^3 2x \cos 2x\,dx$ let $u = \sin 2x$, $du = 2\cos 2x\,dx$

$x = \frac{\pi}{8} \rightarrow u = \frac{1}{\sqrt{2}}$

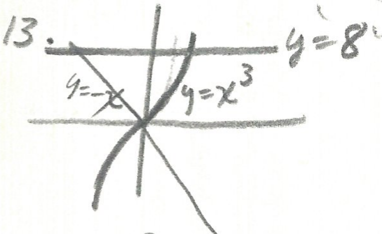
$x = \frac{\pi}{4} \rightarrow u = 1$

$\int_{\frac{1}{\sqrt{2}}}^{1} u^3 \frac{du}{2} = \left.\frac{u^4}{8}\right|_{2^{-1/2}}^{1}$

$= \frac{1}{8}\left[1 - \left(2^{-1/2}\right)^4\right] = \frac{1}{8}\left[1 - 2^{-2}\right]$

$= \frac{1}{8}\left(1 - \frac{1}{4}\right) = \frac{3}{32}$

13.



$A = \int_{-8}^{0} 8 - (-x)\,dx + \int_{0}^{2} (8 - x^3)\,dx$

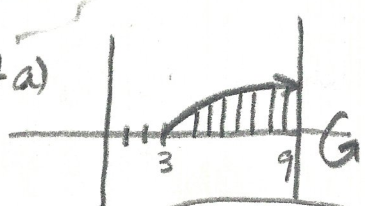
or $A = \int_{0}^{8} \sqrt[3]{y} - (-y)\,dy$

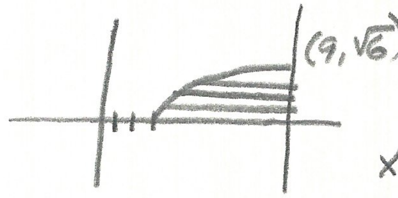
$= \left.\frac{3}{4}y^{4/3} + \frac{y^2}{2}\right|_0^8$

$= \frac{3}{4}\cdot 16 + \frac{64}{2}$

$= 12 + 32 = 44$

14a)





$y = \sqrt{x-3}$

$y^2 = x - 3$

$x = y^2 + 3$

Disk: $\pi\int_3^9 \left(\sqrt{x-3}\right)^2 dx$

$= \pi\int_3^9 (x-3)\,dx$

$= \pi\left[\frac{x^2}{2} - 3x\right]_3^9$

$= \pi\left[\frac{81}{2} - 27 - \frac{9}{2} + 9\right]$

$= \pi\left[\frac{72}{2} - 18\right] = 18\pi$

Shell $= 2\pi\int_0^{\sqrt{6}} \left[9 - (y^2+3)\right] y\,dy$

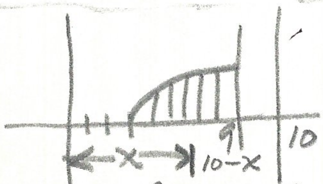
14b) Shell

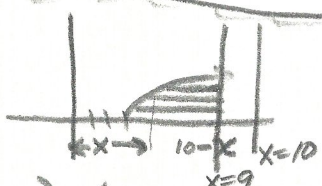
$2\pi\int_3^9 x\sqrt{x-3}\,dx$

Disk $= \pi\int_0^{\sqrt{6}} \left[9^2 - (y^2+3)^2\right] dy$

Do NOT SOLVE 14b)

14c)

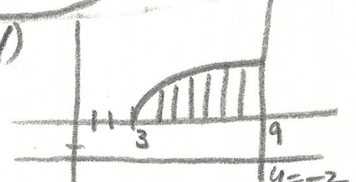


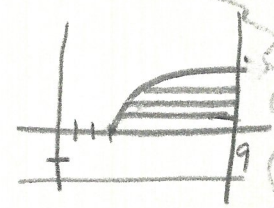


Shell $2\pi\int_3^9 \sqrt{x-3}\,(10-x)\,dx$

Disk $\pi\int_0^{\sqrt{6}} \left[10 - (y^2+3)\right]^2 - 1^2\,dy$

14d)





Disk: $\pi\int_3^9 \left(2 + \sqrt{x-3}\right)^2 dx$

Shell: $2\pi\int_0^{\sqrt{6}} \left[9 - (y^2+3)\right](y+2)\,dy$

16. $a = -9.8$

$V = -9.8t + V_0$

$= -9.8t + 49$

$s = -4.9t^2 + 49t + s_0$

$= -4.9t^2 + 49t + 150$

a) $V = 0 = -9.8t + 49$ (max height)

$t = \frac{49}{9.8} = 5$ sec.

b) $s = -4.9(5)^2 + 49(5) + 150$

$= 272.5$ m.

c) $s = -4.9t^2 + 49t + 150 = 0$

Quadratic formula

$t \approx 12.46$ sec

d) $V(12.46) = -9.8(12.46) + 49$

≈ -73.1 m/sec

17. $y = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}$

$y' = x^2 - \frac{1}{4}x^{-2} = \frac{4x^4 - 1}{4x^2}$

$(y')^2 = \frac{16x^8 - 8x^4 + 1}{16x^4}$

$1 + (y')^2 = \frac{16x^8 - 8x^4 + 1}{16x^4} + \frac{16x^4}{16x^4}$

$= \frac{16x^8 + 8x^4 + 1}{16x^4} = \left(\frac{4x^4 + 1}{4x^2}\right)^2$

$S = \int_1^2 \sqrt{1 + (y')^2}\, dx = \int_1^2 \frac{4x^4 + 1}{4x^2}\, dx$

$= \int_1^2 \left(x^2 + \frac{1}{4}x^{-2}\right) dx$

$= \frac{x^3}{3} - \frac{x^{-1}}{4}\Big|_1^2$

$= \left(\frac{8}{3} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$

$= \frac{61}{24} - \frac{2}{24} = \frac{59}{24}$