CALCULUS I EXAM 6E Sections 5.7 – 6.3 (Larson/Hostetler)

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Show all work on separate paper. CALCULATORS ALLOWED.

You may keep this copy of the test and your answers. (One problem free

In 1–4, find $\frac{dy}{dx}$ and simplify:

1. $y = \ln\left(x + \sqrt{4+x^2}\right)$

2. $y = \ln\sqrt{\frac{4-x^2}{x}}$

3. $y = xe^{x^2-5x}$

4. $y = x^{\sin 2x}$

In 5–7, perform the integration:

5. $\int \frac{dx}{\sqrt{x}\,(1+\sqrt{x})}$ 6. $\int e^{2x}\sqrt{1+e^{2x}}\,dx$ 7. $\int \frac{2e^{x}-2e^{-x}}{e^{x}+e^{-x}}\,dx$

8. Use the fact that $\ln 2 \approx .7$ and $\ln 3 \approx 1.1$ to find approximations without calculators for:

a) $\ln 72$ b) $\ln \sqrt[3]{12}$

9. Find $\frac{dF}{dx}$ for $F(x) = \int_0^x (t^2 - 4t + 3)\,dt$

10. Show the equivalence of $\int \sec x\,dx = \ln|\sec x + \tan x| + C$ and $\int \sec x\,dx = -\ln|\sec x - \tan x| + C$ (You need not verify the integrals!)

11. Find $f^{-1}(x)$ for $f(x) = \frac{-2}{\sqrt{x^2-9}}$. Give domain and range of f and f^{-1}! where $x>3$

12. Find the tangent line and normal line to the graph $y = 2e^{-3x}$ at $(0,2)$

13. Prove that if $y = a^x$, then $\frac{dy}{dx} = a^x \ln a$. (Hint: You must take the ln of both sides. Use of formula sheet formula is not a proof!!)

14. Integrate: $\int \frac{e^{-x}}{1+e^{-x}}\,dx$. Show that the result is equivalent to $x - \ln(e^x+1) + C$.

15. In 1960, the population of a town was 2500, and in 1970 it was 3500. The population increases at a rate proportional to the existing population (i.e., $y = y_0 e^{kt}$). Find the equation of growth, and the population in the year 2000

CALCULUS I EXAM 6E SOLUTIONS

1. $y=\ln\left(x+\sqrt{4+x^2}\right)$

$$\frac{dy}{dx}=\frac{1}{x+\sqrt{4+x^2}}\cdot\left[1+\tfrac{1}{2}(4+x^2)^{-1/2}\cancel{2}x\right]$$

$$=\frac{1}{x+\sqrt{4+x^2}}\cdot\left[1+\frac{x}{\sqrt{4+x^2}}\right]$$

$$=\frac{1}{\cancel{x+\sqrt{4+x^2}}}\cdot\frac{\cancel{\sqrt{4+x^2}+x}}{\sqrt{4+x^2}}$$

$$=\frac{1}{\sqrt{4+x^2}}$$

2. $y=\ln\sqrt{\dfrac{4-x^2}{x}}$

$$y=\tfrac{1}{2}\left[\ln(4-x^2)-\ln x\right]$$

$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{1}{4-x^2}(-2x)-\frac{1}{x}\right]$$

$$=\frac{1}{2}\left[\frac{-2x^2-4+x^2}{x(4-x^2)}\right]$$

$$=\frac{-1}{2}\left(\frac{x^2+4}{x(4-x^2)}\right)$$

Better yet: $\dfrac{x^2+4}{2x(x^2-4)}$

3. $y=x^2e^{-5x}$

$$\frac{dy}{dx}=x^2e^{-5x}(-5)+e^{-5x}2x$$

$$=e^{-5x}(-5x^2+2x)$$

or $xe^{-5x}(2-5x)$

4. $y=x^{\sin 2x}$

$$\ln y=\ln x^{\sin 2x}$$

$$\ln y=(\sin 2x)(\ln x)$$

$$\frac{1}{y}y'=\sin 2x\,\frac{1}{x}+\ln x(\cos 2x)\cdot 2$$

$$y'=x^{\sin 2x}\left(\frac{\sin 2x+2x\ln x\cos 2x}{x}\right)$$

5. $\displaystyle\int\frac{dx}{\sqrt{x}(1+\sqrt{x})}$ let $u=1+\sqrt{x}$, $du=\tfrac{1}{2}x^{-1/2}dx$, $2du=\dfrac{dx}{\sqrt{x}}$

