

Show all work on separate paper. Turn in ALL worksheets.

1. Graph $y = \sqrt{x}$. From this graph, sketch a) $y = \sqrt{x} + 2$; b) $y = -\sqrt{x}$;
c) $y = \sqrt{-x}$; d) $y = \sqrt{x+4}$.
2. Test for symmetry (x axis, y axis, and origin) and give the x and y intercepts:
 $y = x^3 - 4x$.
3. Find the equation of the line joining the points of intersection of
 $y = x^2 - 4x + 3$
 $y = -x^2 + 2x + 3$
4. Find the domain and range for $f(x) = \frac{16}{x^2 - 4x}$.
[Hint: Use a graphing calculator to find the range!]
5. Find the domain and range for $f(x) = \sqrt{x^2 - 3x - 4}$.
6. If $f(x) = \sqrt{x}$ and $g(x) = x^3 + 3x - 6$, find $f(g(x))$ and $g(f(x))$.
7. Graph: $f(x) = \begin{cases} 8 - 2x & \text{if } x > 2 \\ x + 2 & \text{if } x \leq 2 \end{cases}$
8. Given: $f(x) = \begin{cases} 8 - 2x & \text{if } x > 2 \\ x + 2 & \text{if } x \leq 2 \end{cases}$
a) $\lim_{x \rightarrow 2^-} f(x)$ b) $\lim_{x \rightarrow 2^+} f(x)$ c) $\lim_{x \rightarrow 2} f(x)$
d) Is this graph continuous? Explain your answer.
9. Given: $f(x) = \begin{cases} 2x - 8 & \text{if } x > 2 \\ x - 2 & \text{if } x \leq 2 \end{cases}$
a) $\lim_{x \rightarrow 2^-} f(x)$ b) $\lim_{x \rightarrow 2^+} f(x)$ c) $\lim_{x \rightarrow 2} f(x)$
d) Is this graph continuous? Explain your answer.
10. Find $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 3x}$.
11. Find $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^3 + 8}$.

12. Find $\lim_{h \rightarrow 0} \frac{x^2 h - x h^2 + h^3}{h}$.

13. Find $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x}$.

14. Find $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{\theta}$.

15. Given: $f(x) = \frac{|x|}{x}$

a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$ d) Sketch the graph.

16. For $f(x) = 3x^2 - 5x + 2$, find a) $\frac{f(x + \Delta x) - f(x)}{\Delta x}$; b) $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$.

17. For what value(s) of c will the function

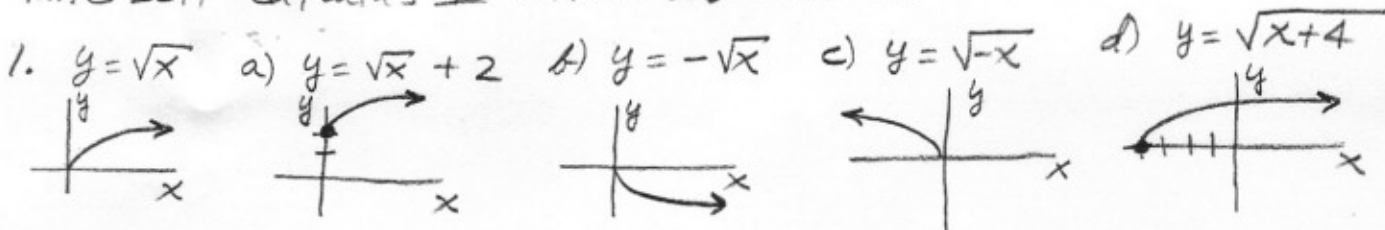
$$f(x) = \begin{cases} cx - 1 & \text{if } x < 4 \\ cx^2 & \text{if } x \geq 4 \end{cases} \text{ be continuous.}$$

18. Give all values for which the function is discontinuous. Distinguish whether points of discontinuity are removable or non-removable discontinuities.

[Note: The graph will be posted soon!]

19. Use the function $f(x) = \frac{x+2}{x^2 - 2x - 8}$ to explain the difference between removable and non-removable discontinuities. Find all vertical asymptotes. Sketch the graph.20. If $f(x) = \frac{1}{2}x + 4$, $a=2$, and $\epsilon = .02$, find L , and find δ such that $|f(x) - L| < \epsilon$ for every x where $0 < |x - a| < \delta$.

