CALCULUS I REVIEW FOR FINAL RAPALJE--SCC at HUNT CLUB

SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers. Where calculators are used, describe window, procedures, etc.

In 1-2, solve for X and graph answers on numberline:

1. Find y such that the distance from (5,y) to (5,1) is 8.
2. Sketch the graph of $16X^2 + 16Y^2 + 16X + 40Y - 7 = 0$.
3. Find all points of intersection: $Y = 1 - X^2$ and $Y = X^4 - 2X^2 + 1$.
4. Find an equation in standard form of the perpendicular bisector of the line that passes through the points (3,-5) and (5,7).
5. Find the domain: $Y = \dfrac{X - 6}{\sqrt{X^2 - 2X}}$
6. If $f(X) = X^3$, find $\dfrac{f(X + \Delta X) - f(X)}{\Delta X)}$.
7. If sec X = -5/3 in QIII, give the exact values of the other five trigonometric functions.
8. If $\pi < \theta < 3\pi/2$, and $\sin \theta = -2/3$, find $\sin 2\theta$ and $\cos 2\theta$.
9. Given lim (2X+1) = 3, find γ such that $|f(X)-L|<0.01$ whenever $0<|X-a|<\gamma$.
10. Find $\lim\limits_{X \to 16} \dfrac{4 - \sqrt{X}}{X - 16}$
11. Let $f(X) = 5/(X-9)$ and $g(X)=X^2$.
   a) Find f[g(X)]
   b) Find all values of X for which f[g(X)] is discontinuous.

12a) Identify all asymptotes for $f(X) = \dfrac{X + 2}{X^2 - 4}$.

   b) Identify any other discontinuities.
   c) Use this function to explain the difference between removable and nonremovable discontinuities.

In 13 - 16, find the derivative f'(X). Be sure answers are simplified and factored completely.

13. $f(X) = 5X\sqrt{X^2 + 4}$

14. $f(X) = \dfrac{3X - 5}{\sqrt[3]{X^2 + 1}}$

15. $f(X) = \tan^3 5X$

16. $f(X) = \sqrt{X^2 + 1}\,(2X - 3)^6$

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17. Find $\frac{dy}{dx}$ by implicit differentiation: $Y = \cos(XY)$.

18. Use the calculator to evaluate the derivative of the function

$f(X) = \dfrac{3X^2}{X^2 - 2X + 1}$ a) at $X = 4$ b) at $X = 1$. Explain your work.

19. Find the equation of the tangent line to the function

$f(X) = X\sqrt{X^2 + 3}$ at $X = -1$. Is there an easy way to find the slope of the line?

20 Water runs into a conical tank at the rate of 2 cu ft per min. The tank stands point down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?

21. Find each of the following limits:

a) $\lim_{X\to 0} \dfrac{\sin 2X}{X}$ b) $\lim_{X\to\infty} \dfrac{\sin 2X}{X}$ c) $\lim_{X\to -\infty} \dfrac{2X}{\sqrt{X^2+4X}}$

22. An open box is to be made from a 12" by 12" piece of material by cutting equal squares from each corner and turning up the sides. Find the dimensions of the largest box that can be made in this manner.

23. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius 10 meters.

24. Evaluate the integral: $\displaystyle\int \frac{x^2 + 3}{\sqrt[3]{x}}\,dx$

25. Find $y = f(x)$ if $f''(x) = 2 - 6x$, $f'(2) = 3$, $f(2) = -1$

26a) Use a geometric formula to find the value of $\int_0^2 4x + 3\,dx$. Sketch

b) Find the value of the integral by algebraic methods.

c) Use the "calc" function of the calculator to find the area.

27. Draw a sketch, use the calculator and find the area under the curve for $\int_0^2 (X+3)\sqrt{X^2+6X}\,dx$ using:

a) Left rectangles with $n = 4$
b) Right rectangles with $n = 4$
c) Trapezoidal Rule with $n = 4$
d) Simpson's Rule with $n/2 = 4$
e) Simpson's Rule with $n = 4$
f) Trapezoidal Rule with $n = 20$.
g) "Calc", "fnint" function of the calculator.

