CALCULUS I EXAM 5A NAME ______________

In 1–6, find the derivatives:

1. $y = \ln\left(\dfrac{x^2+4}{x^3}\right)$

2. $y = \ln\left(x+\sqrt{x^2+9}\right)$ Factor completely and simplify.

3. $y = \dfrac{x^3\sqrt{x^2-9}}{\sqrt[3]{5x+3}}$ Use logarithmic differentiation. You need not find common denom, etc.

4. $y = x^3 e^{x^2}$ Factor completely.

5. $y = 4^{\sin x}$

6. $y = x^{\sin x}$

7. Use implicit differentiation to find $\frac{dy}{dx}$:
$\ln(xy) + 5x = 30.$

In 7–11, find the indefinite integrals:

7. $\displaystyle\int \frac{x+3}{x^2+6x+7}\,dx$

8. $\displaystyle\int \frac{x+3}{(x^2+6x+7)^2}\,dx$

9. $\displaystyle\int \frac{\sin x}{1+\cos x}\,dx$

10. $\displaystyle\int \frac{2x}{(2x+3)^{1/2}}\,dx$

11. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$

12a) Find the exact value b) use calculator to approximate:

$\displaystyle\int_0^1 x^2 e^{x^3}\,dx$

over →

13. Solve for x (give exact values!)

a) $12 = e^{3x+2}$ b) $\ln(3x+2) = 12$

14. Find the inverse of f. Graph f and f^{-1}.

$f(x) = \sqrt{4-x^2}$ $x \leq 0$.

15. If \$10,000 is invested for 5 years at 8% annual interest, find the amount if compounded

a) annually b) semiannually c) daily d) continuously.

16. Determine the amount of money P (present value) that must be invested at 8% annual interest for 5 years to produce an amount of \$10,000 if compounded a) annually b) daily c) continuously.

17. The half life of a substance is 5730 years. If there is initially 12 grams, a) how much is left after 1000 years? b) after 10,000 years?

18. The half life of a substance is 5730 years. If after 1000 years 12 grams remains, find the initial amount.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $y = y_0 e^{kt}$

CALCULUS I EXAM 5A Solutions.

1. $y = \ln\left(\frac{x^2+4}{x^3}\right)$

$y = \ln(x^2+4) - \ln x^3$

$y = \ln(x^2+4) - 3\ln x$

$y' = \frac{1}{x^2+4}\cdot 2x - \frac{3}{x}$

$= \frac{2x^2 - 3(x^2+4)}{x(x^2+4)} = \frac{-x^2-12}{x(x^2+4)}$

2. $y = \ln(x + \sqrt{x^2+9})$

$y' = \frac{1}{x+\sqrt{x^2+9}}\cdot\left[1 + \frac{1}{2}(x^2+9)^{-1/2}\, 2x\right]$

$= \frac{1}{x+\sqrt{x^2+9}}\left[1 + \frac{x}{\sqrt{x^2+9}}\right]$

$= \frac{1}{\cancel{x+\sqrt{x^2+9}}}\left[\frac{\cancel{\sqrt{x^2+9}+x}}{\sqrt{x^2+9}}\right] = \frac{1}{\sqrt{x^2+9}}$

3. $y = \frac{x^3\sqrt{x^2-9}}{\sqrt[3]{5x+3}}$

$\ln y = \ln \frac{x^3(x^2-9)^{1/2}}{(5x+3)^{1/3}}$

$\ln y = 3\ln x + \frac{1}{2}\ln(x^2-9) - \frac{1}{3}\ln(5x+3)$

$\frac{1}{y}y' = \frac{3}{x} + \frac{1}{2}\frac{1}{x^2-9}\cdot 2x - \frac{1}{3}\frac{1}{5x+3}\cdot 5$

$y' = y\left[\frac{3}{x} + \frac{x}{x^2-9} - \frac{5}{3(5x+3)}\right]$

$y' = \frac{x^3\sqrt{x^2-9}}{\sqrt[3]{5x+3}}\left(\frac{3}{x} + \frac{x}{x^2-9} - \frac{5}{3(5x+3)}\right)$

4. $y = x^3 e^{x^2}$

$y' = x^3\cdot e^{x^2}(2x) + e^{x^2}(3x^2)$

$= x^2 e^{x^2}(2x^2+3)$

5. $y = 4^{\sin x}$

$\ln y = \ln 4^{\sin x}$

$\ln y = \sin x(\ln 4)$

$\frac{1}{y}y' = \cos x(\ln 4)$

$y' = 4^{\sin x}\cos x \ln 4$

6. $y = x^{\sin x}$

$\ln y = \ln x^{\sin x}$

$\ln y = (\sin x)(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = \sin x\cdot\frac{1}{x} + \ln x\cdot\cos x$

