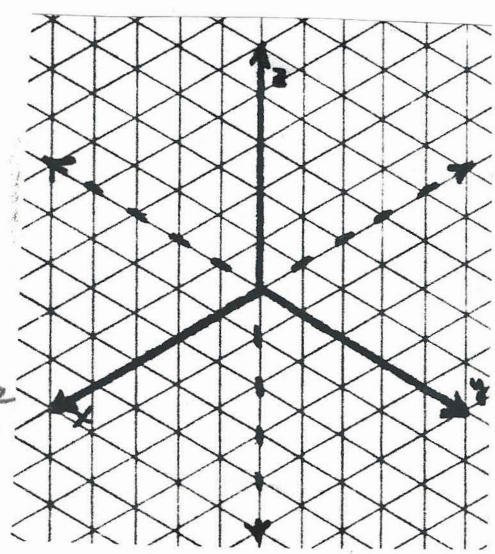
CALCULUS III EXAM 1 A

Show all work on separate paper.
Calculators are permitted.

1. Find a unit vector in the direction of $\langle 2, -3, 4\rangle$.
2. Find the center and radius of the sphere
$$4x^2 + 4y^2 + 4z^2 - 8x - 32y + 8z + 47 = 0$$
3. Determine whether $\vec{u} = -2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{v} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ are parallel, orthogonal, or neither. Justify answer.
4. Find the vector component of $\vec{u} = \langle 0, 4, 1\rangle$ along $\vec{v} = \langle 0, 2, 3\rangle$. Find the vector component of $\vec{u}$ orthogonal to $\vec{v}$.
5. Find the area of $\triangle ABC$ triangle: $A(2,-3,4)$ $B(0,1,2)$ $C(-1,2,0)$
6. Find $\vec{u}\cdot(\vec{v}\times\vec{w})$ for $\vec{u} = \langle 2, 0, 4\rangle$, $\vec{v} = \langle 1, -2, 3\rangle$, $\vec{w} = \langle 3, 2, 1\rangle$
7. Find the equation of a line passing through $(4, -2, 5)$ that is perpendicular to $3x + y - 2z = 8$.
8. Find the equation of the line of intersection of the planes: $3x + 2y - z = 7$ and $x - 4y + 2z = 0$.
9. Find the point of intersection of the line $\frac{x-1}{4} = \frac{y}{2} = \frac{z-3}{6}$ with the plane $2x + 3y - 2z = -8$
10. Find the distance from point $(-1, 2, 3)$ to the line $x = 2 - t$, $y = 1$, $z = 1 + t$.
11. Name and sketch the surface $y = \frac{x^2}{4} + \frac{z^2}{9}$
12. Convert $(-1, \sqrt{3}, -2)$ from rectangular coordinates to cylindrical and spherical.



CALCULUS III Exam 1 Solutions.

1. $\vec{v} = \langle 2, -3, 4 \rangle$

$\|\vec{v}\| = \sqrt{2^2 + (-3)^2 + 4^2}$

$= \sqrt{29}$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\sqrt{29}} \langle 2, -3, 4 \rangle$

2. $4x^2 + 4y^2 + 4z^2 - 8x - 32y + 8z + 47 = 0$

$4(x^2 - 2x + 1) + 4(y^2 - 8y + 16) + 4(z^2 + 2z + 1) = -47 + 7.$

$(x-1)^2 + (y-4)^2 + (z+1)^2 = \dfrac{25}{4}$

$C(1, 4, -1),\ r = 5/2$

3. $\vec{u} = -2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{u} \cdot \vec{v} = -4 + 3 + 1 = 0$

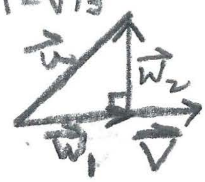
Orthogonal!

