CALCULUS I EXAM 7D (Chapter 8) Dr. RAPALJE

1. Find the tangent of the angle from $L_1: 2x+3y=5$ to $L_2: 3x-2y=10$

2. Find the distance between the parallel lines $3x-2y=1$ and $3x-2y=10$

3. Find the equation of the circle with center at $(2,3)$ and tangent to $3x+4y+2=0$. (Give form $x^2+y^2+Dx+Ey+F=0$)

4. Find the equation of the tangent and normal lines to $x^2+y^2+8x+2y+16=0$ at the point $P(-4,-2)$

5. Find the focus, vertex, directrix and axis of the parabola $y^2+2x-4y+7=0$. Sketch a graph.

6. Find the equation of the ellipse with vertices at $(0,\pm 6)$ and foci at $(0,\pm 4)$.

7. Given the equation $25x^2-144y^2-900=0$, find the vertices, foci, eccentricity and sketch a graph using the central rectangle.

8. Find the equation of the hyperbola with foci at $(2,-6)$ and $(2,8)$ and eccentricity $7/2$. (Any form will be acceptable)

9. Given the equation $3x^2+4y^2-12x+8y+4=0$, find the vertices, foci, eccentricity. Name and sketch the graph.

10. Given a rotated system where $\sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$,
    a) The vertices of a conic in $(\bar{x},\bar{y})$ are $(\pm 4\sqrt{5}, 0)$. Find the (x,y) coordinates.
    b) Find the equation in x,y of the line $\bar{x}=\sqrt{5}$.

11. Determine the rotated form of the conic $16x^2+24xy+9y^2-30x+40y=0$. Name the conic. $[24^2+7^2=25^2]$

(6 points) (Extra Credit) — Give all pertinent information, (such as vertices, focal points, directrix), in the $\bar{x},\bar{y}$ and x,y systems.

CALCULUS I EXAM 7 Solutions — Dr. RAPALJE

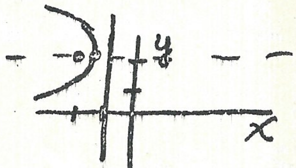
1. $m_1 = -2/3$
$m_2 = 3/2$
Since $m_1 \cdot m_2 = -1$,
lines are $\perp$
$\tan \phi = \infty$ Undefined

2. $3x - 2y = 1$ let $(1,1)$ be point
$3x - 2y - 10 = 0$
$$d = \frac{|3-2-10|}{\sqrt{3^2+(-2)^2}} = \frac{9}{\sqrt{13}}$$

3. $r = \text{dist } (2,3)$ to $3x+4y+2=0$
$$= \frac{|3\cdot 2 + 4\cdot 3 + 2|}{\sqrt{3^2+4^2}} = \frac{20}{5} = 4$$
$(x-2)^2 + (y-3)^2 = 4^2$ or 16
$x^2 + y^2 - 4x - 6y - 3 = 0$

4. $x^2 + y^2 + 8x + 2y + 16 = 0$ at $(-4,-2)$
$2x + 2yy' + 8 + 2y' = 0$
$$y' = -\frac{(x+4)}{y+1} = 0$$
TAN. LINE $y = -2$
NOR. LINE $x = -4$ } By insp.

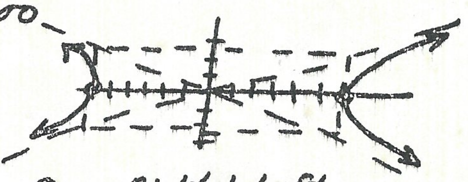
5. $y^2 - 4y = -2x - 7$
$y^2 - 4y + 4 = -2x - 3$
$(y-2)^2 = -2(x + 3/2)$
Parabola, vertex $(-3/2, 2)$
Opens left
$2p = 2$
$p/2 = 1/2$



$F(-2,2)$
Directrix $x = -1$
Axis $y = 2$

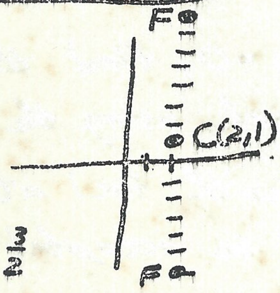
6. $V(0,\pm 6)$ $F(0,\pm 4)$ $C(0,0)$
$a = 6$ $c = 4$
$a^2 = b^2 + c^2$
$36 = b^2 + 16$
$20 = b^2$
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \text{ (major)}$$
$$\frac{x^2}{20} + \frac{y^2}{36} = 1$$
or $9x^2 + 5y^2 = 180$