28. Find the exact area (in simplest radical form) of

$\int_0^2 (X+3)\sqrt{X^2+6X}\,dx$ by "algebraic integration."

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In 29 - 32, evaluate the integrals.

29. $\int \frac{X}{\sqrt{4 - X^2}}\, dx$

30. $\int 3\cos^2 x \sin x \, dx$

31. $\int \frac{dx}{\sqrt{x}(1 + \sqrt{x})^2}$

32. $\int x\sqrt{2x-1}\, dx$

In 33 - 38, find dy/dx.

33. $y = \ln(x + \sqrt{4 + x^2})$

34. $y = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

35. $y = \frac{x^2\sqrt{x^2+4}}{(x^3-4x-6)\sqrt[3]{x^6+16}}$

36. $y = e^{3x}\ln x$

37. $y = \frac{3^{x^2}}{x^2}$

38. $y = x^{3x^2}$

In 39 - 45, evaluate the integral.

39. $\int \frac{dx}{\sqrt{x}(1 + \sqrt{x})}$

40. $\int \frac{dx}{x \ln x}$

41. $\int \frac{e^{2x}}{1 + e^{2x}}\, dx$

42. $\int e^{\tan x}\sec^2 x\, dx$

43. $\int \frac{\cos x}{\sqrt{1 + \sin x}}\, dx$

44. $\int \frac{\cos x}{1 + \sin x}\, dx$

45. $\int \left(x^3 + \frac{1}{x^3} + 3^x + \frac{3}{x} + \frac{x}{3} + 3\right) dx$

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1. $d = \sqrt{(x_1 - x_2)^2 + (y_1 + y_2)^2}$

$8 = \sqrt{(5-5)^2 + (y-1)^2}$

$8 = \sqrt{(y-1)^2}$

$\pm 8 = y - 1$

$\underline{y = 9} \quad \underline{y = -7}$ ✓

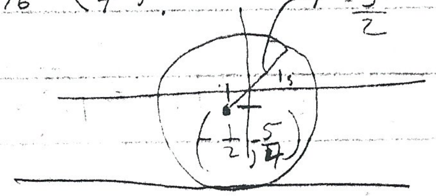
2. $16x^2 + 16x + 16y^2 + 40y - 7 = 0$

$x^2 + x + \frac{1}{4} + y^2 + \frac{10}{4}y + \frac{25}{16} = \frac{7}{16} + \frac{1}{4} + \frac{25}{16}$

$(x + \frac{1}{2})^2 + (y + \frac{5}{4})^2 = \frac{7 + 25 + 4}{16} = \frac{36}{16} = (\frac{6}{4})^2$

$(x + 1/2)^2 + (y + \frac{5}{4})^2 = (\frac{3}{2})^2$ ✓

$C(-1/2, -5/4) \quad r = 3/2$



3. $y = 1 - x^2 \quad y = x^4 - 2x^2 + 1$

$1 - x^2 = x^4 - 2x^2 + 1$

$x^4 - x^2 = 0$

$x^2(x^2 - 1) = 0$

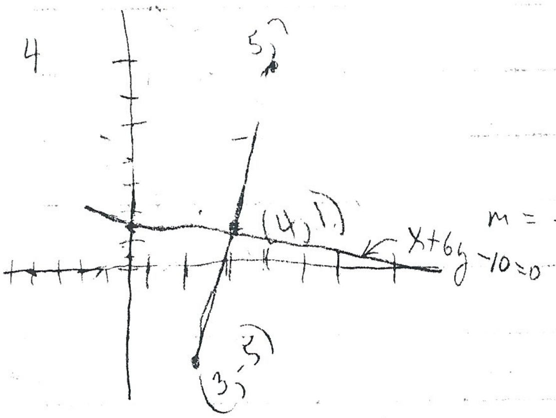
$x = 0 \quad x = 1 \quad x = -1$ ✓

$y = 1 - 0 \quad \underline{(0, 1)}$

$y = 1 - (1)^2 = 0 \quad \underline{(1, 0)}$

$y = 1 - (-1)^2 = 0 \quad \underline{(-1, 0)}$

4.



