# CALCULUS I EXAM 3E Dr. Rapalje NAME

show all work on separate paper. Turn this test in with all worksheets

1. Find the maximum and minimum values of the function (if they exist)

(18) a) $f(x) = 3x^4 - 4x^3$ on $[-1,2]$

b) $f(x) = x^2 - 2x$ on $(0,2)$

c) $f(x) = \frac{x}{x-2}$ on $[1,4)$

2. Find all values of c for which the Mean Value Theorem applies, or explain why the theorem does not apply.

(12) a) $f(x) = x^3 - x^2 - 2x$ on $[-1,1]$

b) $f(x) = 2\sin x + \sin 2x$ on $[0,\pi]$

3. Determine the following limits:

(18) a) $\lim_{x\to\infty} \frac{x}{|x|}$ b) $\lim_{x\to-\infty} \frac{x}{|x|}$ c) $\lim_{x\to\infty} \frac{x^2 - x^3 + 4}{3x^3 + 4x - 1}$

d) $\lim_{x\to\infty} \frac{x^2 - x^3 + 4}{3x^4 + 4x - 1}$ e) $\lim_{x\to\infty} \frac{x^2 - x^3 + 4}{3x^2 + 4x - 1}$ f) $\lim_{x\to\infty} \frac{c}{x}$

4. Determine the limits algebraically – show all work!

(18) a) $\lim_{x\to-\infty} (x + \sqrt{x^2 + 9x})$ b) $\lim_{x\to\infty} (x - \sqrt{x^2 + 9x})$ c) $\lim_{x\to-\infty} \frac{x}{\sqrt{4x^2 - 1}}$

5. Given $y = x^4 - 4x^3 + 16x$

$y' = 4(x+1)(x-2)^2$

$y'' = 12x(x-2)$

(30) a) Find critical values.

b) Intervals increasing/decreasing

c) Points of inflection

d) Intervals concave up/down.

e) Rel max and min.

f) $\lim_{x\to\infty}$ and $\lim_{x\to-\infty}$

g) Discuss vertical tangents and asymptotes.

h) Graph

6. Given $y = 3x^{2/3} - 2x$

$y' = \frac{2(1 - x^{1/3})}{x^{1/3}}$

$y'' = \frac{-2}{3x^{4/3}}$

Answer as in #5.

(8) 7a) Give vertical and horizontal asymptotes for: $y = \frac{4x - 8}{\sqrt{x^2 + 9}}$

b) Give x & y intercepts

104 Total.

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1a) $f(x) = 3x^4 - 4x^3$ on $[-1,2]$

$f'(x) = 12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$x = 0 \quad x = 1$

$f(-1) = 3 + 4 = 7$

$f(2) = 48 - 32 = 16$ Max

$f(0) = 0$

$f(1) = -1$ Min.

b) $f(x) = x^2 - 2x$ on $(0,2)$

$f'(x) = 2x - 2 = 0$

$x = 1$

$f(0) = 0$ Not Included

$f(2) = 0$ Not Included

$f(1) = -1$ Minimum

No Maximum

c) $f(x) = \frac{x}{x-2}$ on $[1,4]$

$\lim_{x \to 2^+} = +\infty$

$\lim_{x \to 2^-} = -\infty$

No Max or Min.

2a) $f(x) = x^3 - x^2 - 2x$ on $[-1,1]$

$f(b) = 1 - 1 - 2 = -2$

$f(a) = -1 - 1 + 2 = 0$

$\frac{f(b)-f(a)}{b-a} = \frac{-2-0}{1-(-1)} = -1$

$f'(x) = 3x^2 - 2x - 2$

$f'(c) = 3c^2 - 2c - 2 = -1$

$3c^2 - 2c - 1 = 0$

$(3c+1)(c-1) = 0$

$c = -\frac{1}{3}$ ~~$c = 1$~~ Not in $(-1,1)$

b) $f(x) = 2\sin x + \sin 2x$ on $[0,\pi]$

$f(b) = 0$, $f(a) = 0$ $\quad \frac{f(b)-f(a)}{b-a} = 0$

$f'(x) = 2\cos x + 2\cos 2x$

$f'(c) = 2\cos c + 2(2\cos^2 c - 1) = 0$

$2(2\cos^2 c + \cos c - 1) = 0$

$(2\cos c - 1)(\cos c + 1) = 0$

$\cos c = \frac{1}{2}$ $\quad \cos c = -1$

$c = \frac{\pi}{3}$, ~~$\frac{5\pi}{3}$~~ ~~$c = \pi$~~ Not in $(0,\pi)$

3a) $\lim_{x\to\infty} \frac{x}{|x|} = \lim_{x\to\infty} \frac{x}{x} = 1$

b) $\lim_{x\to-\infty} \frac{x}{|x|} = \lim_{x\to-\infty} \frac{x}{-x} = -1$

c) $\lim_{x\to\infty} \frac{\frac{1}{x^3}(x^2 - x^3 + 4)}{\frac{1}{x^3}(3x^3 + 4x - 1)} = \frac{\frac{1}{x}\to 0 - 1 + \frac{4}{x^3}\to 0}{3 + \frac{4}{x^2}\to 0 - \frac{1}{x^3}\to 0} = -\frac{1}{3}$

d) $\lim_{x\to\infty} \frac{x^2 - x^3 + 4}{3x^4 + 4x - 1} = \frac{\frac{1}{x}\to 0 - 1 + \frac{4}{x^3}\to 0}{3x + \frac{4}{x^2}\to 0 - \frac{1}{x^3}\to 0}$

