MAC 2311 EXAM 3B **R^2** **NAME ____________________________**

Show all work on separate paper. Turn in ALL worksheets.
When you use the calculator, say so, and explain what you did.

1. Find the maximum and minimum values of the function (if they exist):
   a) $f(x) = 3x^4 - 4x^3$ on [-1, 2].
   b) $f(x) = x^2 - 2x$ on (0, 2).
   c) $f(x) = \dfrac{x}{x-2}$ on [1, 4).

2. Find the values of c for which the Mean Value Theorem applies, or explain why the theorem does not apply.
   a) $f(x) = 3x^4 - 4x^3$ on [-1, 1].
   b) $f(x) = \dfrac{1}{x-3}$ on [0, 6].

3. Evaluate the limits:
   a) $\lim\limits_{x \to \infty} \dfrac{x}{|x|}$ b) $\lim\limits_{x \to -\infty} \dfrac{x}{|x|}$ c) $\lim\limits_{x \to \infty} \dfrac{c}{x}$
   d) $\lim\limits_{x \to \infty} \dfrac{-x^3 + x^2 + 4}{3x^3 + 4x - 1}$ e) $\lim\limits_{x \to \infty} \dfrac{-x^3 + x^2 + 4}{3x^2 + 4x - 1}$ f) $\lim\limits_{x \to \infty} \dfrac{-x^3 + x^2 + 4}{3x^4 + 4x - 1}$

4. Given $f'(x) = \dfrac{x^2 + 2x - 3}{(x+1)^2}$, use the first derivative test to find all critical values, identify x coordinates of relative maximum and minimum points, and give intervals in which the graph is increasing and decreasing.

5. Given $f(x) = \dfrac{x}{x^2+1}$, $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$, $f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}$, find the x-coordinates of all points of inflection. Give intervals in which the graph is concave up and concave down. What conditions are necessary to have a point of inflection?

In 6-7, sketch the graphs, find x and y intercepts, horizontal and vertical tangents, vertical asymptotes, and $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Describe your window.

6. Sketch the graph of $f(x) = \dfrac{x^2 - 2x + 4}{x - 2}$

7. Sketch the graph of $f(x) = \dfrac{x}{x^2 + 1}$.

8. Given $f(x) = -x^3 + 5$, set up a formula to find the root by Newton's Method. Beginning with x=1, draw a sketch (or an outline of steps) to illustrate how Newton's Method works, and give a list of x values leading to the root. If you used a calculator method or program, describe what you did.

9. Given $f(x) = x^2 - 4x + 6$, find Δf and df when $x = 5$ and $\Delta x = dx = 0.1$.

10. A farmer plans to fence a rectangular garden adjacent to a river. The garden must contain 1250 square feet. What dimensions would require the least amount of fencing if no fence is required along the river. How many feet of fence are needed?

11. Equal squares are to be cut from each corner of a piece of tin measuring 6 meters by 6 meters, and the edges are then turned up to form a box with no lid. What are the dimensions of the box with maximum volume, and what is the volume of the box?

CALCULUS I EXAM 3B Solutions

1a) $f(x) = 3x^4 - 4x^3$ on $[-1,2]$

$f'(x) = 12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$x=0 \quad x=1$

$f(0) = 0$

$f(1) = -1$ MIN

$f(-1) = 7$

$f(2) = 16$ MAX

b) $f(x) = x^2 - 2x$ on $(0,2)$

$f'(x) = 2x - 2 = 0$

$x = 1$

$f(1) = -1$ MIN

$f(0) = 0$ Not incl.

$f(2) = 0$ Not incl.

No MAX

c) $f(x) = \dfrac{x}{x-2}$ $[1,4)$

Asymptote at $x=2$ within the interval.

$\lim_{x \to 2^-} = -\infty$

$\lim_{x \to 2^+} = +\infty$

No MAX, No MIN

2a) $f(x) = 3x^4 - 4x^3$ on $[-1,1]$

$f'(x) = 12x^3 - 12x^2$

$f'(c) = 12c^3 - 12c^2$

$f(b) = f(1) = -1$

$f(a) = f(-1) = 7$

$f'(c) = \dfrac{f(b) - f(a)}{b-a}$

$\dfrac{f(b)-f(a)}{b-a} = \dfrac{-1-7}{1-(-1)} = -\dfrac{8}{2} = -4$

$12c^3 - 12c^2 = -4$

$12c^3 - 12c^2 + 4 = 0$

$3c^3 - 3c^2 + 1 = 0$

ROOT: $c \approx -.475$ within $(-1,1)$

b) Thm of the Mean does not apply since $f(x)$ is not continuous - Asymptote at $x=3$ within $[0,6]$

