CALCULUS I Exam 1C Dr. RAPALJE

Show all work on separate paper. Calculators are allowed. Turn in all work sheets. You may keep this test and your answers if you wish.

1. Let $A=(-\infty,5)$ and $B=[-1,10]$. Give interval notation for:
   a) $A \cup B$ b) $A \cap B$ c) $(A-B) \cup (B-A)$

In 2–5, solve and graph on a numberline:

2. $x^2-5x \le 6$ 3. $\left|\frac{4-3x}{5}\right| > 2$ 4. $\frac{4}{3-x} \le 2$

5. $x \ge \frac{3}{x-2}$

6. Given points $A(-3,2)$ and $B(5,-8)$:
   a) Find the midpoint of $\overline{AB}$
   b) Find the slope of $\overline{AB}$
   c) Find (in simplest radical form) the distance between A and B.

7. Find the equation of the tangent line to $(x-1)^2+y^2=10$ at $(-2,1)$.

In 8–10, find the domain and range:

8. $y=x^2-4x+5$ 9. $y=\frac{4x^2}{x^2-9}$ 10. $y=\frac{1}{\sqrt{4-x^2}}$

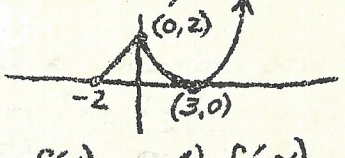
11. If $f(x)=x^2+3x$, find $f(x+\Delta x)$ and $\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

12. If $f(x)=\frac{2x}{x-4}$ and $g(x)=\frac{x+5}{x}$, find $f \circ g$, $g \circ f$, and the domain of each.

13. Let $P(x)=\frac{1}{x}$, $Q(x)=1-x$, $R(x)=P \circ Q$. Find $R \circ R \circ R$.

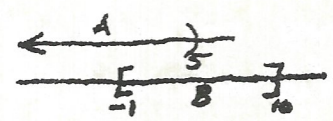
14. a) Graph $f(x)=x^3$
    b) $f(x)=(x-2)^3$
    c) $f(x)=-(x-2)^3+4$

15. Given the graph of $f(x)$. Graph:



a) $-f(x)$ b) $f(-x)$ c) $f(x+2)-3$
(Label points)

CALCULUS I Exam 1C Solutions — Dr. RAPALJE

1. 

a) $A \cup B = (-\infty, 10]$

b) $A \cap B = [-1, 5)$

c) $A - B = (-\infty, -1)$

$B - A = [5, 10]$

$(A-B) \cup (B-A) = (-\infty, -1) \cup [5, 10]$

2. $x^2 - 5x - 6 \le 0$

$(x-6)(x+1) = 0$

$x = 6 \quad x = -1$

Between Endpts.

$[-1, 6]$

3. $\left|\frac{4-3x}{5}\right| > 2$

(Extremes!)

$\frac{4-3x}{5} > 2$ or $\frac{4-3x}{5} < -2$

$4 - 3x > 10$ $\quad$ $4 - 3x < -10$

$-3x > 6$ $\quad$ $-3x < -14$

$x < -2$ $\quad$ $x > 14/3$

$(-\infty, -2) \cup \left(\frac{14}{3}, \infty\right)$

4. $\frac{4}{3-x} \le 2$

(Use interval test or cases.)

$4 = 2(3-x)$

$4 = 6 - 2x$

$2x = 2$

$x = 1, \quad x = 3$

Endpoints.

$(-\infty, 1] \cup (3, \infty)$

5. $x \ge \frac{3}{x-2}$

Again interval test or cases.

$x = \frac{3}{x-2} \qquad x = 2$ Endpt.

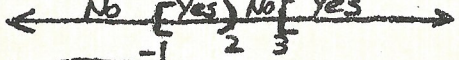
$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

Endpts.