$$=\int\frac{2\,du}{u}$$

$$=2\ln u+C$$

$$2\ln(1+\sqrt{x})+C$$

6. $\displaystyle\int e^{2x}\sqrt{1+e^{2x}}\,dx$ let $u=1+e^{2x}$, $du=2e^{2x}dx$, $\dfrac{du}{2}=e^{2x}dx$

$$=\int u^{1/2}\frac{du}{2}$$

$$=\frac{2u^{3/2}}{3\cdot 2}+C$$

$$=\frac{1}{3}(1+e^{2x})^{3/2}+C$$

7. $\displaystyle\int\frac{2e^x-2e^{-x}}{e^x+e^{-x}}dx$ let $u=e^x+e^{-x}$, $du=(e^x-e^{-x})dx$

$$=\int\frac{2\,du}{u}=2\ln u+C$$

$$=2\ln(e^x+e^{-x})+C$$

8a) $\ln 72=\ln 9\cdot 8$

$$=\ln 3^2+\ln 2^3$$

$$=2\ln 3+3\ln 2$$

$$=2(1.1)+3(.7)$$

$$=2.2+2.1=4.3$$

b) $\ln\sqrt[3]{12}$

$$=\tfrac{1}{3}\ln 2^2 3$$

$$=\tfrac{1}{3}\left[2\ln 2+\ln 3\right]$$

$$=\tfrac{1}{3}\left[1.4+1.1\right]$$

$$=\frac{2.5}{3}=.833$$

9. $F=\displaystyle\int_0^x(t^2-4t+3)\,dt$

$$\frac{dF}{dx}=x^2-4x+3$$

10. $-\ln|\sec x-\tan x|=\ln\dfrac{1}{|\sec x-\tan x|}\dfrac{|\sec x+\tan x|}{|\sec x+\tan x|}$

$$=\ln\frac{|\sec x+\tan x|}{\sec^2x-\tan^2x}$$

$$=\ln\frac{|\sec x+\tan x|}{1}$$

11. $f(x)=\dfrac{-2}{\sqrt{x^2-9}}$ D: $x>3$ Right half; R: $y<0$ (Lower half)

$$y=\frac{-2}{\sqrt{x^2-9}}$$

$$y^2=\frac{4}{x^2-9}$$

$$x^2y^2-9y^2=4$$

$$x^2y^2=4+9y^2$$

$$x^2=\frac{9y^2+4}{y^2}$$

$$x=\pm\frac{\sqrt{9y^2+4}}{y}$$

$$x=-\frac{\sqrt{9y^2+4}}{y}$$ Use the upper left half

$$f^{-1}(x)=-\frac{\sqrt{9x^2+4}}{x}$$

D: $x<0$ (Left only)

R: $f^{-1}>3$

12. $y=2e^{-3x}$, $(0,2)$

$$\frac{dy}{dx}=-6e^{-3x}$$

$m=-6$ y int $=2$.

TANGENT LINE:

$$y=-6x+2$$

NORMAL LINE:

$m=\frac{1}{6}$ y int $=2$.

$$y=\tfrac{1}{6}x+2$$

13. $y=a^x$

$$\ln y=\ln a^x$$

$$\ln y=x\ln a$$

$$\frac{1}{y}\frac{dy}{dx}=\ln a$$

$$\frac{dy}{dx}=y\ln a=a^x\ln a$$

14. $\displaystyle\int\frac{e^{-x}}{1+e^{-x}}dx$ let $u=1+e^{-x}$, $du=-e^{-x}dx$

$$=\int-\frac{du}{u}=-\ln(1+e^{-x})+C$$

$$=\ln\frac{1}{1+e^{-x}}+C$$

$$=\ln\frac{e^x\cdot 1}{e^x[1+e^{-x}]}+C$$

$$=\ln\frac{e^x}{e^x+1}+C=\ln e^x-\ln(e^x+1)+C$$

$$=x-\ln(e^x+1)+C$$

15. $y=y_0e^{kt}$

$$y=2500e^{kt}$$

$$3500=2500e^{10k}$$

$$1.4=\frac{7}{5}=e^{10k}=(e^k)^{10}$$

$$e^k=(1.4)^{1/10}$$

or $k=\frac{1}{10}\ln(1.4)\approx .0336$

$$y=2500e^{\frac{\ln 1.4}{10}t}$$

or $y=2500(e^k)^t$

$$y=2500(1.4)^{t/10}$$

Either way, when $t=40$

$y=2500e^{40\left(\frac{\ln 1.4}{10}\right)}$ OR $y=2500(1.4)^4$

$=9604$ $=9604$

(Use calculator value – DON'T ROUND OFF)