3a) $\lim_{x\to\infty} \dfrac{x}{|x|} = \lim_{x\to\infty} \dfrac{x}{x} = 1$

b) $\lim_{x\to-\infty} \dfrac{x}{|x|} = \lim_{x\to-\infty} \dfrac{x}{-x} = -1$

c) $\lim_{x\to\infty} \dfrac{c}{x} = 0$

d) $\lim_{x\to\infty} \dfrac{(-x^3+x^2+4)\cdot\frac{1}{x^3}}{(3x^3+4x-1)\frac{1}{x^3}}$

$= \lim_{x\to\infty} \dfrac{-1 + \cancel{1/x}^{\to 0} + \cancel{4/x^3}^{\to 0}}{3 + \cancel{4/x^2}^{\to 0} - \cancel{1/x^3}^{\to 0}} = -\dfrac{1}{3}$

e) $\lim_{x\to\infty} \dfrac{(-x^3+x^2+4)\ 1/x^2}{(3x^2+4x-1)\ 1/x^2}$

$= \lim_{x\to\infty} \dfrac{-x + 1 + \cancel{4/x^2}^{\to 0}}{3 + \cancel{4/x}^{\to 0} - \cancel{1/x^2}^{\to 0}} = -\infty$

f) $\lim_{x\to\infty} \dfrac{(-x^3+x^2+4)\frac{1}{x^3}}{(3x^4+4x-1)\frac{1}{x^3}}$

$= \lim_{x\to\infty} \dfrac{-1 + \cancel{1/x}^{\to 0} + \cancel{4/x^3}^{\to 0}}{-3x + \cancel{4/x^2}^{\to 0} - \cancel{1/x^3}^{\to 0}}$

$= \lim_{x\to\infty} \dfrac{-1}{-3x} \to 0$

4. $f'(x) = \dfrac{x^2+2x-3}{(x+1)^2} = \dfrac{(x+3)(x-1)}{(x+1)^2}$

Critical values: $f'(x) = 0$ or ∞ where $f(x)$ is defined.

Critical values $x=-3,\ x=1$

Note: $f(x)$ is probably not defined at $x=-1$.

x:	-3		-1		1	
f':	$+\ 0$	$-$	∞	$-$	0	$+$

MAX ↓ at $x=-3$, MIN ↓ at $x=1$

Rel Max at $x=-3$ Rel Min at $x=1$

Increasing: $(-\infty,-3) \cup (1,\infty)$

Decreasing: $(-3,-1) \cup (-1,1)$

5. $f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3} = 0$

$x=0,\quad x=\pm\sqrt{3}$

x	$-\sqrt{3}$	0	$\sqrt{3}$
f	$-\frac{\sqrt{3}}{10}$	0	$\frac{\sqrt{3}}{10}$
f''	$-\ 0\ +$	$0\ -$	$0\ +$

Pts of Inflection: $(-\sqrt{3}, -\frac{\sqrt{3}}{10})$ $(0,0)$ and $(\sqrt{3}, \sqrt{3}/10)$

$x=0 \quad x=\pm\sqrt{3}$

x	$-\sqrt{3}$	0	$\sqrt{3}$
f	$-\frac{\sqrt{3}}{10}$	0	$\frac{\sqrt{3}}{10}$

f'' $\quad - \; 0 \; + \; 0 \; - \; 0 \; +$

Pts of Inflection: $(-\sqrt{3}, -\frac{\sqrt{3}}{10})$ $(0,0)$ $(\sqrt{3}, \frac{\sqrt{3}}{10})$

Concave Up: $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$

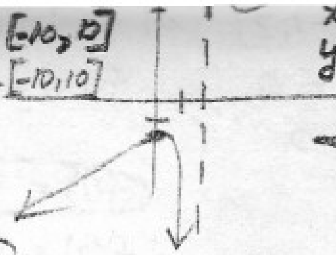
Concave Down: $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$

Pt of Inflec: ① $f(x)$ must exist

② $f''(x)=0$ or Undefined.

③ Concavity changes

$x=[-10,10]$
$y=[-10,10]$



No x intercepts.

