CALCULUS I FINAL EXAM FORM E DR. KAPALJE

Show all work on separate paper.

1a) $\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1}$ b) $\lim_{x \to \infty} \frac{2x+5}{3x^2+1}$ c) $\lim_{x \to \infty} \frac{5+4x-2x^2}{3x^2+1}$

2. Use the limit definition (4 step procedure) to find the derivative of $y = \frac{3}{x^2}$.

3. Find $\frac{dy}{dx}$ for $y = (7x-2)^3(2x+5)^4$. Express in factored form.

4. Find $\frac{dy}{dx}$ for $y = \frac{(2x-5)^3}{\sqrt{3x+2}}$. Express in factored form.

5. Find $\frac{dy}{dx}$ for $2x^2 - 3xy + 6y^2 = 4$

6. A lighthouse 2 miles from a shoreline is rotating at 5 revolutions per minute. How fast in miles per hour is the light moving down the shoreline when it is 1 mile down the shoreline?

7. For $y = 3x^4 - 4x^3 + 6$; $y' = 12x^3 - 12x^2$; $y'' = 36x^2 - 24x$, a) find intervals increasing/decreasing b) concave up/down c) rel max d) rel min e) points of inflection f) graph.

8. $\int x\sqrt{x-3}\,dx$

9. $\int_3^7 x\sqrt{x-3}\,dx$

10. $\int \tan^4 5x \sec^2 5x\,dx$

11. Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

12. $\int \frac{(\ln x)^2}{x}\,dx$

13. Find $\frac{dy}{dx}$ for $y = x^2 e^{-4x}$ (Factored form.)

14. Find $\frac{dy}{dx}$ for $y = (\sin 2x)^x$

15. $\int e^{2x}\sqrt{1-e^{2x}}\,dx$

16. A population, now 100, grows to 300 in 5 days. If growth rate is proportional to the population ($y = y_0 e^{kt}$), find the equation of growth and the population after 60 days.

17.

1a) $\lim_{x\to -1} \frac{2x^2-x-3}{x+1} = \lim_{x\to -1}\frac{(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}} = -5$

b) $\lim_{x\to\infty} \frac{\frac{1}{x}(2x+5)}{\frac{1}{x}(3x^2+1)} = \lim_{x\to\infty}\frac{2+\frac{5}{x}}{3x+\frac{1}{x}} = 0$

c) $\lim_{x\to\infty} \frac{\frac{1}{x^2}(5+4x-2x^2)}{\frac{1}{x^2}(3x^2+1)} = \lim_{x\to\infty}\frac{\frac{5}{x^2}+\frac{4}{x}-2}{3+\frac{1}{x^2}} = -\frac{2}{3}$

2. $y=f(x)=\frac{3}{x^2}$ $f(x+\Delta x)=\frac{3}{(x+\Delta x)^2}$

$f(x+\Delta x)-f(x)=\frac{3}{(x+\Delta x)^2}-\frac{3}{x^2}$

$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{3x^2-3(x+\Delta x)^2}{x^2\cdot(x+\Delta x)^2}\cdot\frac{1}{\Delta x}$

$= \frac{\cancel{3x^2}-\cancel{3x^2}-6x\Delta x-3(\Delta x)^2}{x^2\Delta x(x+\Delta x)^2}$

$= \frac{\cancel{\Delta x}(-6x-3\Delta x)}{x^2\cancel{\Delta x}(x+\Delta x)^2}$

$\lim_{\Delta x\to 0} = \frac{-6x}{x^2\cdot x^2} = -\frac{6}{x^3}$

3. $y=(7x-2)^3(2x+5)^4$

$\frac{dy}{dx} = (7x-2)^3 4(2x+5)^3\cdot 2 + (2x+5)^4 3(7x-2)^2\cdot 7$

$=(7x-2)^2(2x+5)^3[8(7x-2)+21(2x+5)]$

$=(7x-2)^2(2x+5)^3(56x-16+42x+105)$

$=(7x-2)^2(2x+5)^3(98x+89)$

4. $y=\frac{(2x-5)^3}{\sqrt{3x+2}}$

$\frac{dy}{dx} = \frac{\sqrt{3x+2}\;3(2x-5)^2\cdot 2-(2x-5)^3\frac{1}{2}(3x+2)^{-1/2}\cdot 3}{3x+2}$

$= \frac{(3x+2)^{-1/2}(2x-5)^2\left[6(3x+2)-\frac{3}{2}(2x-5)\right]}{(3x+2)}$

$= \frac{(2x-5)^2[36x+24-6x+15]}{2(3x+2)^{3/2}}$

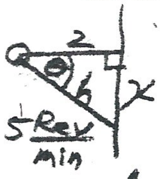
$= \frac{(2x-5)^2(30x+39)}{2(3x+2)^{3/2}}$ or $\frac{3(10x+13)(2x-5)^2}{2(3x+2)^{3/2}}$

5. $2x^2-3xy+6y^2=4$

$4x-3x\frac{dy}{dx}+y(-3)+12y\frac{dy}{dx}=0$

$4x-3y=(3x-12y)\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{4x-3y}{3x-12y}$

6. 

$\tan\theta=\frac{x}{2}$

$x=2\tan\theta$

$\frac{dx}{dt}=2\sec^2\theta\frac{d\theta}{dt}$

$=2\cdot\frac{5}{4}\cdot 10\pi$

$=25\pi$ mi/min.

(Fast light!)