$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\left(\frac{3+5}{2}, \frac{-5+7}{2}\right) = (4, +1)$

$m = \frac{(-7) - (-5)}{(5-3)} = \frac{+12}{2} = +6 \quad \perp \text{ sd} = \frac{1}{-6}$

$(y - 1) = \frac{1}{-6}(x - 4)$

$-6y + 6 = x - 4$

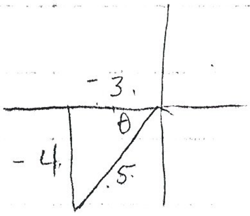
$\underline{x + 6y - 10 = 0}$ ✓

5 $y = \frac{x-6}{\sqrt{x^2-2x}} = \frac{(x-6)}{\sqrt{x(x-2)}}$ $\underline{(-\infty, 0)\ (+2, \infty)}$

6 $f(x) = x^3$ $\frac{(x+\Delta x)^3 - x^3}{\Delta x} = \frac{\cancel{x^3} + 3x\Delta x^2 + 3\Delta x^2 x^2 + \Delta x^3 - \cancel{x^3}}{\Delta x}$

$= 3x^2 + 3x(\Delta x) + (\Delta x)^2$

7

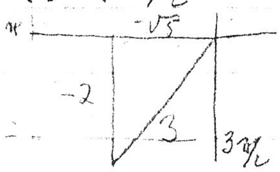


$\sin\theta = -4/5$
$\cos\theta = -3/5$
$\tan\theta = +4/3$
$\cot\theta = 3/4$
$\sec\theta = -5/3$
$\csc\theta = -5/4$

$\cos\theta = \frac{-3}{5}$

$x^2 + 9 = 25$
$x^2 = 16$
$x = 4$

8 $\pi < \theta < 3\pi/2$



$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = \underline{\frac{4\sqrt{5}}{9}}$

$\cos 2\theta = 2\left(\frac{-\sqrt{5}}{3}\right)^2 - 1$
$= \frac{2\cdot 5}{9} - 1 = \frac{10}{9} - \frac{9}{9} = \underline{\frac{1}{9}}$

$x^2 + 4 = 9$
$x^2 = 5$
$x = \sqrt{5}$

9 $\lim_{x \to 1} (2x+1) = 3$ $|f(x) - L| < 0.01$

$|(2x+1) - 3| < 0.01$

$2x - 2 < 0.01$

$|x-1| < 0.005$

$\underline{0 < |x-1| < 0.005}$

10 $\lim_{x \to 16} \frac{4-\sqrt{x}}{x-16} = \lim_{x \to 16} \frac{-\cancel{(\sqrt{x}-4)}}{(\sqrt{x}+4)\cancel{(\sqrt{x}-4)}} = \lim_{x \to 16} \frac{-1}{\sqrt{x}+4} = \frac{-1}{4+4} = \underline{-\frac{1}{8}}$

11 $f(x) = 5/(x-9)$ $g(x) = x^2$ $f(g\,x) = \frac{5}{x^2-9}$

$f(g(x))$ is discontinuous $\underline{x = +3,\ x = -3}$

12 a) $f(x) = \frac{x+2}{(x+2)(x-2)}$ discontinuous at $x = -2$, $x = 2$

b) $x = -2$ can be removed by dividing out $x+2$

$\underline{x-2}$ cannot be removed.

