CALCULUS I EXAM 6D (CHAPTER 7) Dr. RAPALJE

1. Evaluate: $\sum_{i=7}^{35}(3i-5)$ Given: $\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$

2. Find the largest and smallest values that the integral $\int_{-2}^{3}\frac{2+x}{\sqrt{1+x^2}}\,dx$ can have. (Remember, do <u>not</u> integrate, but find max and min of $f(x)$ on $[-2,3]$.)

3. Evaluate: $\int_{-2}^{4}(x^3-2)\,dx$

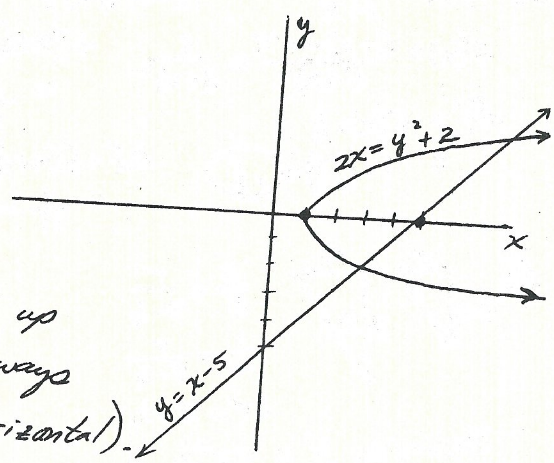
4. Evaluate: $\int_{1}^{3}\frac{t^3+2t-1}{\sqrt[3]{t}}\,dt$

5. Evaluate: $\int_{1}^{5}\frac{1}{x^3}\,dx$

6. Solve: $\int(4x-12)^7\,dx$

7. Evaluate: $\int_{1}^{2}(2x+1)\sqrt{x^2+x+1}\,dx$

8. Solve: $\int x^5\sqrt{x^3+1}\,dx$



9. Given the graph of the region, set up the integral to find the area in two ways using dx (vertical slices) and dy (horizontal). DO <u>NOT</u> SOLVE. (unless you want to!)

10. Find the area bounded by $y=x^2$ and $y=-x^2+4x$

CALC I EXAM 6D (Chap 7) Solutions Dr. RAPALJE

1. $\sum_{i=7}^{35} 3i-5 = \sum_{j=1}^{29} 3(j+6)-5$

$j = i-6$

$i = j+6$

$= \sum_{j=1}^{29} 3j+13$

$= 3\sum_{j=1}^{29} j + \sum_{j=1}^{29} 13$

$= 3\frac{(29)(30)}{2} + 29 \cdot 13$

$= 1305 + 377 = 1682$

2. $f(x) = \frac{2+x}{\sqrt{1+x^2}}$

$f'(x) = \frac{\sqrt{1+x^2} - (2+x)\frac{1}{2}2x(1+x^2)^{-1/2}}{(1+x^2)}$

$= \frac{(1+x^2)^{-1/2}\left[1+x^2-2x-x^2\right]}{1+x^2}$

$= \frac{1-2x}{(1+x^2)^{3/2}} = 0$ at $x = 1/2$

$f(1/2) = \frac{5/2}{\sqrt{5/4}} = \frac{5}{2} \cdot \frac{2}{\sqrt{5}} = \sqrt{5}$ Max

$f(-2) = 0$ Min

$f(3) = \frac{5}{\sqrt{10}} = \frac{\sqrt{10}}{2}$

Max Area $= \boxed{5\sqrt{5}}$ Min Area $= \boxed{0}$

3. $\int_{-2}^{4}(x^3-2)\,dx = \frac{x^4}{4} - 2x\Big|_{-2}^{4}$

$= (64-8)-(4+4)$

$= 64-16 = 48$

4. $\int_1^3 \frac{t^3+2t-1}{\sqrt[3]{t}}\,dt = \int_1^3 \left(t^{8/3} + 2t^{2/3} - t^{-1/3}\right)dt$

$= \frac{3t^{11/3}}{11} + \frac{2\cdot 3t^{5/3}}{5} - \frac{3t^{2/3}}{2}\Big|_1^3$

$= 3t^{2/3}\left(\frac{t^3}{11} + \frac{2t}{5} - \frac{1}{2}\right)\Big|_1^3$

$= 3\sqrt[3]{9}\left(\frac{27}{11} + \frac{6}{5} - \frac{1}{2}\right) - 3\left(\frac{1}{11} + \frac{2}{5} - \frac{1}{2}\right)$

$= \frac{3\sqrt[3]{9}\,(270+132-55) - 3(10+44-55)}{110}$

$= \frac{3}{110}\left(347\sqrt[3]{9} + 1\right)$

or $\frac{1041\sqrt[3]{9}+3}{110}$ or 19.72.