$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \ln x\cos x\right]$

$= x^{\sin x}\left(\frac{\sin x}{x} + \ln x\cos x\right)$

7. $\ln(xy) + 5x = 30$

$\ln x + \ln y + 5x = 30$

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + 5 = 0$

$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x} - 5 = \frac{-1-5x}{x}$

$\frac{dy}{dx} = \frac{-y(5x+1)}{x}$

7. (Sorry!) $\int \frac{x+3}{x^2+6x+7}\,dx$ let $u = x^2+6x+7$

$du = (2x+6)\,dx$

$\frac{du}{2} = (x+3)\,dx$

$= \frac{1}{2}\int\frac{du}{u}$

$= \frac{1}{2}\ln u + C$

$= \frac{1}{2}\ln|x^2+6x+7| + C$

or $\ln\sqrt{x^2+6x+7} + C$

8. $\int \frac{(x+3)}{(x^2+6x+7)^2}\,dx$

$= \frac{1}{2}\int\frac{du}{u^2} = \frac{1}{2}\int u^{-2}\,du$

$= \frac{1}{2}\frac{u^{-1}}{-1} + C$

$= -\frac{1}{2}\frac{1}{u} + C$

$= -\frac{1}{2(x^2+6x+7)} + C$

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9. $\int \frac{\sin x}{1+\cos x}\,dx$ let $u = 1+\cos x$, $du = -\sin x\,dx$, $-du = \sin x\,dx$

$= \int \frac{-du}{u}$

11.) $= -\ln u + c = -\ln|1+\cos x| + c$

12a) $\int_0^1 x^2 e^{x^3}\,dx$ let $u = x^3$, $du = 3x^2\,dx$, $\frac{du}{3} = x^2\,dx$

$\int e^u \frac{du}{3}$

$= \frac{1}{3}e^u = \frac{1}{3}e^{x^3}\Big|_0^1$

$= \frac{1}{3}(e-1)$

b) 2nd, calc, fnInt F5

$x^2e^{x^3}, x, 0, 1)$

$= .57276$

10. $\int \frac{2x}{(2x+3)^{1/2}}\,dx$ let $u = 2x+3$, $du = 2\,dx$, $\frac{du}{2} = dx$, $2x = u-3$

$= \int \frac{u-3}{u^{1/2}}\frac{du}{2}$

$= \frac{1}{2}\int (u^{1/2} - 3u^{-1/2})\,du$

$= \frac{1}{2}\left[\frac{2}{3}u^{3/2} - 3\cdot\frac{2u^{1/2}}{1}\right] + c$

$= \frac{1}{3}(2x+3)^{3/2} - 3(2x+3)^{1/2} + c$

or $\frac{1}{3}(2x+3)^{1/2}\left[(2x+3) - 9\right] + c$

$= \frac{1}{3}(2x+3)^{1/2}(2x-6) + c$

$= \frac{2}{3}(2x+3)^{1/2}(x-3) + c$

11. $\int \frac{e^{1/x}}{x^2}\,dx$ let $u = x^{-1}$, $du = -x^{-2}\,dx$

$= \int e^u(-du)$ $-du = \frac{dx}{x^2}$

$= -e^u + c = -e^{1/x} + c$

13a) $12 = e^{3x+2}$

$\ln 12 = \ln e^{3x+2}$

$\ln 12 = 3x+2$

$3x = \ln 12 - 2$

$x = \frac{1}{3}(\ln 12 - 2)$

b) $\ln(3x+2) = 12$

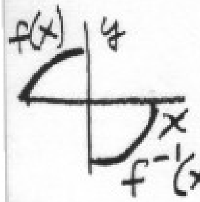
$e^{\ln(3x+2)} = e^{12}$

$3x+2 = e^{12}$

$3x = e^{12} - 2$

$x = \frac{1}{3}(e^{12} - 2)$

14. $f(x) = \sqrt{4-x^2}$ $x \le 0$, $y \ge 0$



Solve for x: $y = \sqrt{4-x^2}$

$y^2 = 4-x^2$

$x^2 = 4-y^2$

$x = \pm\sqrt{4-y^2}$

$D_f: [-2, 0]$

$R_f: [0, 2]$

Interchange: $y = \pm\sqrt{4-x^2}$

$D_{f^{-1}}: [0, 2]$

$f^{-1}(x) = \sqrt{4-x^2}$ $f^{-1}(x) = -\sqrt{4-x^2}$

Choose this

$R_{f^{-1}}: [-2, 0]$

15. $A = P(1+\frac{r}{n})^{nt}$

a) $A = 10{,}000(1+.08)^5 = \$14693.28$

b) $A = 10{,}000(1+.04)^{10} = \$14802.44$

c) $A = 10{,}000(1+\frac{.08}{365})^{5(365)} = \$14917.59$

d) $A = 10{,}000\,e^{5(.08)} = \$14918.25$

16a) $P = \frac{10{,}000}{(1+.08)^5}$

$= \$6805.83$

b) $P = \frac{10{,}000}{(1+.08/365)^{5(365)}}$

$= \$6703.49$

c) $P = \frac{10{,}000}{e^{5(.08)}} = \$6703.20$

17. $y = y_0 e^{kt}$

$\frac{1}{2}y_0 = y_0 e^{k5730}$

$\ln\frac{1}{2} = \ln e^{5730k}$

$5730k = \ln\frac{1}{2}$

$k = \frac{\ln .5}{5730} \approx -1.2096 \times 10^{-4}$

a) $y = 12e^{1000k} = 10.63g$

b) $y = 12e^{10000k} = 3.58g$

18. $y = y_0 e^{kt}$

$\frac{1}{2}y_0 = y_0 e^{5730k}$

$\ln\frac{1}{2} = \ln e^{5730k}$.

$k = \frac{\ln .5}{5730}$ (see #17.)

$12 = y_0 e^{1000k}$

$y_0 = \frac{12}{e^{1000k}}$

$y_0 = 13.54g.$