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13 $f'(x) = 5\left[(\sqrt{x^2+4}) + x\left(\tfrac{1}{2}\right)(x^2+4)^{-\frac{1}{2}}2x\right]$

$= 5\left[\dfrac{x^2+4+x^2}{\sqrt{x^2+4}}\right] = \dfrac{5(2x^2+4)}{\sqrt{x^2+4}} = \underline{\dfrac{10(x^2+2)}{\sqrt{x^2+4}}}$ ✓

14 $f(x) = \dfrac{3x-5}{(x^2+1)^{1/3}}$ $f'(x) = \dfrac{(x^2+1)^{\frac{1}{3}}(3) - (3x-5)(+\frac{1}{3})(x^2+1)^{-\frac{2}{3}}2x}{(x^2+1)^{2/3}}$

$= \dfrac{\cancel{(x^2+1)^{1/3}}\left(3 - \frac{2}{3}x(3x-5)(x^2+1)^{-\frac{3}{3}}\right)}{(x^2+1)^{1/3}} = \dfrac{3 - \dfrac{2}{3}\dfrac{x(3x-5)}{3(x^2+1)}}{(x^2+1)^{1/3}}$

$= \dfrac{\dfrac{9x^2+9-6x^2+10x}{3(x^2+1)}}{(x^2+1)^{1/3}} = \underline{\dfrac{3x^2+10x+9}{3(x^2+1)^{4/3}}}$ ✓

15 $f(x) = \tan^3 5x$

$f'(x) = 3(\tan^2 5x)\sec^2 5x\,(5)$

$= \underline{15\tan^2 5x\,\sec^2 5x}$ ✓

16 $f(x) = \sqrt{x^2+1}\,(2x-3)^6$

$f'(x) = \frac{1}{2}(x^2+1)^{-1/2}(2x)(2x-3)^6 + \sqrt{x^2+1}\,(6)(2x-3)^5 2$

$= \dfrac{x(2x-3)^6}{(x^2+1)^{1/2}} + \dfrac{12(x^2+1)(2x-3)^5}{(x^2+1)^{1/2}} = \dfrac{(2x-3)^5(2x^2-3x+12x^2+12)}{(x^2+1)^{1/2}}$

$= \underline{\dfrac{(2x-3)^5(14x^2-3x+12)}{(x^2+1)^{1/2}}}$ ✓

17 $y = \cos xy$

$y' = -\sin xy\,(1y + xy')$

$y' = -y\sin xy - xy'\sin xy$

$y' + xy'\sin xy = -y\sin xy$

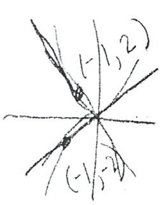
$y'(1 + x\sin xy) = -y\sin xy$

$\underline{y' = \dfrac{-y\sin xy}{1 + x\sin xy}}$ ✓

18 $f(x) = \frac{3x^2}{x^2-2x+1}$ a) $-\frac{8}{9} \approx -.88\cancel{889}$. b) the $f'(x)$ is discontinuous at this point

$= \frac{(x-1)^2 6x - 3x^2(2x-2)}{(x-1)^4}$

$= \frac{\cancel{x^3} - 12x^2 + 6x - \cancel{6x^3} + 6x^2}{(x-1)^4} = \frac{-6x(x-1)}{(x-1)^4} = \frac{-6x}{(x-1)^4}$ at $x=4$ $= \frac{-24}{27} = \frac{-8}{9}$



19 $f(x) = x\sqrt{x^2+3}$ at $f(-1) = -1\sqrt{4} = -2$

$f'(x) = \sqrt{x^2+3} + x(\frac{1}{2})(2x)(x^2+3)^{-1/2}$

$= \frac{x^2+3+x^2}{(x^2+3)^{1/2}} = \frac{2x^2+3}{(x^2+3)^{1/2}}$

at -1 $\frac{2(-1)^2+3}{((-1)^2+3)^{1/2}} = \frac{5}{2}$

or der1$(x\sqrt{(x^2+3)}, x, -1) = 2.5 = \frac{5}{2}$

$y-(-2) = \frac{5}{2}(x-(-1))$

$y+2 = \frac{5}{2}(x+1)$

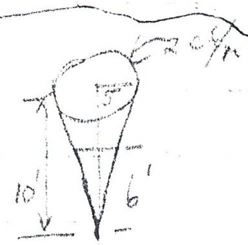
$2y+4 = 5x+5$ or $y = \frac{5}{2}x + \frac{1}{2}$