5. $\int_1^5 x^{-3}\,dx = \frac{x^{-2}}{-2}\Big|_1^5$

$= -\frac{1}{2}\left(\frac{1}{25} - 1\right)$

$= -\frac{1}{2}\left(-\frac{24}{25}\right) = \frac{12}{25}$

6. $\int (4x-12)^7\,dx$

$u = 4x-12$

$du = 4\,dx$

$dx = \frac{du}{4}$

$= \int u^7 \frac{du}{4}$

$= \frac{1}{32}u^8 + C$

$= \frac{1}{32}(4x-12)^8 + C.$

$= \frac{1}{32}\left(2^2(x-3)\right)^8 + C$

$= \frac{1}{32} \cdot 2^{16}(x-3)^8 + C$

$= 2^{11}(x-3)^8 + C.$

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7. $\int_1^2 (2x+1)\sqrt{x^2+x+1}\,dx$ let $u=x^2+x+1$, $du=(2x+1)\,dx$

$= \int_3^7 u^{1/2}\,du = \frac{2u^{3/2}}{3}\Big|_3^7$

$= \frac{2}{3}\left(7\sqrt{7}-3\sqrt{3}\right)$

8. $\int x^5\sqrt{x^3+1}\,dx$ $u=x^3+1$, $du=3x^2\,dx$, $x^2\,dx=\frac{du}{3}$, $x^3=u-1$

$= \int (u-1)\,u^{1/2}\,\frac{du}{3}$

$= \frac{1}{3}\int u^{3/2}-u^{1/2}\,du$

$= \frac{1}{3}\left[\frac{2u^{5/2}}{5}-\frac{2u^{3/2}}{3}\right]+C.$

$= \frac{2}{3}\,\frac{3(x^3+1)^{5/2}-5(x^3+1)^{3/2}}{15}$

$= \frac{2}{45}(x^3+1)^{3/2}\left(3(x^3+1)-5\right)$

$= \frac{2(x^3+1)^{3/2}(3x^3-2)}{45}$

9. $2x=y^2+2$ $y=x-5$

$x=y+5$

$2y+10=y^2+2$

$y^2-2y-8=0$

$(y+2)(y-4)=0$

$y=-2$ $y=4$

$x=3$ $x=9$

vertical: $2\int_1^3 \sqrt{2x-2}\,dx+\int_3^9\left(\sqrt{2x-2}-x+5\right)dx$

horiz. $\int_{-2}^4\left(y+5-\frac{y^2+2}{2}\right)dy$

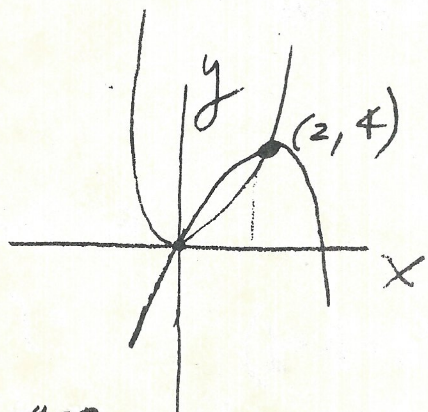
10. $y=x^2$ $y=-x^2+4x$

$x^2=-x^2+4x$

$2x^2-4x=0$

$2x(x-2)=0$

$x=0$ $x=2$



$y=x^2$ Vertex $x=-\frac{b}{2a}=0,\ y=0.$

$y=-x^2+4x$ $x=\frac{-4}{2(-1)}=2$

$\int_0^2(-x^2+4x)-x^2\,dx$

$= \int_0^2 -2x^2+4x\,dx = \frac{-2x^3}{3}+2x^2\Big|_0^2 = -\frac{16}{3}+8 = -\frac{16}{3}+\frac{24}{3}$

$= 8/3$