4. $\vec{u} = \langle 0, 4, 1 \rangle \quad \vec{u} \cdot \vec{v} = 11$

$\vec{v} = \langle 0, 2, 3 \rangle \quad \|\vec{v}\| = \sqrt{13}$

$\text{Proj}_{\vec{v}} \vec{u} = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} = \vec{w}_1$

a) $= \dfrac{11}{13} \langle 0, 2, 3 \rangle$



b) $\vec{w}_2 = \vec{u} - \vec{w}_1$

$= \langle 0, 4, 1 \rangle$

$- \langle 0, \frac{22}{13}, \frac{33}{13} \rangle$

$= \langle 0, \frac{30}{13}, -\frac{20}{13} \rangle$

5. $A = (2, -3, 4)$

$B = (0, 1, 2)$

$C = (-1, 2, 0)$

$\vec{BA} = \langle 2, -4, 2 \rangle$

$\vec{CA} = \langle 3, -5, 4 \rangle$

$\vec{BA} \times \vec{CA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 2 \\ 3 & -5 & 4 \end{vmatrix}$

$= \hat{i}(-16+10) - \hat{j}(8-6) + \hat{k}(-10+12)$

$= -6\hat{i} - 2\hat{j} + 2\hat{k}$

Area $= \frac{1}{2} \|\vec{BA} \times \vec{CA}\| = \frac{1}{2}\sqrt{36+4+4} = \sqrt{11}$

6. $\vec{u} \cdot (\vec{v} \times \vec{w})$

$= \begin{vmatrix} 2 & 0 & 4 \\ 1 & -2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$

$= 2(-2-6) - 0 + 4(2+6)$

$= 2(-8) + 4(8) = 16$

7. Plane $\perp$ $3x + y - 2z = 8$

through $(4, -2, 5)$

$\dfrac{x-4}{3} = \dfrac{y+2}{1} = \dfrac{z-5}{-2}$

or $x = 3t + 4$

$y = t - 2$

$z = -2t + 5$

8. $2(3x + 2y - z = 7)$

$x - 4y + 2z = 0$

$7x = 14$

$x = 2$

$6 + 2y - z = 7$

$z = 2y - 1$

Let $y = t$:

$x = 2$

$y = t$

$z = 2t - 1$

Answer not unique.

9. $\dfrac{x-1}{4} = \dfrac{y}{2} = \dfrac{z-3}{6}$

$x = 4t + 1$

$y = 2t$

$z = 6t + 3$

$2x + 3y - 2z = -8$

$2(4t+1) + 3(2t) - 2(6t+3) = -8$

$8t + 2 + 6t - 12t - 6 = -8$

$2t = -4$

$t = -2$

$x = -7 \quad y = -4 \quad z = -9$

10. Dist $P(-1, 2, 3)$ to line

Let $Q(2, 1, 1)$ be pt on line.

$\vec{PQ} = \langle 3, -1, -2 \rangle$

$\vec{u} = \langle -1, 0, 1 \rangle$

$\vec{PQ} \times \vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -2 \\ -1 & 0 & 1 \end{vmatrix}$

$= \hat{i}(-1) - \hat{j}(1) + \hat{k}(-1)$

$D = \dfrac{\|\vec{PQ} \times \vec{u}\|}{\|\vec{u}\|}$

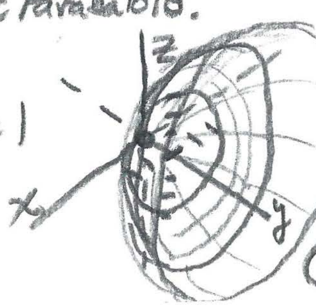
$= \dfrac{\sqrt{1+1+1}}{\sqrt{1+1}} = \sqrt{\dfrac{3}{2}}$

$= \sqrt{1.5}$

11. Elliptic Paraboloid.

$y = 0 \quad (0,0,0)$

$y = 1 \quad \dfrac{x^2}{4} + \dfrac{z^2}{9} = 1$



12. (x, y, z)

$(-1, \sqrt{3}, -2)$

$r = \sqrt{1+3} = 2$

$\theta = 2\pi/3$

$\rho = \sqrt{1+3+4}$

$\rho = 2\sqrt{2}$

$\phi = 5\pi/4$

12a) $(2, 2\pi/3, -2)$

b) $(2\sqrt{2}, 2\pi/3, 3\pi/4)$