$5x - 2y + 1 = 0$

$y-2 = -\frac{5}{2}(x+1)$

$2y-4 = -5x-5$

$5x+2y+1=0$

20 

$\frac{r}{5} = \frac{h}{10}$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\frac{h^2}{4}h$

$V = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$2 = \frac{\pi}{4}(6)^2\frac{dh}{dt}$

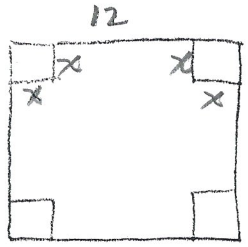
$\frac{8}{36\pi} = \frac{4}{3\pi} = \frac{dh}{dt}$ ft/m

$\frac{2}{9\pi}$

21 a) $\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{x\to 0} 2\left(\frac{\sin 2x}{2x}\right) = 2$

b) $\lim_{x\to\infty} \frac{\sin 2x}{x} = 0$

c) $\lim_{x\to-\infty} \frac{2x}{\sqrt{x^2+4x}} = \lim_{x\to-\infty} \frac{2x}{|x|\sqrt{1+4/x}} = -2$

22 

$V = (12-2x)(x)(12-2x) = x(144 - 48x + 4x^2)$
$= 144x - 48x^2 + 4x^3$

$\frac{dV}{dx} = 144 - 96x + 12x^2 = 0$ $x^2 - 8x + 12$

$\cancel{(x-6)}(x-2) = 0$

$6+6=0$ $x = 2$

$12 - 4 = 8$ $8'' \times 8'' \times 2''$

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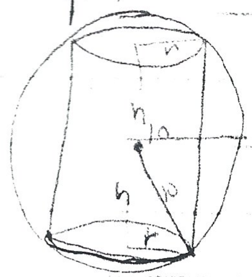


$V = \pi r^2 (2h)$ $\qquad r^2 + h^2 = 10^2$

$V = \pi (100 - h^2)(2h)$ $\qquad r^2 = 10^2 - h^2$

$V = \pi (200h - 2h^3)$

$\frac{dV}{dh} = \pi (200 - 6h^2) = 0$

$6h^2 = 200$

$h = \sqrt{\frac{100}{3}}$ ✓

$V = \pi \left(100 - \left(\frac{100}{3}\right)\right)\left(2\sqrt{\frac{100}{3}}\right)$

$= \underline{2418.3992 \ ft^3}$

24 $\int \frac{x^2+3}{\sqrt[3]{x}} dx = \int (x^{5/3} + 3(x)^{-1/3}) dx = \frac{3}{8}x^{8/3} + 3\cdot\frac{3}{2}x^{2/3} + c$

$= \underline{\frac{3}{8}x^{8/3} + \frac{9}{2}x^{2/3} + c}$ ✓

25 $y'' = 2 - 6x$

$y' = 2x - 3x^2 + c$ $\qquad 3 = 2\cdot 2 - 3(2)^2 + c$

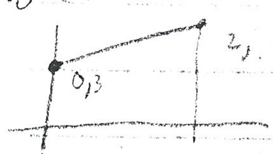
$y' = 2x - 3x^2 + 11$ $\qquad 11 = c$

$y = (1)x^2 - (1)x^3 + 11x + c$ ✓ $\qquad -1 = (2)^2 - (+2)^3 + 11(+2) + c$

$-1 = 4 - 8 + 22 + c$

$\underline{= x^2 - x^3 + 11x - 19}$ ✓ $\qquad -19 = c$

26 b) $\int_0^2 (4x+3)\,dx = 2x^2 + 3x \Big|_0^2 = 2(2)^2 + 3(2) = \underline{14}$ ✓

a) 