$[-1, 2) \cup [3, \infty)$

6. $A(-3, 2) \quad B(5, -8)$

a) Midpt $(1, -3)$

b) $m = \frac{-8-2}{5+3} = -\frac{5}{4}$

c) $dist = \sqrt{8^2 + 10^2}$

$= \sqrt{164}$

$= 2\sqrt{41}$

7. Circle center $(1, 0)$ to $(-2, 1)$

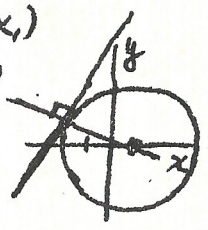
$slope = \frac{1}{-3}$

Tangent line $\perp$, $slope = 3$

$y - y_1 = m(x - x_1)$

$y - 1 = 3(x + 2)$

$y = 3x + 7$



8. $y = x^2 - 4x + 5$

D: all real x

$x^2 - 4x + 5 - y = 0$

$x = \frac{4 \pm \sqrt{16 - 4(5-y)}}{2}$

$= \frac{4 \pm \sqrt{4y - 4}}{2}$

R: $4y - 4 \ge 0$

$y \ge 1$

9. $y = \frac{4x^2}{x^2 - 9}$

D: all $x \ne \pm 3$

$x^2 y - 9y = 4x^2$

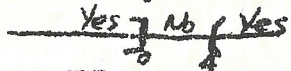
$x^2 y - 4x^2 = 9y$

$x^2(y-4) = 9y$

$x = \pm\sqrt{\frac{9y}{y-4}}$

R: $y \ne 4, \quad \frac{9y}{y-4} \ge 0$

Endpts $y = 0, \; y = 4$



$(-\infty, 0] \cup (4, \infty)$

10. $y = \frac{1}{\sqrt{4 - x^2}}$

D: $4 - x^2 > 0$

$x^2 - 4 < 0$

$(-2, 2)$

R: $y = \frac{1}{\sqrt{\quad}}, \; \text{so } y > 0$

$y^2 = \frac{1}{4 - x^2}$

$4y^2 - x^2 y^2 = 1$

$\frac{4y^2 - 1}{y^2} = \frac{x^2 y^2}{y^2}$

$x = \pm\sqrt{\frac{4y^2 - 1}{y^2}}$

Endpts $0, \pm\frac{1}{2}$, but $y > 0$

Range: $y \ge \frac{1}{2}$

CALCULUS I Exam 1C Solutions (Con't) DR. RAPALJE

11. $f(x) = x^2 + 3x$

NOTE: $f(x+\Delta x) \neq f(x) + \Delta x$ and $f(x+\Delta x) \neq f(x) + f(\Delta x)$

$f(x+\Delta x) = (x+\Delta x)^2 + 3(x+\Delta x)$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{x^2 + 2x\Delta x + \Delta x^2 + 3x + 3\Delta x - x^2 - 3x}{\Delta x}$$

$$= \frac{\Delta x(2x + \Delta x + 3)}{\Delta x} = 2x + \Delta x + 3$$

12. $f(x) = \frac{2x}{x-4}$ $g(x) = \frac{x+5}{x}$

$$f \circ g = \frac{2\left(\frac{x+5}{x}\right)}{\frac{x+5}{x} - 4} = \frac{2x+10}{x+5-4x} = \frac{2x+10}{5-3x}$$

D: $x \neq 0, \frac{5}{3}$

$$g \circ f = \frac{\frac{2x}{x-4} + 5}{\frac{2x}{x-4}} = \frac{2x + 5x - 20}{2x} = \frac{7x-20}{2x}$$

D: $x \neq 4, 0$

13. $P(x) = \frac{1}{x}$ $Q(x) = 1 - x$

#13. ALSO SEE BELOW

$R(x) = P \circ Q = \frac{1}{1-x}$

$$R \circ R = \frac{1}{1 - \frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}}$$

$$= \frac{1-x}{-x} = \frac{x-1}{x}$$

$(R \circ R) \circ R$ [Subst. R in R∘R]

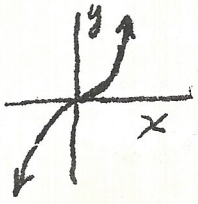
$$\frac{\frac{1}{1-x} - 1}{\frac{1}{1-x}} = \frac{1 - 1 + x}{1} = x$$

-OR-

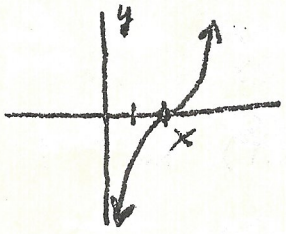
$R \circ (R \circ R)$ [Subst. R∘R in R]

$$\frac{1}{1 - \frac{x-1}{x}} = \frac{1}{\frac{x - x + 1}{x}} = x$$

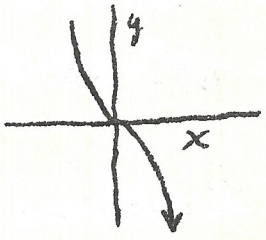
14 a) $f(x) = x^3$



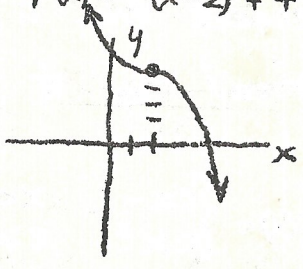
b) $f(x) = (x-2)^3$



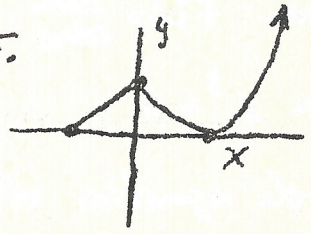
c) $f(x) = -x^3$



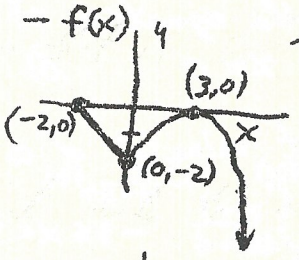
$f(x) = -(x-2)^3 + 4$



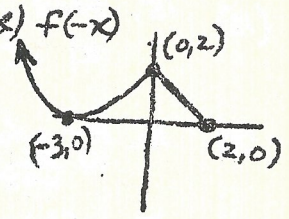
15.



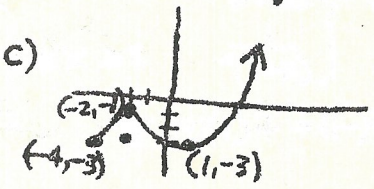
a) $-f(x)$



b) $f(-x)$



c)



13 – ALSO –

$R = \frac{1}{1-x}$ $R \circ R = \frac{1}{1 - \frac{1}{1-x}}$ (R^2) $R \circ R \circ R = \frac{1}{1 - \frac{1}{1-x}}$ (R^3)