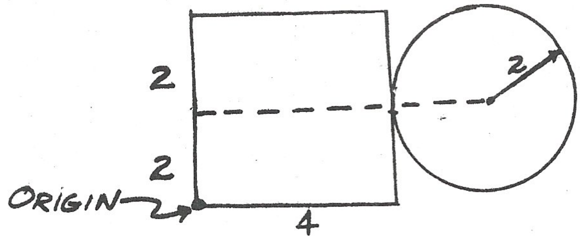
MS 229 EXAM 2

Thought for today: For, behold, I create new heavens and a new earth, and the former shall not be remembered, nor come into mind. Isaiah 65:17.

* = SET UP, SIMPLIFY, BUT DO ~~NOT~~ INTEGRATE.

1. Find the center of mass from the specified origin.



* 2. Find the center of mass of the region bounded by $y = x^3$ and $y = \sqrt{x}$.

* 3. Find the center of mass of the arc $y = \ln|x|$ for $1 \le x \le 3$.

* 4. Find the center of mass of the solid generated by revolving R about the y axis, where R is the region bounded by the $y = 0$ $x = 3$ $y = \frac{x}{\sqrt{4-x}}$.

* 5. The arc $4x^2 + y^2 = 36$ is revolved about the x axis. Find the center of mass of the surface generated if $0 \le x \le 3$ $y \ge 0$.

6. Use the Theorem of Pappus to find the volume generated by R revolved about the x axis where R is the region between $y = x$ and $y = x^3$ $0 \le x \le 1$

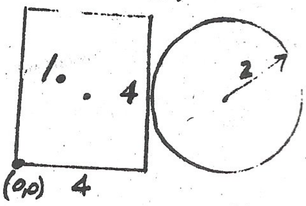
7. A cannon is fired at an angle of 30°. with the horizontal and with an initial velocity of 1000 ft/sec. Determine how long before the cannonball strikes the ground and how far it will go.

8. Use Simpson's rule to approximate the integral
$\int_0^3 x\sqrt{16+x^2}\,dx$ $2n = 6$ 4 Decimals round to 3.
Check by regular integration.

E.C. A tank contains 800 gallons of brine in which 400 lb. of salt are dissolved. Pure water is run in and brine drained out at 20 gal per min. How long before exactly half the salt is gone?

MS 229 Exam 2 Solutions

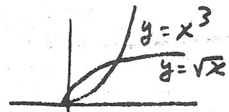
1\.



$$\bar{x} = \frac{2\cdot 16 + 6\cdot 4\pi}{16+4\pi} = \frac{4(8+6\pi)}{4(4+\pi)} = \boxed{\frac{8+6\pi}{4+\pi}}$$

$$\bar{y} = \frac{2\cdot 16 + 2\cdot 4\pi}{16+4\pi} = \boxed{2}$$

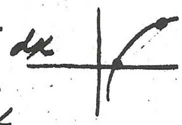
2\.



$$\bar{x} = \frac{\int_0^1 x(\sqrt{x}-x^3)\,dx}{\int_0^1(\sqrt{x}-x^3)\,dx} = \boxed{\frac{\int_0^1 (x^{3/2}-x^4)\,dx}{\int_0^1 (x^{1/2}-x^3)\,dx}}$$

$$\boxed{\bar{y} = \frac{\frac{1}{2}\int_0^1 (x-x^6)\,dx}{\int (x^{1/2}-x^3)\,dx}}$$

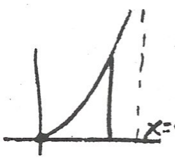
3\. $y=\ln|x| \quad 1\le x\le 3$



$$\bar{x} = \frac{\int_1^3 x\sqrt{1+\frac{1}{x^2}}\,dx}{\int_1^3\sqrt{1+\frac{1}{x^2}}\,dx} = \boxed{\frac{\int_1^3\sqrt{x^2+1}\,dx}{\int_1^3 \frac{1}{x}\sqrt{x^2+1}\,dx}}$$

$$\bar{y} = \frac{\int_1^3 \ln|x|\sqrt{1+\frac{1}{x^2}}\,dx}{\int_1^3\sqrt{1+\frac{1}{x^2}}\,dx} = \boxed{\frac{\int_1^3 \frac{\ln x}{x}\sqrt{x^2+1}\,dx}{\int_1^3 \frac{1}{x}\sqrt{x^2+1}}}$$

4\.



about y axis. $\boxed{\bar{x}=0}$

[Shell method for volume]

$$\bar{y} = \frac{\int y\,dV}{\int dV}$$

$$\bar{y} = \frac{\int x y^2\,dx}{\int x y\,dx} = \frac{\int_0^3 x\frac{x^2}{4-x}\,dx}{\int_0^3 \frac{x^2}{\sqrt{4-x}}\,dx} = \boxed{\frac{\int_0^3 \frac{x^3}{4-x}\,dx}{\int_0^3\frac{x^2}{\sqrt{4-x}}\,dx}}$$

There are other ways, also.