$A = \frac{2}{2}((3+11)) = \underline{14}$ ✓

$A_{TRAP} = \frac{1}{2}(B+b)h$

$= \frac{1}{2}(3+11)\cdot 2$

$= 14$

c) fnint$(4x+3, x, 0, 2) = \underline{14}$ ✓

27 a) 15.9931 ✓

b) 25.9931 ✓

c) 20.9931 ✓

d) 21.2601 ✓

e) 21.3072 ✗ 21.13

f) 21.2933 ✓

g) 21.3333 ✓

$-\text{OR}-$ $du = (2x+6)\,dx$ $\int u^{1/2}\frac{du}{2} = \frac{1}{2}\cdot\frac{2}{3}u$

$\frac{du}{2} = (x+3)\,dx$ $= \frac{1}{3}(x^2+6x)^{3/2}\Big|_0^2 = \frac{1}{3}\cdot 16^{3/2} = \frac{64}{3}$

28 $\int_0^2 (x+3)\sqrt{x^2+6x}\,dx = \frac{1}{2}\int_0^2 (x^2+6x)^{\frac{1}{2}}(2x+6)\,dx$ p6.

$= \frac{1}{2}\cdot\frac{2}{3}(x^2+6x)^{\frac{3}{2}}\Big|_0^2 = \frac{1}{3}\left(2^2+6(2)\right)^{\frac{3}{2}} = \frac{1}{3}(16)(16)^{1/2} = \frac{16}{3}\cdot 4 = \frac{64}{3} = 21\frac{1}{3}$

29 $\int \frac{x}{\sqrt{4-x^2}}\,dx = -\frac{1}{2}\int \frac{du}{u^{1/2}}$ $u = 4-x^2$, $du = -2x$

$-\frac{1}{2}\int u^{-\frac{1}{2}}du = -\frac{1}{2}\left(\frac{2}{1}\right)u^{\frac{1}{2}} + C = -\sqrt{4-x^2}+C$

30 $\int 3\cos^2 x\sin x\,dx = -3\int \cos^2 x(-\sin x)\,dx$

$= -3\left(\frac{1}{3}\right)\cos^3 x + C = -\cos^3 x + C$

31 $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$ $u = 1+\sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$ $2\int \frac{du}{u^2} = 2\int u^{-2}du = 2(-1)u^{-1}+c$

$-2\frac{1}{1+\sqrt{x}} + c = \frac{-2}{1+\sqrt{x}} + C$

32 $\int x\sqrt{2x-1}\,dx$ $u = 2x-1$, $du = 2dx$, $\frac{1}{2}(u+1) = x$

$\frac{1}{2}\int (u+1)(u)^{\frac{1}{2}}\frac{du}{2}$

$\frac{1}{4}\left[\int u^{\frac{3}{2}}du + \int u^{\frac{1}{2}}du\right]$

$\frac{1}{4}\left(\frac{2}{5}\right)u^{\frac{5}{2}} + \frac{1}{4}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + c =$

$= \frac{u^{5/2}}{10} + \frac{u^{3/2}}{6} + c =$

$= \frac{(2x-1)^{\frac{5}{2}}}{10} + \frac{(2x-1)^{\frac{3}{2}}}{6} + c$

$= (2x-1)^{\frac{3}{2}}\left(\frac{2x-1}{10} + \frac{1}{6}\right) + c$

$\frac{3(2x-1)}{3\cdot 10} + \frac{5\cdot 1}{5\cdot 6} = \frac{6x-3+5}{30} = \frac{6x+2}{30} = \frac{3x+1}{15}$

$\frac{1}{15}(2x-1)^{3/2}(3x+1) + c$

33 $y = \ln(x+\sqrt{4+x^2})$ $y' = \frac{1+\frac{1}{2}(4+x^2)^{-\frac{1}{2}}(2x)}{x+\sqrt{4+x^2}} =$

$\frac{\frac{\sqrt{4+x^2}}{\sqrt{4+x^2}} + \frac{x}{\sqrt{4+x^2}}}{x+\sqrt{4+x^2}} = \frac{x+\sqrt{4+x^2}}{\sqrt{4+x^2}(x+\sqrt{4+x^2})} = \frac{1}{\sqrt{4+x^2}}$

