CALCULUS I EXAM 40 ~~Chapter 6~~ Dr. RAPALJE

p.1 (See p.2 for FORMULAS and hints!)

1. Determine if the theorem of the mean holds for $f(x) = \frac{3x-2}{2x+2}$ for $0 \le x \le 2$. If so, find all x_0 for which the theorem is valid. If not, explain why the theorem is not valid.

2. For $f(x) = x^4 - 4x^2$, find the maximum and minimum values if they exist, for the interval:
   a) $0 < x < 2$
   b) $-1 \le x \le 4$

   } Give <u>coordinates</u> of points.

3. Given the table and the fact that $\lim\limits_{x \to -\infty} = 0^-$ and $\lim\limits_{x \to +\infty} = -\infty$

x	-3	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
f		*		1		0		1	
f'	−	*	−	−	−	*	+	0	−
f''	−	*	+	0	−	*	−	−	−

   a) Identify relative maximums, relative minimums, points of inflection, asymptotes, and vertical tangent points.
   b) Give intervals in which function increases and decreases.
   c) Give intervals of concave up and concave down.
   d) Graph it.

4. If $f(x) = x^{4/3} + 4x^{1/3}$ or $x^{1/3}(x+4)$

$$f'(x) = \frac{4(x+1)}{3x^{2/3}}$$

$$f''(x) = \frac{4(x-2)}{9x^{5/3}},$$

Complete the table:

x	-4	-1	$-\frac{1}{2}$	0	1	2	3
f							
f'							
f''							

5. Find Δf and df for $f(x) = x^2 + 3x$ if $x = 1$ and $h = .1$

6. A piece of wire of length L is cut into two parts, one of which is bent into a square, the other into a circle. How should the wire be cut so that the sum of the areas is a minimum?

7. Sand is pouring from a spout at the rate of 4 m³/min and falling on a conical pile whose diameter at the base is always three times the altitude. At what rate is the altitude increasing when the altitude is 3 m?

Theorem of mean: If f is continuous for $a \le x \le b$ and $f'(x)$ exists for each x between a + b, then there is an x_0 between a + b such that $f'(x_0) = \frac{f(b) - f(a)}{b - a}$.

$df = f'(x) \cdot h \qquad \Delta f = f(x+h) - f(x) \qquad \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

Volume of cone $= \frac{1}{3}\pi r^2 h$; Lateral Surface area $= \pi r \sqrt{r^2 + h^2}$

Sphere $V = \frac{4}{3}\pi r^3$ Surface area $= 4\pi r^2$.

MAX or MIN

1. Draw figure.
2. Assign letters to each quantity mentioned in problem.
3. Select quantity to be max or min; express as function of other variables.
4. Eliminate all quantities but one, so as to have function of one variable.
5. Differentiate, set equal to zero.

RELATED RATES

1. Draw diagram. Label quantities which remain constant.
2. Denote variables by letters. Find relation among variables.
3. Take differentials of both sides, divide by dt.
4. Insert special numerical quantities.

1. Thm of mean is valid.

$f(x) = \frac{3x-2}{2x+2} \quad 0 \le x \le 2$

$f'(x) = \frac{(2x+2)\cdot 3 - (3x-2)\,2}{(2x+2)^2}$

$= \frac{6x+6-6x+4}{(2x+2)^2}$

$= \frac{10}{4(x+1)^2}$

$f(b) = \frac{6-2}{4+2} = \frac{4}{6} = \frac{2}{3}$

$f(a) = -1$

$f'(x_0) = \frac{f(b)-f(a)}{b-a}$

$\frac{10}{4(x+1)^2} = \frac{2/3+1}{2-0}$

$20 = \frac{5}{3}\,4\cdot(x+1)^2$

$(x+1)^2 = 3$

$x+1 = \pm\sqrt{3}$

$x = -1 \pm \sqrt{3}$,

Between 0 & 2: $x_0 = -1+\sqrt{3}$

2. $f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x$

$= 4x(x^2-2) = 0$

at $x=0 \quad x=\pm\sqrt{2}$

a) $f(0) = 0$ open endpt.

$f(2) = 0$ open endpt.

$f(\sqrt{2}) = 4-8 = -4$

Rel min $(\sqrt{2}, -4)$
No max on $0<x<2$

b) $-1 \le x \le 4$

$f(-1) = 1-4 = -3$

$f(4) = 256-64 = 192$

$f(\sqrt{2}) = -4$

Min $= (\sqrt{2}, -4)$
Max $= (4, 192)$

3 a) Rel max: $x=1$

Rel min: $x=0$

Inflect: $x=-1$

asymp: $x=-2$

Vert tan: $x=0$

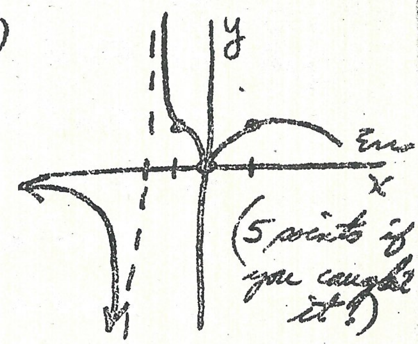
b) Increase: $[0,1]$

Decrease: $(-\infty,0]\,[1,\infty)$

c) Concave up: $[-2,-1]$

down: $(-\infty,-2]\,[1,\infty)$

d)



(5 points if you caught it!)

4. $f = x^{1/3}(x+4)$

$f' = \frac{4(x+1)}{3x^{2/3}}$

$f'' = \frac{4(x-2)}{9x^{5/3}}$

x	-4	-1	$-\frac{1}{2}$	0	1	2	3
f		-3		0	5	$5\frac{1}{2}$	
f'	$-$	0	$+$	$\times$	$+$	$+$	$+$
f''	$+$	$+$	$+$	$\times$	$-$	0	$+$

5. $f(x) = x^2+3x \quad x=1,\ h=.1$

$f'(x) = 2x+3$

$f(x+h) = f(1+.1)$

$= f(1.1) = 1.21+3.3$

$= 4.51$

$f(x) = f(1) = 4$

$\Delta f = f(x+h) - f(x) =$

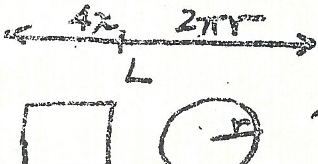
$= 4.51 - 4 = .51$

$df = f'(x)\cdot h$

$= (2x+3)\cdot h$

$= 5(.1) = .5$

6.



$x = \frac{L-2\pi r}{4}$

Sum of areas $= x^2 + \pi r^2$

$SA = \left(\frac{L-2\pi r}{4}\right)^2 + \pi r^2$

$= \frac{L^2 - 4\pi r L + 4\pi^2 r^2}{4} + \pi r^2$

$SA' = -\pi L + 2\pi^2 r + 2\pi r = 0$

$2\pi r(\pi+1) = \pi L$

$2\pi r = \frac{L\pi}{(\pi+1)}$ from end

$\frac{L}{(\pi+1)}$ from other end

$x = \frac{L}{4(\pi+1)}$

$r = \frac{L}{2(\pi+1)}$ or

7. $\frac{dV}{dt} = 4\ m^3/min$

$V = \frac{1}{3}\pi r^2 h$

diam. $= 3h = 2r$

$r = \frac{3h}{2}$

$V = \frac{1}{3}\pi \cdot \frac{9h^2}{4}\cdot h$

$= \frac{3\pi}{4} h^3$

$\frac{dV}{dt} = \frac{9\pi h^2}{4}\frac{dh}{dt}$

$4 = \frac{9\pi(9)}{4}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{16}{81\pi}\ m/min.$