$\frac{d\theta}{dt}=5\frac{Rev}{min}$

$=10\pi\frac{Rad}{min.}$

When $x=1$

$h=\sqrt{2^2+1^2}$

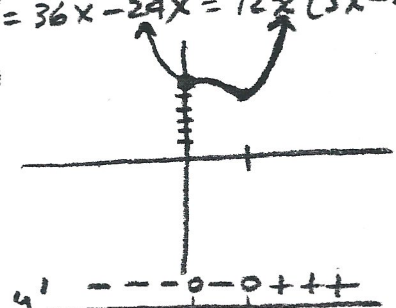
$=\sqrt{5}$

$\sec\theta=\frac{\sqrt{5}}{2}$

7. $y=3x^4-4x^3+6$

$y'=12x^3-12x^2=12x^2(x-1)$

$y''=36x^2-24x=12x(3x-2)$

f) 

a) Increasing: $(1,\infty)$

Decreasing $(-\infty,1)$

b) Concave up: $(-\infty,0)\cup(2/3,\infty)$

down: $(0, 2/3)$

c) Rel max: None

d) Rel Min: $(1,5)$

e) Inflect: $(0,6)$, $(2/3, 5.4)$

8. $\int x\sqrt{x-3}\,dx$ Let $u=x-3$, $du=dx$

$\int(u+3)u^{1/2}du$

$=\int(u^{3/2}+3u^{1/2})du$

$=\frac{2u^{5/2}}{5}+\cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2}$

$=\frac{2}{5}(x-3)^{5/2}+2(x-3)^{3/2}+c$

9. $\int_3^7 x\sqrt{x-3}\,dx$

$=\frac{2}{5}(x-3)^{5/2}+2(x-3)^{3/2}\Big|_3^7$

$=\frac{2}{5}\cdot 4^{5/2}+2\cdot 4^{3/2}$

$=\frac{64}{5}+16=\frac{144}{5}$

10. $\int\tan^4 5x\sec^2 5x\,dx$ Let $u=\tan 5x$, $du=5\sec^2 5x\,dx$

$=\int u^4\frac{du}{5}$

$=\frac{1}{25}u^5+c$

$=\frac{1}{25}\tan^5 5x+c$

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FORM E
Solutions p 2

11. $y = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

$y = \frac{1}{2}\ln(4+x^2) - \ln x$

$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{4+x^2}\cdot 2x - \frac{1}{x}$

$= \frac{x}{4+x^2} - \frac{1}{x}$

$= \frac{x^2-4-x^2}{x(4+x^2)} = \frac{-4}{x(x^2+4)}$

12. $\int \frac{(\ln x)^2}{x}\,dx$ Let $u = \ln x$, $du = \frac{1}{x}dx$

$= \int u^2\,du = \frac{u^3}{3} + C$

$= \frac{1}{3}(\ln x)^3 + C$

13. $y = x^2 e^{-4x}$

$\frac{dy}{dx} = x^2\cdot e^{-4x}(-4) + e^{-4x}2x$

$= e^{-4x}(-4x^2+2x)$

or $2xe^{-4x}(-2x+1)$

14. $y = (\sin 2x)^x$

$\ln y = \ln(\sin 2x)^x$

$\ln y = x\ln(\sin 2x)$

$\frac{1}{y}\frac{dy}{dx} = x\cdot\frac{1}{\sin 2x}\cdot\cos 2x\cdot 2 + \ln(\sin 2x)$

$\frac{dy}{dx} = y\left[2x\cot 2x + \ln(\sin 2x)\right]$

$= (\sin 2x)^x\left(2x\cot 2x + \ln(\sin 2x)\right)$

15. $\int e^{2x}\sqrt{1-e^{2x}}\,dx$

Let $u = 1-e^{2x}$

$du = -2e^{2x}dx$

$\frac{du}{-2} = e^{2x}dx$

$= \int u^{1/2}\frac{du}{-2}$

$= \frac{1}{-2}\frac{2u^{3/2}}{3} + C$

$= -\frac{1}{3}(1-e^{2x})^{3/2} + C$

16. $y = 100e^{kt}$ ⟶ OR ⟶ $y = 100(e^k)^t$

$y = 300$ when $t = 5$, so $300 = 100e^{5k}$

$3 = e^{5k}$ ⟶ OR ⟶ $3 = (e^k)^5$

$\ln 3 = \ln e^{5k}$

$\ln 3 = 5k$

$k = \frac{1}{5}\ln 3$

$y = 100e^{(\frac{1}{5}\ln 3)t}$

$t=60$, $y = 100e^{\frac{1}{5}\ln 3\cdot 60}$

$= 100e^{[(\ln 3)\cdot 12]}$

$=$

$3^{1/5} = e^k$

so $y = 100(3^{1/5})^t$

$y = 100\cdot 3^{\frac{1}{5}t}$

$t=60$; $y = 100\cdot 3^{\frac{60}{5}}$

$y = 100\cdot 3^{12}$

$=$

17. Given $\frac{dy}{dt} = ky$

$\frac{dy}{y} = k\,dt$

$\ln y = kt + C$

$e^{\ln y} = e^{(kt+C)}$

$y = e^{kt}e^{C}$

$t=0$, $y = y_0 = e^C$, so $y = y_0e^{kt}$

$\boxed{\text{NOTE: } (\ln 3)\cdot 12 \neq \ln 36 \quad (\ln 3)\cdot 12 = 12\ln 3 = \ln 3^{12}}$