34 $y = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$ $y' = \frac{\frac{x\left(\frac{1}{2}\right)(4+x^2)^{-\frac{1}{2}}(2x) - (1)\sqrt{4+x^2}}{x^2}}{\frac{\sqrt{4+x^2}}{x}} = \frac{\frac{x^2}{\sqrt{4+x^2}} - \frac{4+x^2}{\sqrt{4+x^2}}}{x^2}\cdot\frac{x}{\sqrt{4+x^2}}$

$= \frac{\frac{x^2-4-x^2}{\sqrt{4+x^2}}}{x\sqrt{4+x^2}} = \frac{-4}{\sqrt{4+x^2}}\cdot\frac{1}{x\sqrt{4+x^2}} = \frac{-4}{x(4+x^2)}$

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35. $y = \dfrac{x^2\sqrt{x^2+4}}{(x^3-4x-6)\sqrt[3]{x^6+16}}$ $\qquad \ln y = 2\ln x + \frac{1}{2}\ln(x^2+4) - \ln(x^3-4x-6) - \frac{1}{3}\ln(x^6+16)$

$$\frac{y'}{y} = 2\left(\frac{1}{x}\right) + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2+4}\right) - \frac{3x-4}{x^3-4x-6} - \frac{1}{\cancel{3}}\left(\frac{\cancel{6}2x^5}{x^6+16}\right)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{x}{x^2+4} - \frac{3x-4}{x^2-4x-6} - \frac{2x^5}{x^6+16}$$

$$y' = \frac{x^2\sqrt{x^2+4}}{(x^3-4x-6)\sqrt[3]{x^6+16}}\left(\frac{2}{x} + \frac{x}{x^2+4} - \frac{3x-4}{x^2-4x-6} - \frac{2x^5}{x^6+16}\right)$$

36. $y = e^{3x}\ln x$

$$y' = 3e^{3x}\ln x + e^{3x}\left(\frac{1}{x}\right)$$
$$= e^{3x}\left(3\ln x + \frac{1}{x}\right)$$

37. $y = \dfrac{3^{x^2}}{x^2}$

$$y' = \frac{x^{\cancel{4}}(\ln 3\ 3^{x^2}(2x)) - 2\cancel{x}(3^{x^2})}{x^{\cancel{4}3}} = \frac{2(3)^{x^2}(x^2\ln 3 - 1)}{x^3}$$

38. $y = x^{3x^2}$

$$\ln y = 3x^2\ln x$$
$$\frac{y'}{y} = 6x\ln x + 3x^2\frac{1}{x}$$
$$y' = x^{3x^2}(6x\ln x + 3x) = 3x^{3x^2+1}(2\ln x + 1)$$

39. $\displaystyle\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$ $\qquad u = 1+\sqrt{x}$, $\quad du = \frac{1}{2}\frac{dx}{\sqrt{x}}$

$$2\int \frac{du}{u} = 2\ln|u| + c = 2\ln|1+\sqrt{x}| + c = \ln(1+\sqrt{x})^2 + c$$

40. $\displaystyle\int \frac{1}{\ln x}\frac{dx}{x} =$ $\qquad u = \ln x$, $\quad du = \frac{dx}{x}$

$$\int \frac{du}{u} = \ln u + c = \ln|\ln(x)| + c$$

41. $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}dx =$ $\qquad u = 1+e^{2x}$, $\quad du = 2e^{2x}dx$ $\qquad$ or $\frac{1}{2}\ln(1+e^{2x}) + c$ (Why??)

$$\frac{1}{2}\int \frac{2e^{2x}dx}{1+e^{2x}} = \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|1+e^{2x}| + c$$

42. $\int e^{\tan x} \sec^2 x \, dx = \underline{e^{\tan x} + C}$

43. $\int \frac{\cos x \, dx}{(1+\sin x)^{1/2}}$ $\quad u = 1+\sin x$, $du = \cos x \, dx$

$\int \frac{du}{u^{1/2}} = \int u^{-1/2} du = 2u^{\frac{1}{2}} + C = \underline{2(1+\sin x)^{\frac{1}{2}} + C}$

