CALCULUS I EXAM 5E Dr. RAPALJE

Sections 5.1 – 5.6 (Larson + Hostetler)

Show all work as necessary. Calculators, approved formula sheets allowed.

In 1 – 4, evaluate the indefinite integrals:

1. $\int \frac{x^2+3x-6}{\sqrt{x}}\,dx$

2. $\int (\sec^2\theta - \sin 3\theta)\,d\theta$

3. $\int (x^2-1)^7 x\,dx$

4. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

In 5 – 7, evaluate the definite integrals:

5. $\int_1^4 \frac{x-4}{\sqrt{x}}\,dx$

6. $\int_0^3 (3x^2+x-2)\,dx$

7. $\int_0^2 x\sqrt{4-x^2}\,dx$

8. Find the area bounded by $y=-x^2+3x$ and $y=0$.

9. Find the average value of $f(x)=3x^2-2x$ on $[1,4]$.

10. Find all values of c that satisfy the Mean Value Theorem for Integrals for #9.

11. Find the value of $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\frac{2}{n}$

12. Use a geometric formula to calculate $\int_0^r \sqrt{r^2-x^2}\,dx$, where r = constant

13. If $\int_0^3 f(x)\,dx=4$ and $\int_3^8 f(x)\,dx=10$

a) $\int_3^0 f(x)\,dx =$ ____ b) $\int_0^8 f(x)\,dx =$ ____ c) $\int_8^3 -2f(x)\,dx$

In 14 – 15, use the limit process (Σ) to find the area between the given function and the x axis over the given interval.

14. $y=x^2+4$ on $[0,2]$

15. $y=-\frac{1}{2}x+6$ on $[2,4]$

CALCULUS I EXAM 5E Solutions Dr. RAPALJE

1. $\int \frac{x^2+3x-6}{\sqrt{x}}\,dx$

$= \int (x^{3/2} + 3x^{1/2} - 6x^{-1/2})\,dx$

$= \frac{2}{5}x^{5/2} + 2x^{3/2} - 12x^{1/2} + C$

2. (6) $\int (\sec^2\theta - \sin 3\theta)\,d\theta$

$= \tan\theta + \frac{1}{3}\cos 3\theta + C$

3. (6) $\int (x^2-1)^7 x\,dx$

$u = x^2 - 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int u^7 \frac{du}{2}$

$= \frac{u^8}{16} + C$

$= \frac{1}{16}(x^2-1)^8 + C$

4. $\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\,dx$ Let $u = 1+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2}dx$

$2du = \frac{dx}{\sqrt{x}}$

$= \int \frac{1}{u^2}\,2\,du$

$= 2\int u^{-2}\,du$

$= 2\left[\frac{u^{-1}}{-1}\right] + C = \frac{-2}{1+\sqrt{x}} + C$

5. (7) $\int_1^4 \frac{x-4}{\sqrt{x}}\,dx$

$= \int_1^4 (x^{1/2} - 4x^{-1/2})\,dx$

$= \frac{2x^{3/2}}{3} - \frac{24}{1}x^{1/2}\Big|_1^4$

$= \left(\frac{2}{3}\cdot 4^{3/2} - 8\cdot 4^{1/2}\right) - \left(\frac{2}{3} - 8\right)$

$= \frac{16}{3} - 16 - \frac{2}{3} + 8$

$= \frac{14}{3} - 8 = -\frac{10}{3}$

6. (7) $\int_0^3 (3x^2 + x - 2)\,dx$

$= x^3 + \frac{x^2}{2} - 2x\Big|_0^3$

$= 27 + \frac{9}{2} - 6$

$= 21 + \frac{9}{2}$

$= \frac{42+9}{2} = \frac{51}{2}$

7. $\int_0^2 x\sqrt{4-x^2}\,dx$ Let $u = 4 - x^2$

$du = -2x\,dx$

If $x=0$, $u=4$

If $x=2$, $u=0$

$= \int_4^0 u^{1/2}\,\frac{du}{-2}$

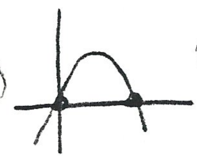
$= \frac{1}{2}\int_0^4 u^{1/2}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^4 = \frac{1}{3}\cdot 4^{3/2} = \frac{8}{3}$

8. (7) $y = -x^2 + 3x$

$0 = x(-x+3)$

$x=0 \quad x=3$



$\int_0^3 (-x^2 + 3x)\,dx$

$= -\frac{x^3}{3} + \frac{3x^2}{2}\Big|_0^3$

$= -9 + \frac{27}{2} = \frac{9}{2}$

9. (7) $\frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{3}\int_1^4 (3x^2 - 2x)\,dx$

$= \frac{1}{3}\left[x^3 - x^2\Big|_1^4\right]$

$= \frac{1}{3}\left[(64-16) - (1-1)\right]$

$= \frac{48}{3} = 16$

10. $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

$3c^2 - 2c = 16$

$3c^2 - 2c - 16 = 0$

$(3c-8)(c+2) = 0$

$c = 8/3$ $\quad \cancel{c = -2}$

Not in $[1,4]$

11. (7) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right)\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)$

$= \lim_{n\to\infty} \frac{2}{n}\cdot n + \frac{8}{n^2}\,\frac{n(n+1)}{2} + \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$= 2 + 4 + \frac{8}{3} = \frac{26}{3}$

12. (5) $y = \sqrt{r^2 - x^2} \quad 0 \le x \le r$

Quarter-circle.

$A = \frac{\pi r^2}{4}$

13. a) -4

b) $4 + 10 = 14$

c) $(-2)(-10) = 20$

14. (7) $y = x^2 + 4 \quad [0,2]$

$\Delta x = \frac{2}{n} \quad x_i = \frac{2i}{n}$

$f(x_i) = \frac{4i^2}{n^2} + 4$

$A = \lim_{n\to\infty} \sum \left(\frac{4i^2}{n^2} + 4\right)\frac{2}{n}$

$= \lim \left(\frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{8}{n}\cdot n\right)$

$= \frac{8}{3} + 8$

$= \frac{32}{3}$

15. (7) $y = -\frac{1}{2}x + 6 \quad [2,4]$

$\Delta x = \frac{2}{n} \quad x_i = 2 + \frac{2i}{n}$

$f(x_i) = -\frac{1}{2}\left(2 + \frac{2i}{n}\right) + 6$

$= -1 - \frac{i}{n} + 6$

$= 5 - \frac{i}{n}$

$A = \lim_{n\to\infty} \sum \left(5 - \frac{i}{n}\right)\frac{2}{n}$

$= \frac{10}{n}\cdot n - \frac{2}{n^2}\,\frac{n(n+1)}{2}$

$= 10 - 1$

$= 9$