$= \lim_{x\to\infty} \frac{-1}{3x} = 0$

e) $\lim_{x\to\infty} \frac{\frac{1}{x^2}(x^2 - x^3 + 4)}{\frac{1}{x^2}(3x^2 + 4x - 1)} = \lim_{x\to\infty} \frac{1 - x + \frac{4}{x^2}\to 0}{3 + \frac{4}{x}\to 0 - \frac{1}{x^2}\to 0} = -\infty$

f) $\lim_{x\to\infty} \frac{c}{x} = 0$

4a) $\lim_{x\to\infty} \left(x + \sqrt{x^2 + 9x}\right)$

$= \infty + \infty = \infty$

b) $\lim_{x\to\infty} \left(x - \sqrt{x^2+9x}\right) \frac{(x + \sqrt{x^2+9x})}{(x + \sqrt{x^2+9x})}$

$= \infty - \infty$ Indeterm.

$= \lim_{x\to\infty} \frac{x^2 - x^2 - 9x}{(x + \sqrt{x^2(1 + 9/x)})}$

$= \lim_{x\to\infty} \frac{-9x}{x + x\sqrt{1 + 9/x}}$

$= \lim_{x\to\infty} \frac{-9x}{x(1 + \sqrt{1 + 9/x})} = -\frac{9}{2}$

c) $\lim_{x\to-\infty} \frac{x}{\sqrt{4x^2 - 1}}$

$= \lim_{x\to-\infty} \frac{x}{\sqrt{x^2(4 - 1/x^2)}}$

$= \lim_{x\to-\infty} \frac{x}{|x|} \cdot \frac{1}{\sqrt{4 - 1/x^2}}$

$= -1 \cdot \frac{1}{2} = -\frac{1}{2}$

5. $y = x^4 - 4x^3 + 16x$

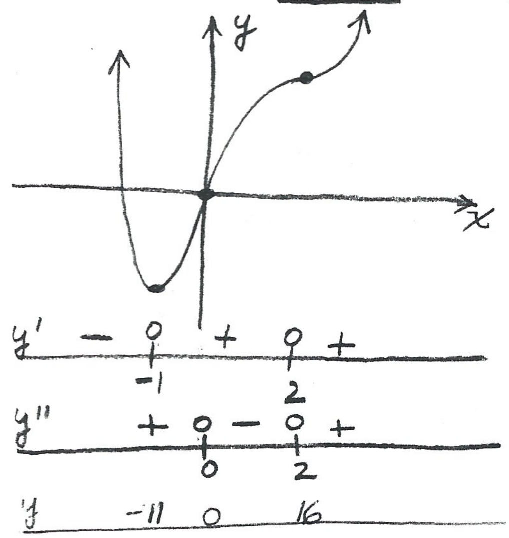
$y' = 4(x+1)(x-2)^2 = 0$

a) at $x = -1$, $x = 2$ Critical Values.

$y'' = 12x(x-2) = 0$

$x = 0$ $x = 2$ Possible inflec. pts.

h)



b) Increasing $(-1, \infty)$; Decreasing $(-\infty, -1)$

c) Points of inflection: $(0,0)$ $(2,16)$

d) Concave up: $(-\infty, 0) \cup (2, \infty)$
down: $(0, 2)$

e) Rel max None. Rel Min: $(-1, -11)$

f) $\lim_{x \to +\infty} = \infty$ ; $\lim_{x \to -\infty} = \infty$

g) None.

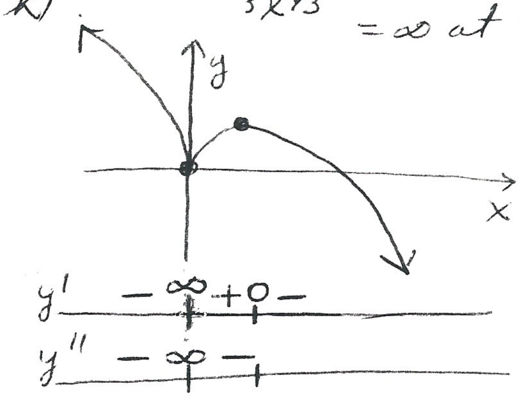
6. $y = 3x^{2/3} - 2x$

$y' = \dfrac{2(1 - x^{1/3})}{x^{1/3}} = 0$ at $x = 1$
$= \infty$ at $x = 0$

a) Critical points $(0,0)$ $(1,1)$

$y'' = \dfrac{-2}{3x^{4/3}} \neq 0$
$= \infty$ at $x = 0$.

h)



b) Increasing: $(0,1)$ Decr: $(-\infty, 0) \cup (1, \infty)$

c) No points of inflection.

d) Concave down $(-\infty, \infty)$

e) Rel Max: $(1,1)$; Rel min $(0,0)$

f) $\lim_{x \to \infty} = -\infty$ ; $\lim_{x \to -\infty} = +\infty$

g) Vertical tangent at $(0,0)$
No vert. asymptotes.

7. $y = \dfrac{4x - 8}{\sqrt{x^2 + 9}}$

a) Vertical asymptotes: Denom $\neq 0$ None

Horiz. asymptotes: $\lim_{x \to \infty} \dfrac{x(4 - 8/x)}{|x|\sqrt{1 + 9/x^2}} = 4$

$\lim_{x \to -\infty} \dfrac{x(4 - 8/x)}{|x|\sqrt{1 + 9/x^2}} = -4$

$y = 4$, $y = -4$

b) Intercepts: $x = 0$, $y = -8/3$ $(0, -8/3)$

$y = 0$, $x = 2$ $(2, 0)$