44. $\int \frac{\cos x \, dx}{1+\sin x}$ $\quad u = 1+\sin x$, $du = \cos x \, dx$ $\quad \int \frac{du}{u} = \ln u + C = \underline{\ln(1+\sin x) + C}$

45. $\int \left(x^3 + \frac{1}{x^3} + 3^x + \frac{3}{x} + \frac{x}{3} + 3\right) dx$

x^{-3}

$\frac{x^4}{4} + \frac{x^{-2}}{-2} + \frac{3^x}{\ln 3} + 3\ln|x| + \frac{1}{3}\frac{x^2}{2} + 3x + C$

$\underline{\frac{x^4}{4} - \frac{1}{2x^2} + \frac{3^x}{\ln 3} + 3\ln|x| + \frac{x^2}{6} + 3x + C}$

Review For Final   pg 1   Leif Erickson

1. $d = \sqrt{(x_1 - x_2)^2 + (y_1 + y_2)^2}$

$8 = \sqrt{(5-5)^2 + (y-1)^2}$

$8 = \sqrt{(y-1)^2}$

$\pm 8 = y - 1$

$\underline{y = 9} \quad \underline{y = -7}$ ✓

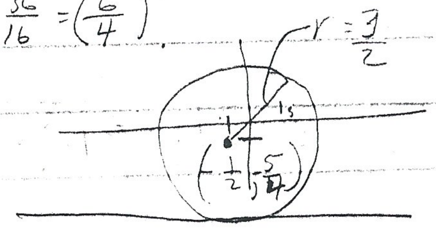
2. $16x^2 + 16x + 16y^2 + 40y - 7 = 0$

$x^2 + x + \frac{1}{4} + y^2 + \frac{10}{4}y + \frac{25}{16} = \frac{7}{16} + \frac{1}{4} + \frac{25}{16}$

$(x + \frac{1}{2})^2 + (y + \frac{5}{4})^2 = \frac{7 + 25 + 4}{16} = \frac{36}{16} = \left(\frac{6}{4}\right)^2$

$(x + 1/2)^2 + (y + \frac{5}{4})^2 = \left(\frac{3}{2}\right)^2$ ✓

$C(-1/2, -5/4) \quad r = 3/2$



3. $y = 1 - x^2 \quad y = x^4 - 2x^2 + 1$

$1 - x^2 = x^4 - 2x^2 + 1$

$x^4 - x^2 = 0$

$x^2(x^2 - 1) = 0$

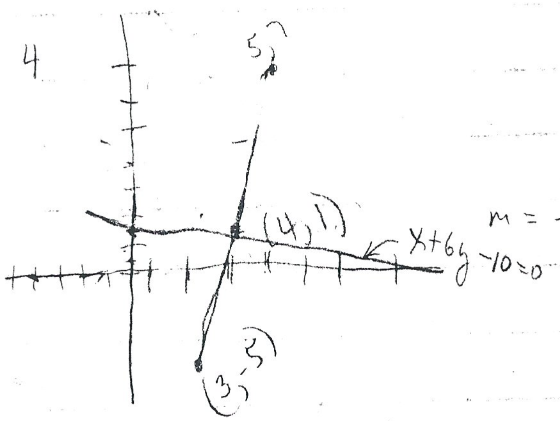
$x = 0 \quad x = 1 \quad x = -1$ ✓

$y = 1 - 0 \quad \underline{(0, 1)}$

$y = 1 - (1)^2 = 0 \quad \underline{(1, 0)}$

$y = 1 - (-1)^2 = 0 \quad \underline{(-1, 0)}$

4.



$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\left(\frac{3+5}{2}, \frac{-5+7}{2}\right) = (4, +1)$

$m = \frac{(-7) - (-5)}{(5-3)} = \frac{+12}{2} = +6 \quad \perp \text{sd} = -\frac{1}{6}$

$(y - 1) = \frac{1}{-6}(x - 4)$

$-6y + 6 = x - 4$

$\underline{x + 6y - 10 = 0}$ ✓