7. $25x^2 - 144y^2 = 900$
$$\frac{x^2}{36} - \frac{4y^2}{25} = 1$$
$$\frac{x^2}{36} - \frac{y^2}{25/4} = 1$$

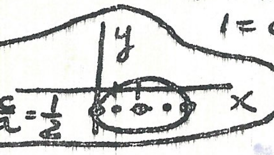


Opens Right + Left.
$a^2 = 36$ $b^2 = 25/4$
$c^2 = a^2 + b^2 = \frac{169}{4}$ } $a = 6$, $b = 5/2$, $c = 13/2$
$V(\pm 6, 0)$ $F(\pm 13/2, 0)$
$$e = \frac{c}{a} = \frac{13/2}{6} = 13/12 = 1.08$$

8. $F(2,-6)$ $F(2,8)$
$C(2,1)$
(Center is halfway)
$2c = 14$, so $c = 7$.
$e = \frac{c}{a} = \frac{3}{2}$ so $\frac{7}{a} = \frac{3}{2}$
$a = \frac{14}{3}$
$c^2 = a^2 + b^2$
$49 = \frac{196}{9} + b^2$ so $b^2 = \frac{245}{9}$
Opens up + down!
$$\frac{(y-1)^2}{a^2} - \frac{(x-2)^2}{b^2} = 1$$
$$\frac{(y-1)^2}{196/9} - \frac{(x-2)^2}{245/9} = 1$$
$$\frac{9(y-1)^2}{196} - \frac{9(x-2)^2}{245} = 1$$



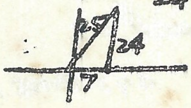
9. $3x^2 - 12x + 4y^2 + 8y = -4$
$3(x^2 - 4x + 4) + 4(y^2 + 2y + 1) = -4 + 12 + 4$
$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{3} = 1 \text{ Ellipse.}$$
$a^2 = b^2 + c^2$
$4 = 3 + c^2$
$1 = c$
Center $(2,-1)$ $a = 2$
$V(0,-1)$ $V(4,-1)$
$F(1,-1)$ $F(3,-1)$ $e = \frac{c}{a} = \frac{1}{2}$



10a) $(\bar{x},\bar{y}) = (\pm 4\sqrt{5}, 0)$ $\sin\theta = \frac{2}{\sqrt{5}}$ $\cos\theta = \frac{1}{\sqrt{5}}$
$x = \bar{x}\cos\theta - \bar{y}\sin\theta$
$x = \pm 4\sqrt{5}\cdot\frac{1}{\sqrt{5}} - 0\cdot\frac{2}{\sqrt{5}}$
$= \pm 4$
$y = \bar{x}\sin\theta + \bar{y}\cos\theta$
$= \pm 4\sqrt{5}\cdot\frac{2}{\sqrt{5}} + 0\cdot\frac{1}{\sqrt{5}}$
$= \pm 8$
$(4,8)$; $(-4,-8)$

b) $\bar{x} = \sqrt{5}$
$\bar{x} = x\cos\theta + y\sin\theta$
$\sqrt{5} = x\cdot\frac{1}{\sqrt{5}} + y\frac{2}{\sqrt{5}}$
$5 = x + 2y$
$x + 2y - 5 = 0$

11. $A = 16$, $B = 24$, $C = 9$, $D = -30$, $E = 40$, $F = 0$ }
$\cot 2\theta = \frac{16-9}{24} = \frac{7}{24}$ $\cos 2\theta = \frac{7}{25}$



$\cos\theta = \sqrt{\frac{1 + 7/25}{2}} = 4/5$ $\sin\theta = \sqrt{\frac{1 - 7/25}{2}} = 3/5$

$\bar{A} = 25$, $\bar{B} = 0$, $\bar{C} = 0$, $\bar{D} = 0$, $\bar{E} = 50$, $\bar{F} = 0$ }
$25\bar{x}^2 + 50\bar{y} = 0$ Parabola
$\bar{x}^2 + 2\bar{y} = 0$ or $\bar{x}^2 = -2\bar{y}$
$V(0,0)$
$F(\bar{x},\bar{y}) = (0, -1/2)$ $F(x,y) = (3/10, -2/5)$
D: $\bar{y} = 1/2$ $6x - 8y + 5 = 0$