TERM II, 1996

CALCULUS II EXAM 1 Dr. RAPALJE--SCC at HUNT CLUB

SHOW ALL WORK ON SEPARATE PAPER. Justify and circle all answers.
Where calculators are used, describe window, procedures, etc.
Gold Sheet, Supplement to Gold Sheet, Trig Sheet are allowed.

In 1 - 6, find dy/dx.

1. $y = x^2 e^{x^3}$ (Factor completely!)

2. $y = x^{\sin x}$

3. $y = \ln(x + \sqrt{4 + x^2})$

4. $y = \ln\left(\dfrac{x^2}{\sqrt{4+x^2}}\right)$

5. $y = x \arcsin(x) + \sqrt{1 - x^2}$

6. $y = x \arctan(2x) - \dfrac{1}{4}\ln(1 + 4x^2)$

In 7 - 18 evaluate the integral.

7. $\displaystyle\int \frac{dx}{x \ln x}$

8. $\displaystyle\int \frac{\ln x \, dx}{x}$

9. $\displaystyle\int x^2 e^{x^3}\, dx$

10. $\displaystyle\int \frac{e^{2x}}{4 + e^{2x}}\, dx$

11. $\displaystyle\int \frac{1}{9 + 4x^2}\, dx$

12. $\displaystyle\int \frac{x\, dx}{9 + 4x^2}$

13. $\displaystyle\int \frac{x\, dx}{\sqrt{9 + 4x^2}}$

14. $\displaystyle\int \frac{1}{\sqrt{9 - 4x^2}}\, dx$

15. $\displaystyle\int \frac{\arcsin x}{\sqrt{1 - x^2}}\, dx$

16. $\displaystyle\int \frac{dx}{x^2 + 6x + 25}$ Complete Sq.

17. $\displaystyle\int \frac{x\, dx}{x^2 + 6x + 25}$

18. $\displaystyle\int \frac{x - 1}{\sqrt{4x - x^2}}\, dx$

In 19 - 20, find the decimal value using the calculator.
Then find the exact value using algebraic methods.

19. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin x}\, dx$

20. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x}\, dx$

CALCULUS II Exam 1 Solutions

1. $y = x^2 e^{x^3}$

$y' = x^2 e^{x^3}(3x^2) + e^{x^3}(2x)$

$= x e^{x^3}(3x^3+2)$

2. $y = x^{\sin x}$

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$

$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \ln x \cos x\right)$

3. $y = \ln\left(x+\sqrt{4+x^2}\right)$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{4+x^2}} \cdot \left(1+\frac{1}{2}(4+x^2)^{-1/2} 2x\right)$

$= \frac{1}{x+\sqrt{4+x^2}}\left(1+\frac{x}{\sqrt{4+x^2}}\right)$

$= \frac{1}{x+\sqrt{4+x^2}} \cdot \left(\frac{\sqrt{4+x^2}+x}{\sqrt{4+x^2}}\right)$

$= \frac{1}{\sqrt{4+x^2}}$

4. $y = \ln\left(\frac{x^2}{\sqrt{4+x^2}}\right) = \ln x^2 - \ln\sqrt{4+x^2}$

$y = 2\ln x - \frac{1}{2}\ln(4+x^2)$

$\frac{dy}{dx} = 2 \cdot \frac{1}{x} - \frac{1}{\cancel{2}}\frac{1}{4+x^2} \cdot \cancel{2}x$

$= \frac{2}{x} - \frac{x}{4+x^2} = \frac{8+2x^2-x^2}{x(4+x^2)}$

$= \frac{8+x^2}{x(4+x^2)}$

5. $y = x \arcsin x + \sqrt{1-x^2}$

$= x\frac{1}{\sqrt{1-x^2}} + \arcsin x + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= \cancel{\frac{x}{\sqrt{1-x^2}}} + \arcsin x - \cancel{\frac{x}{\sqrt{1-x^2}}} = \arcsin x$

6. $y = x\arctan(2x) - \frac{1}{4}\ln(1+4x^2)$

$\frac{dy}{dx} = x \cdot \frac{1 \cdot 2}{1+4x^2} + \arctan 2x - \frac{1}{4}\frac{1}{1+4x^2} \cdot 8x$

$= \cancel{\frac{2x}{1+4x^2}} + \arctan 2x - \cancel{\frac{2x}{1+4x^2}} = \arctan 2x$

7. $\int \frac{dx}{x\ln x}$ Let $u = \ln x$

$du = \frac{1}{x}dx$

$\int \frac{du}{u} = \ln u + c$

$= \ln(\ln x) + c$

8. $\int \frac{\ln x\, dx}{x}$ Let $u = \ln x$

$du = \frac{1}{x}dx$

$= \int u\, du$

$= \frac{u^2}{2} + c = \frac{1}{2}(\ln x)^2 + c$

9. $\int x^2 e^{x^3} dx$ Let $u = x^3$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$= \int e^u \frac{du}{3}$