MAC 2311 Calculus I EXAM 1 Solutions



2. $y = x^3 - 4x$
 x axis (use -y)
 $-y = x^3 - 4x$ No!
 y axis (use -x)
 $y = (-x)^3 - 4(-x)$
 $y = -x^3 + 4x$ No!
 Origin (use -x, -y)
 $-y = (-x)^3 - 4(-x)$
 $-y = -x^3 + 4x$
 $y = x^3 - 4x$ Yes

y int (x=0)
 $(0,0)$
 x int: $y=0$
 $0 = x^3 - 4x$
 $0 = x(x^2 - 4)$
 $0 = x(x-2)(x+2)$
 $x = 0, 2, -2$

3. $y = x^2 - 4x + 3$
 $y = -x^2 + 2x + 3$
 $x^2 - 4x + 3 = -x^2 + 2x + 3$
 $2x^2 - 6x = 0$
 $2x(x-3) = 0$
 $x = 0, x = 3$
 $y = 3, y = 9 - 12 + 3$
 $y = 0$

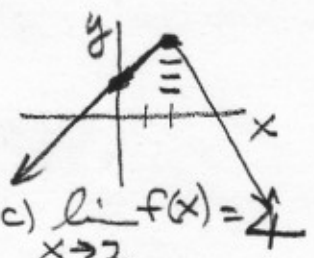
Intersection =
 $(0,3) (3,0)$
 $m = \frac{3-0}{0-3} = -1$
 y int = $3 = b$
 $y = mx + b$
 $y = -x + 3$

(Also, use calculator to find points of intersection!)

4. $y = \frac{16}{x^2 - 4x}$
 $D: x^2 - 4x \neq 0$
 $D: x \neq \text{all real } x \neq 0, 4$
 $R: (-\infty, 0) \cup (0, 4) \cup (4, \infty)$
 $R: (-\infty, -4] \cup (0, \infty)$

5. $f(x) = \sqrt{x^2 - 3x - 4}$
 $(x-4)(x+1)$
 $D: (-\infty, -1] \cup [4, \infty)$
 $R: [0, \infty)$

7. $f(x) = \begin{cases} 8-2x & \text{if } x > 2 \\ x+2 & \text{if } x \leq 2 \end{cases}$
 8a) $\lim_{x \rightarrow 2^-} = 4$ b) $\lim_{x \rightarrow 2^+} = 4$
 and $f(2) = 4$.
 d) Continuous



10. $\lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x(x-3)}$
 $= \frac{6}{3} = 2$

yes! Because
 1. $f(2)$ exists
 2. $\lim_{x \rightarrow 2} f(x)$ exists
 3. $\lim_{x \rightarrow 2} f(x) = f(2)$

9. $f(x) = \begin{cases} 2x-8 & \text{if } x > 2 \\ x-2 & \text{if } x \leq 2 \end{cases}$
 a) $\lim_{x \rightarrow 2^-} = 0$ b) $\lim_{x \rightarrow 2^+} = -4$
 c) $\lim_{x \rightarrow 2} \text{DNE}$ d) Disco!
 $\lim_{x \rightarrow 2} \text{DNE!}$

11. $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 + 8}$
 $\lim_{x \rightarrow -2} \frac{(x-2)(x+2)}{(x+2)(x^2 - 2x + 4)} = \frac{-4}{4+4+4}$
 $= \frac{-4}{12} = -\frac{1}{3}$

12. $\lim_{h \rightarrow 0} \frac{x^2 h - x h^2 + h^3}{h}$
 $= \lim_{h \rightarrow 0} \frac{h(x^2 - xh + h^2)}{h}$
 x^2

13. $\lim_{x \rightarrow 0} \frac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{x(\sqrt{4+x}+2)}$
 $\lim_{x \rightarrow 0} \frac{4+x-4}{x(\sqrt{4+x}+2)}$
 $\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{4+x}+2)}$
 $\lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{4}$

14. $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{\theta}$
 $\lim_{\theta \rightarrow 0} \frac{3(\sin \theta)}{3\theta} = 1$
 $= 3$