CALCULUS I EXAM 4 A R^2 NAME _____
SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers. Where calculators are used, describe window, procedures, etc.

1. **Evaluate the integral:** $\int \frac{x^3 + 3}{x^2}\, dx$

2. **Find y = f(x) if f"(x) = x + 2, f'(0) = 3, f(0) = -1**

3. **Use the formulas to find** $\lim_{n \to \infty} \sum_{i=1}^{n} (1 + \frac{2i}{n})^2 (\frac{2}{n})$

4. **Use a calculator method to find the area under the graph $y=1-x^2$ over [-1, 1]. Explain the method and the steps used to find area.**

5a) **Use a geometric formula to find the value of** $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$. Draw a figure, give exact value.

b) **Use the "calc" function of the calculator to find the approximate area. (Use this as a check!)**

6. **Given** $\int_0^{10} f(X)\, dx = 62$, $\int_4^{10} f(X)\, dx = 33$, and $\int_0^{10} g(X)\, dx = 74$.

Find: a) $\int_0^4 f(X)\, dx$ b) $\int_{10}^{0} f(X)\, dx$ c) $\int_0^{10} [2f(X) - g(X)]\, dx$.

7. **Evaluate the definite integral (by algebraic process):** $\int_4^9 \frac{X - 2}{\sqrt{X}}\, dx$. Check by calculator methods. Explain how.

8. **Given** $F(X) = \int_3^x \sqrt{t^2 + 4}\, dt$, find F'(X).

9. **Find the average value of f(X) = sin X on [0,π].**
[Hint: Use $f(c) = \frac{1}{b - a} \int_a^b F(X)\, dx$ unless you know a better way.]

10. **Draw a sketch, use the calculator and find the area under the**

curve for $\int_0^4 X\sqrt{X^2+4}\,dx$ using:

a) Left rectangles with n = 4

b) Right rectangles with n = 4

c) Trapezoidal Rule with n = 4

d) Simpson's Rule with n/2 = 4

e) Simpson's Rule with n = 4

f) Trapezoidal Rule with n = 20.

g) "Calc", "fnint" function of the calculator.

11. Find the exact area (in radical form) of $\int_0^4 X\sqrt{X^2+4}\,dx$ by "algebraic integration."

In 12 - 14, evaluate the integrals.

12. $\int X^2\sqrt{4 - X^3}\,dx$

13. $\int \sin^3 X \cos X\,dx$

14. $\int \frac{(1 + \sqrt{X})^3}{\sqrt{X}}\,dx$

CALCULUS I EXAM 4A Solutions

1. $\int \frac{x^3+3}{x^2}\,dx = \int (x+3x^{-2})\,dx$

$= \frac{x^2}{2} + \frac{3x^{-1}}{-1} + C$

$= \boxed{\frac{x^2}{2} - \frac{3}{x} + C}$

3. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right)\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)$

$= \lim_{n\to\infty} \left[\frac{2}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$

$= \lim_{n\to\infty} \left[\frac{2}{n}\cdot n + \frac{8}{n^2}\frac{n(n+1)}{2} + \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$

$= \lim_{n\to\infty} \left[2 + 4\left(\frac{n+1}{n}\right) + \frac{8}{6}\frac{(n+1)(2n+1)}{n^2}\right]$ $\quad 2n^2+3n+1$

$= 2 + 4 + \frac{8}{6}\cdot 2$

$= 6 + \frac{8}{3} = \boxed{\frac{26}{3}}$

6a) $\int_0^4 f(x)\,dx = \int_0^{10} f(x)\,dx - \int_4^{10} f(x)\,dx$

$= 6 - 3 = \boxed{3}$

b) $\int_{10}^{0} f(x)\,dx = -\int_0^{10} f(x)\,dx = \boxed{-6}$

c) $\int_0^{10} [2f(x) - g(x)]\,dx = 2\int_0^{10} f(x)\,dx - \int_0^{10} g(x)\,dx$

$= 2\cdot 6 - 7 = \boxed{5}$

7. $\int_4^9 \frac{x-2}{\sqrt{x}}\,dx = \int_4^9 (\sqrt{x} - 2x^{-1/2})\,dx$

$= \frac{2}{3}x^{3/2} - 2\cdot\frac{2}{1}x^{1/2}\Big|_4^9$

$= 2x^{1/2}\left[\frac{1}{3}x - 2\right]_4^9$

$= 2\cdot 3\left[3-2\right] - 2\cdot 2\left[\frac{4}{3} - 2\right]$

$= 6 - 4\cdot(-\frac{2}{3})$

$= 6 + \frac{8}{3} = \frac{26}{3} = \boxed{8\frac{2}{3}}$

CALCULATOR = 8.666666...