$= \frac{1}{3}e^u + c = \frac{1}{3}e^{x^3} + c$

10. $\int \frac{e^{2x}}{4+e^{2x}}dx$ Let $u = 4+e^{2x}$

$du = 2e^{2x}dx$

$\frac{du}{2} = e^{2x}dx$

$= \int \frac{1}{u}\frac{du}{2}$

$= \frac{1}{2}\ln u + c$

$= \frac{1}{2}\ln(4+e^{2x}) + c$

11. $\int \frac{1}{9+4x^2}dx$ Let $u^2 = 4x^2$ $a^2 = 9$

$u = 2x$ $a = 3$

$du = 2\,dx$

$\int \frac{1}{a^2+u^2}\frac{du}{2}$

$= \frac{1}{2} \cdot \frac{1}{a}\arctan\frac{u}{a} + c$

$= \frac{1}{6}\arctan\frac{2x}{3} + c$

12. $\int \frac{x\,dx}{9+4x^2}$ Let $u = 9+4x^2$

$du = 8x\,dx$

$\frac{du}{8} = x\,dx$

$= \int \frac{1}{u}\frac{du}{8}$

$= \frac{1}{8}\ln u + c = \frac{1}{8}\ln(9+4x^2) + c$

13. $\int \frac{x\,dx}{\sqrt{9+4x^2}}$ Let $u = 9+4x^2$, $du = 8x\,dx$, $\frac{du}{8} = x\,dx$

$= \int u^{-1/2}\frac{du}{8}$

$= \frac{1}{8}\frac{2}{1}u^{1/2} + c = \frac{1}{4}(9+4x^2)^{1/2} + c$ or $\frac{1}{4}\sqrt{9+4x^2} + c$

14. $\int \frac{1}{\sqrt{9-4x^2}}dx = \int \frac{1}{\sqrt{a^2-u^2}}\frac{du}{2}$

$a^2 = 3$, $u^2 = 4x^2$, $u = 2x$, $du = 2dx$, $\frac{du}{2} = dx$

$= \frac{1}{2}\arcsin\frac{u}{a}$

$= \frac{1}{2}\arcsin\frac{2x}{3}$

15. $\int \frac{\arcsin x}{\sqrt{1-x^2}}dx$ Let $u = \arcsin x$, $du = \frac{1}{\sqrt{1-x^2}}dx$

$= \int u\,du$

$= \frac{u^2}{2} + c = \frac{1}{2}(\arcsin x)^2 + c$

16. $\int \frac{dx}{x^2+6x+9+16} = \int \frac{dx}{(x+3)^2+4^2}$

$= \frac{1}{a}\arctan\frac{u}{a} + c$

$= \frac{1}{4}\arctan\left(\frac{x+3}{4}\right) + c$

17. $\int \frac{x\,dx}{x^2+6x+9+16} = \int \frac{x\,dx}{(x+3)^2+16}$

Let $u = x+3$, $x = u-3$, $dx = du$

$= \int \frac{(u-3)\,du}{u^2+16}$

$= \int \left(\frac{u}{u^2+16} - \frac{3}{u^2+16}\right)du$

Let $v = u^2+16$, $dv = 2u\,du$, $\frac{dv}{2} = u\,du$

$= \int \frac{\frac{dv}{2}}{v} - \int \frac{3}{u^2+16}du$

$= \frac{1}{2}\ln v - \frac{3}{4}\arctan\frac{u}{4} + c$

$= \frac{1}{2}\ln(u^2+16) - \frac{3}{4}\arctan\frac{x+3}{4} + c$

$= \frac{1}{2}\ln(x^2+6x+25) - \frac{3}{4}\arctan\frac{x+3}{4} + c$

19. $\int_0^{\pi/2}\frac{\cos x}{1+\sin x}dx$ Let $u = 1+\sin x$, $du = \cos x\,dx$

$\int \frac{du}{u} = \ln u$

$= \ln(1+\sin x)\Big|_0^{\pi/2}$

$= \ln 2 - \ln 1$

$= \ln 2 = .6931$

18. $\int \frac{x-1}{\sqrt{4x-x^2}}dx = \int \frac{x-1}{\sqrt{-(x^2-4x\quad)}}dx$

Let $u = x-2$, $du = dx$, $x = u+2$

$= \int \frac{x-1}{\sqrt{4-(x^2-4x+4)}}dx$

$= \int \frac{u+2-1}{\sqrt{4-u^2}}du$

Let $v = 4-u^2$, $dv = -2u\,du$, $-\frac{dv}{2} = u\,du$

$= \int \frac{u\,du}{\sqrt{4-u^2}} + \int \frac{1}{\sqrt{4-u^2}}du$

$= -\frac{1}{2}\int v^{-1/2}dv + \arcsin\frac{u}{2} + c$

$= -\frac{1}{2}\cdot\frac{2}{1}v^{1/2} + \arcsin\frac{u}{2} + c$

$= -\sqrt{4x-x^2} + \arcsin\left(\frac{x-2}{2}\right) + c$

20. $\int_0^{\pi/2}\frac{\cos x}{1+\sin^2 x}dx$

Let $u = \sin x$, $du = \cos x\,dx$

$= \int \frac{du}{1+u^2}$

$= \arctan u$

$= \arctan \sin x\Big|_0^{\pi/2}$

$= \arctan 1 - \arctan 0$

$= \pi/4 = .7854$