CALCULUS III EXAM 2A Chapter 13 4/H

Show all work! Attach all worksheets.
Calculators encouraged.

1. Let $\vec{r}(t) = \sin 3t\,\hat{i} + \cos 3t\,\hat{j} + 3t\,\hat{k}$.

a) Find $\|\vec{r}(t)\|$ b) Find $\vec{v}(t)$ c) Find $\|\vec{v}(t)\|$

2. Find $\vec{r}(t)$, given $r'(t) = 2t\,\hat{j} + \sqrt{t}\,\hat{k}$, $r(0) = \hat{i} + \hat{j}$.

3. A projectile is fired from a 100 ft. tower at an angle of 30°. If the initial velocity is 75 ft/sec, find the maximum height and range of the projectile. (Assume $g = 32\ ft/sec^2$)

4. Given $\vec{r}(t) = 4t\,\hat{i} + 3\cos t\,\hat{j} + 3\sin t\,\hat{k}$, find $\vec{T}(t)$, $\vec{N}(t)$, $a_{\vec{T}}$, $a_{\vec{N}}$ at $t = \pi/2$.

5. Find the arclength of the circular helix ("a" and "c" are constant) $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + ct\,\hat{k}$, for $0 \le t \le 2\pi$.

E.C. Use this result to find the length of a copper tube that wraps 3 times around a cylinder with diameter 12 inches and length 100 inches.

# CALCULUS III EXAM 2A Solutions

1. $\vec{r}(t) = \sin 3t\,\hat{i} + \cos 3t\,\hat{j} + 3t\,\hat{k}$

a) $\|\vec{r}(t)\| = \sqrt{\sin^2 3t + \cos^2 3t + 9t^2}$

$= \sqrt{1+9t^2}$

b) $\vec{v}(t) = 3\cos 3t\,\hat{i} - 3\sin 3t\,\hat{j} + 3\hat{k}$

c) $\|\vec{v}(t)\| = \sqrt{9\cos^2 3t + 9\sin^2 3t + 9}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

2. $\vec{r}\,'(t) = 2t\,\hat{j} + \sqrt{t}\,\hat{k}$ $\quad \vec{r}(0) = \hat{i} + \hat{j}$

$\vec{r}(t) = c_1\hat{i} + (t^2 + c_2)\hat{j} + \left(\frac{2}{3}t^{3/2} + c_3\right)\hat{k}$

$0 + c_2 = 1$ $\quad c_2 = 1$

$0 + c_3 = 0$ $\quad c_3 = 0$

$\vec{r}(t) = \hat{i} + (t^2+1)\hat{j} + \frac{2}{3}t^{3/2}\hat{k}$

3. $\vec{r}(t) = (v_0\cos\theta)t\,\hat{i} + \left[h + (v_0\sin\theta)t - \frac{1}{2}gt^2\right]\hat{j}$

$= 75\frac{\sqrt{3}}{2}t\,\hat{i} + \left(100 + 75\cdot\frac{1}{2}t - \frac{1}{2}\cdot 32t^2\right)\hat{j}$

Max height when $\frac{dy}{dt} = 0$. $\quad y = 100 + \frac{75}{2}t - 16t^2$

$\frac{dy}{dt} = \frac{75}{2} - 32t = 0$

$t = \frac{75}{64} = 1.17$ sec.

$y = 100 + \frac{75}{2}\left(\frac{75}{64}\right) - 16\left(\frac{75}{64}\right)^2$

$= 121.97$ ft.

Hits ground when $y = 0$.

$100 + \frac{75}{2}t - 16t^2 = 0$

$32t^2 - 75t - 200 = 0$

$t = \frac{75 \pm \sqrt{75^2 + 4(32)200}}{64}$

$= 3.93$ sec.

$x = \frac{75\sqrt{3}}{2}(3.93)$

$= 255.45$ ft.

4. $\vec{r}(t) = 4t\,\hat{i} + 3\cos t\,\hat{j} + 3\sin t\,\hat{k}$

$\vec{r}\,'(t) = 4\hat{i} - 3\sin t\,\hat{j} + 3\cos t\,\hat{k}$

$\|\vec{r}\,'(t)\| = \sqrt{16 + 9\sin^2 t + 9\cos^2 t} = 5$

$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{4}{5}\hat{i} - \frac{3}{5}\sin t\,\hat{j} + \frac{3}{5}\cos t\,\hat{k}$

$\vec{T}\,'(t) = -\frac{3}{5}\cos t\,\hat{j} - \frac{3}{5}\sin t\,\hat{k}$

$\|\vec{T}\,'(t)\| = \sqrt{\frac{9}{25}\cos^2 t + \frac{9}{25}\sin^2 t} = \frac{3}{5}$

$\vec{N}(t) = \frac{\vec{T}\,'(t)}{\|\vec{T}\,'(t)\|} = -\cos t\,\hat{j} - \sin t\,\hat{k}$

$\vec{T}(\pi/2) = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$ $\quad \vec{N}(\pi/2) = -\hat{k}$

$\vec{a}(t) = -3\cos t\,\hat{j} - 3\sin t\,\hat{k}$ $\quad \vec{a}(\pi/2) = -3\hat{k}$

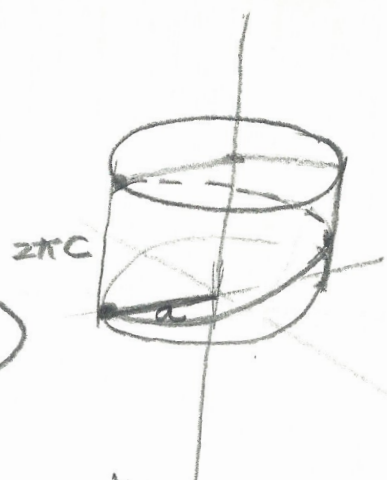
$a_{\vec{T}} = \vec{a}\cdot\vec{T} = 0$ $\quad a_{\vec{N}} = \vec{a}\cdot\vec{N} = 3$

$\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + ct\,\hat{k}$

$\vec{r}'(t) = -a\sin t\,\hat{i} + a\cos t\,\hat{j} + c\,\hat{k}$

$$s = \int_0^{2\pi} \sqrt{a^2\sin^2 t + a^2\cos^2 t + c^2}\,dt.$$

$$= \int_0^{2\pi} \sqrt{a^2 + c^2}\,dt = \boxed{2\pi\sqrt{a^2+c^2}}$$




E.C.

$$s = 3 \cdot 2\pi\sqrt{6^2 + \left(\frac{100}{6\pi}\right)^2}$$

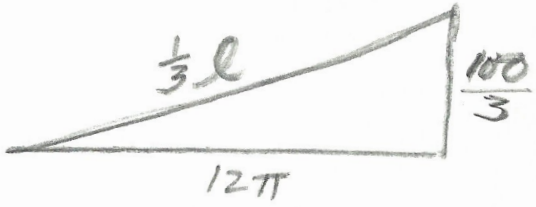
$$= 150.97''$$

radius $= a. = 6$

height $= 2\pi c = \frac{100}{3}$

$c = \frac{100}{6\pi}$

OR




$$\frac{1}{3}\ell = \sqrt{(12\pi)^2 + \left(\frac{100}{3}\right)^2}$$

$$\ell = 3\sqrt{144\pi^2 + \frac{10000}{9}}$$

$$= \sqrt{9(144)\pi^2 + 10{,}000}$$

$$= 150.97''$$