2. $f''(x) = x+2$

$f'(x) = \frac{x^2}{2} + 2x + C_1, \quad f'(0) = 3$

$3 = 0 + 0 + C_1 \quad C_1 = 3$

$f'(x) = \frac{x^2}{2} + 2x + 3$

$f(x) = \frac{x^3}{6} + x^2 + 3x + C_2 \quad f(0) = -1$

$-1 = 0 + 0 + 0 + C_2$

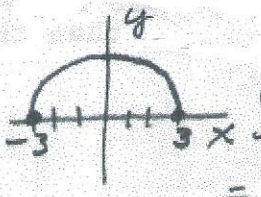
$\boxed{f(x) = \frac{x^3}{6} + x^2 + 3x - 1}$

4. CALC | fnint | $1-x^2$ | [5] | [X] | [5] | -1 | [5] | 1 | [D]

.3333...

$= \boxed{4/3}$

5a)



$\int_{-3}^{3} \sqrt{9-x^2}\,dx$

= Semicircle.

$A = \frac{1}{2}\pi r^2 = \boxed{\frac{9\pi}{2}}$

b) CALC | fnint | $\sqrt{(9-x^2)}$ | [5] | [X] | [5] | -3 | [5] | 3 | [D]

$= \boxed{14.137}1672535$

$\frac{9\pi}{2} = \boxed{14.137}1669412$

8. $F(x) = \int_3^x \sqrt{t^2+4}\,dt$

$F'(x) = \boxed{\sqrt{x^2+4}}$ TOO EASY!!

9. $f(x) = \sin x \quad [0, \pi]$

Bobbruny = [PROG] [NAMES] [FNAVE]

Set func = sin X a=0, b=π,

Ave = .636619772368

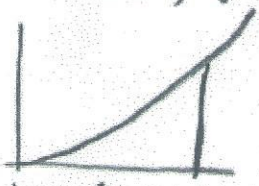
Otherwise: $\frac{1}{\pi}\int_0^{\pi} \sin x\,dx$

$= \frac{1}{\pi}\left[-\cos x\right]_0^{\pi} = \frac{1}{\pi}\left[-(-1)+1\right]$

$= \frac{2}{\pi} = .636619772368$

EX 4A

10. PROG, NAMES, More ↓ Aarea, X=-1 to 5, y=0 to 30.



Interval: 0, ENTER, 4

Area = 27.1475730333

F2 (Rect), F3(L/R), Sub=4

a) 18.7096

b) 36.59812, F5 (EXIT)

c) F3 TRAP = 27.6538

d) F4 SIMP = 27.1470

e) N=4, N/2 = 2; SIMP = 27.1377

f) TRAP, N=20 ⇒ 27.1677.

g) Same as "Area" 27.1475730333

27.1476

11. $\int_0^4 x\sqrt{x^2+4}\,dx$

Let $u = x^2+4$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

If $x=0$, $u=4$

If $x=4$, $u=20$

$\int_4^{20} u^{1/2}\frac{du}{2}$

$= \frac{1}{2}\frac{2}{3}u^{3/2}\Big|_4^{20}$

$= \frac{1}{3}u^{3/2}\Big|_4^{20}$

$= \frac{1}{3}\left(20^{3/2} - 4^{3/2}\right)$

$= \frac{1}{3}\left(20\sqrt{20} - 4\sqrt{4}\right)$

$= \frac{1}{3}\left(40\sqrt{5} - 8\right)$

$= 27.1475730333$

$= 27.1476$

12. $\int x^2\sqrt{4-x^3}\,dx$

Let $u = 4-x^3$

$du = -3x^2\,dx$

$\frac{du}{-3} = x^2\,dx$

$= \int u^{1/2}\frac{du}{-3}$

$= -\frac{1}{3}\frac{2}{3}u^{3/2}$

$= -\frac{2}{9}\left(4-x^3\right)^{3/2} + C$

13. $\int \sin^3 x\cos x\,dx$ Let $u = \sin x$

$du = \cos x\,dx$

$= \int u^3\,du$

$= \frac{u^4}{4} + C = \frac{\sin^4 x}{4} + C$

14. $\int \frac{(1+\sqrt{x})^3\,dx}{\sqrt{x}}$ Let $u = 1+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2}dx$

$2du = \frac{dx}{\sqrt{x}}$

$= \int u^3(2du)$

$= 2\left(\frac{u^4}{4}\right) + C$

$= \frac{1}{2}\left(1+\sqrt{x}\right)^4 + C$