Y int: $(0,-2)$

Horiz tangents:

$(2,0)$ $(4,6)$

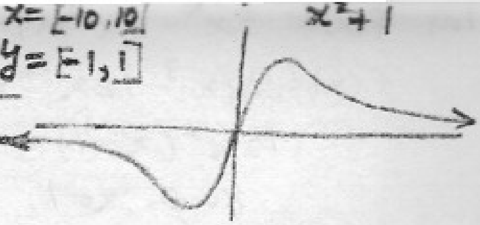
(use calculator or derivative $=0$)

Vertical Asymp: $x=2$

$\lim\limits_{x\to\infty} = \infty$ $\quad \lim\limits_{x\to-\infty} = -\infty$

$x=[-10,10]$
$y=[-1,1]$

x^2+1



x int, y int $=(0,0)$

Horiz tangents:

$(1,\frac{1}{2})$ $(-1,-\frac{1}{2})$

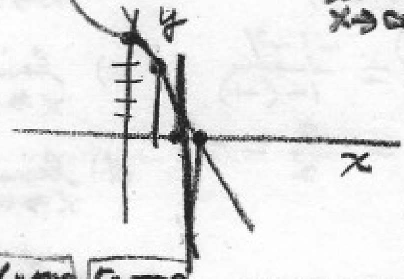
No vertical asymp. or vertical tangent.

$\lim\limits_{x\to\infty}=0 \quad \lim\limits_{x\to-\infty}=0$

8. $f(x)=-x^3+5$

$f'(x)=-3x^2$

$x_{n+1}=x_n-\frac{f(x)}{f'(x)}$



Begin at $x_1=1$; go to $f(1)=4$ on the graph; Draw tangent line; follow tangent line to x axis. $x_2=$ x intercept of tangent line. Go to $f(x_2)$. Draw tangent line; tangent line to x axis. $x_3=$ x intercept of tangent line. $x_n \to$ root of $f(x)$,

Calculator: $\boxed{1}$ $\boxed{\text{STO}}$ $\boxed{x\text{var}}$ $\boxed{\text{ENTER}}$

$\boxed{x\text{var}}$ $\boxed{-}$ $\boxed{(-x^3+5)}$ $\boxed{\div}$ $\boxed{(-3x^2)}$ $\boxed{\text{STO}}$ $\boxed{x\text{var}}$ $\boxed{\text{ENTER}}$ → 2.3333333333

$\boxed{\text{ENTER}}$ 1.8616780454

$\boxed{\text{ENTER}}$ 1.72200188006

$\boxed{\text{ENTER}}$ 1.7100597366

$\boxed{\text{ENTER}}$ 1.70997595078

$\boxed{\text{ENTER}}$ 1.70997594668

NOTE: ROOT of $f(x)=-x^3+5=0$

$-x^3=-5$

$x^3=5$

$x=\sqrt[3]{5}$

9. $f(x)=x^2-4x+6 \quad x=5 \quad \Delta x = dx = 0.1$

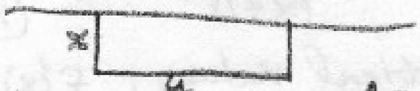
$f'(x)=2x-4 \quad f(5)=11 \quad f(5.1)=11.61$

$df=f'(x)\,dx \quad f'(5)=10-4=6$

$=6(.1)=0.6 \quad \Delta f = f(x+\Delta x)-f(x)$

$=11.61-11=0.61$

10.



$A=xy=1250$

$y=\frac{1250}{x}$

Fence $=2x+y$

$F=2x+1250x^{-1}$

$F'=2-1250x^{-2}=0$

$2-\frac{1250}{x^2}=0$

$2=\frac{1250}{x^2}$

$2x^2=1250$

$x^2=625$

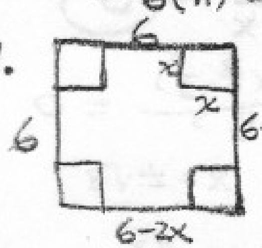
$x=25$ ft $\;\rightarrow\; y=\frac{1250}{25}$

$y=50$ ft

$F=2x+y$ $\quad$ Total Fence

$=2(25)+50=100$ ft

11.



$V=x(6-2x)^2$

$=x(36-24x+4x^2)$

$=36x-24x^2+4x^3$

$V'=36-48x+12x^2=0$

$12(3-4x+x^2)=0$

$(3-x)(x-1)=0$

$x=3 \quad x=1$

$V=0 \quad V=1\cdot 4^2=16\text{ m}^3$