5\. $4x^2+y^2=36$
about x axis
$0\le x\le 3,\ y\ge 0$

$\bar{y}=0$

$$\bar{x} = \frac{\int x f(x)\,ds}{\int f(x)\,ds}$$

$8x+2yy'=0$

$y' = -\frac{4x}{y}$ $\quad (y')^2 = \frac{16x^2}{36-4x^2} = \frac{4x^2}{9-x^2}$

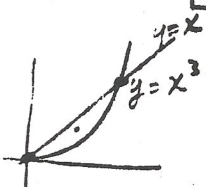
$ds = \sqrt{1+\frac{4x^2}{9-x^2}}\,dx = \sqrt{\frac{9+3x^2}{9-x^2}}\,dx$ $\quad f(x)=\sqrt{36-4x^2} = 2\sqrt{9-x^2}$

$f(x)\,ds = 2\sqrt{9-3x^2}\,dx$

$$= \frac{\int_0^3 x\,2\sqrt{9+3x^2}\,dx}{\int_0^3 2\sqrt{9+3x^2}\,dx} = \frac{\int_0^3 x\sqrt{9+3x^2}\,dx}{\int_0^3\sqrt{9+3x^2}\,dx}$$

or $\boxed{\dfrac{\int_0^3 x\sqrt{3+x^2}\,dx}{\int_0^3\sqrt{3+x^2}\,dx}}$

6\.



$$A = \int_0^1 x-x^3\,dx = \frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1 = \frac{1}{2}-\frac{1}{4} = \frac{1}{4}$$

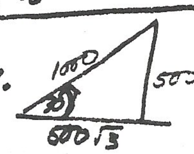
$$\bar{y} = \frac{\frac{1}{2}\int_0^1 x^2-x^6\,dx}{\frac{1}{4}} = \frac{\frac{1}{2}\left[\frac{x^3}{3}-\frac{x^7}{7}\Big|_0^1\right]}{\frac{1}{4}}$$

$$= \frac{\frac{1}{2}\left[\frac{1}{3}-\frac{1}{7}\right]}{\frac{1}{4}} = 2\cdot\frac{7-3}{21} = \frac{8}{21}$$

$$V = \text{Area}\cdot 2\pi\bar{y} = \frac{1}{4}\cdot 2\pi\cdot\frac{8}{21} = \boxed{\frac{4\pi}{21}}$$

E.C. Ans. $t = 40\ln 2 = 27.7$ min.

7\.



$a_y = -32$

$a_x = 0$

$v_x = 500\sqrt{3}$ $\quad v_y = -32t+500$

$x = 500\sqrt{3}\,t$ $\quad y = -16t^2+500t$

$t(-16t+500)$

$x = 500\sqrt{3}\,\frac{125}{4}$

$= \boxed{\frac{62{,}500\sqrt{3}}{4}\text{ ft}}$ $\quad y=0$ at $t=\frac{500}{16} = \boxed{\frac{125}{4}}$

8\. $f(x) = x\sqrt{16+x^2}$

$h = \frac{3-0}{6} = .5$

$f(0) = 0.0000$
$4f(.5) = 8.0623$
$2f(1.0) = 8.2462$
$4f(1.5) = 25.6320$
$2f(2.0) = 17.8885$
$4f(2.5) = 47.1699$
$f(3.0) = 15.0000$

$\frac{1}{6}(121.9989)$
$= 20.33315$
$= \boxed{20.333}$

$\int_0^3 x\sqrt{16+x^2}\,dx$ $\quad u=16+x^2$, $du = 2x\,dx$

$= \int u^{1/2}\frac{du}{2}$

$= \frac{2}{3}\frac{(16+x^2)^{3/2}}{2}\Big|_0^3$

$= \frac{1}{3}\left[(25)^{3/2}-(16)^{3/2}\right] = \frac{1}{3}\cdot 61$

$= \boxed{20.333}$