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CALCULUS III EXAM 4A 15.1 – 15.5 L + H (4th Ed.)

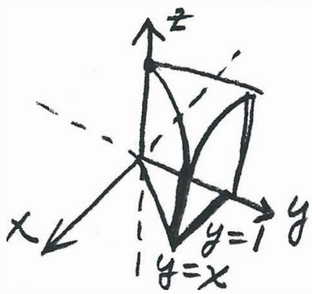
Show all work on separate paper.

Sketch all necessary regions.

1. Reverse the order of integration and evaluate

$$\int_0^1 \int_x^1 \cos y^2 \, dy \, dx$$

2. Find the volume of the solid shown in the figure, (Use double integral), bounded by $z = 1 - xy$, $y = x$, $y = 1$, and the xy plane.



3. Evaluate by changing to polar coordinates:

$$\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx$$

4. Use a double integral to find the mass of the region bounded by $y = \ln x$, $y = 0$, $x = 1$, $x = e$, where the density is given by $\rho = \frac{k}{x}$. SET UP ONLY to find the center of mass $(\bar{x}, \bar{y})$.

5. Find the surface area of the portion of the surface $z = xy$ that is inside $x^2 + y^2 = 1$. (Graph is not necessary!)

$$\bar{x} = \frac{\iint x\rho \, dA}{\iint \rho \, dA} \qquad \bar{y} = \frac{\iint y\rho \, dA}{\iint \rho \, dA}$$

Surface Area $= \iint \sqrt{1 + [f_x]^2 + [f_y]^2} \, dA$

$$\iint dA = \iint r \, dr \, d\theta.$$

CALCULUS III EXAM 4A Solutions

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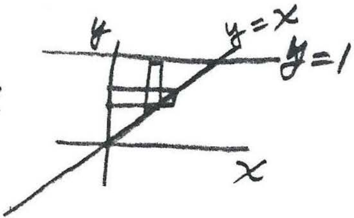
1. $\int_0^1\int_x^1 \cos y^2\,dy\,dx$



$= \int_0^1\int_0^y \cos y^2\,dx\,dy.$

$= \int_0^1 (\cos y^2)\,x\Big|_0^y\,dy$

$= \int_0^1 y\cos y^2\,dy$ let $u=y^2$, $du=2y\,dy$, $\frac{du}{2}=y\,dy.$

$= \int_0^1 \cos u\,\frac{du}{2}$

$= \frac{1}{2}\sin u\Big|_0^1 = \frac{1}{2}\sin 1$

2. $V=\iint f(x,y)\,dA.$

$= \int_0^1\int_x^1 (1-xy)\,dy\,dx$

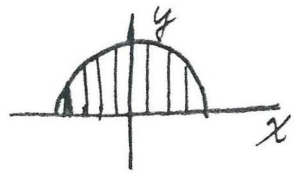
or $\int_0^1\int_0^y (1-xy)\,dx\,dy$

$= \int_0^1 \left[x-\frac{yx^2}{2}\Big|_0^y\right]dy$

$= \int_0^1 \left(y-\frac{y^3}{2}\right)dy$

$= \frac{y^2}{2}-\frac{y^4}{8}\Big|_0^1 = \frac{1}{2}-\frac{1}{8} = \frac{3}{8}$

3. $\int_{-1}^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx$



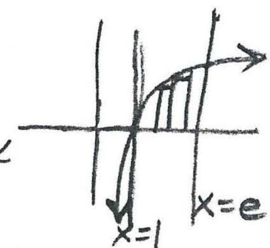
$= \int_0^\pi\int_0^1 e^{r^2}\,r\,dr\,d\theta$ $u=r^2$, $du=2r\,dr$

$= \int_0^\pi\int_0^1 e^u\,\frac{du}{2}\,d\theta$

$= \int_0^\pi \frac{1}{2}e^u\Big|_0^1\,d\theta = \int_0^\pi \frac{1}{2}(e-1)\,d\theta$

$= \frac{\pi}{2}(e-1)$

4. $m=\iint \rho\,dA$



$= \int_1^e\int_0^{\ln x} \frac{k}{x}\,dy\,dx$

$= \int_1^e \frac{k}{x}\ln x\,dx$ let $u=\ln x$, $du=\frac{1}{x}dx$

$= \int_0^1 k\,u\,du$ if $x=1$, $u=0$; $x=e$, $u=1.$

$= k\,\frac{u^2}{2}\Big|_0^1 = \frac{k}{2}$

or $m=\int_0^1\int_{e^y}^e \frac{k}{x}\,dx\,dy$, since $y=\ln x$ means $x=e^y$

$= \int_0^1 k\ln x\Big|_{e^y}^e\,dy$

$= \int_0^1 k(\ln e-\ln e^y)\,dy$

$= \int_0^1 k(1-y)\,dy = k\left(y-\frac{y^2}{2}\right)\Big|_0^1 = \frac{k}{2}$

$\bar{x} = \dfrac{\int_1^e\int_0^{\ln x} k\,dy\,dx}{k/2}$ $\quad \bar{y} = \dfrac{\int_1^e\int_0^{\ln x} \frac{ky}{x}\,dy\,dx}{k/2}$

5. $z=xy$ $\quad (x^2+y^2=1$ unit circle!)

$z_x=y \quad z_y=x$

S.A. $= \iint \sqrt{1+x^2+y^2}\,dA$

$= \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\,r\,dr\,d\theta$ let $u=1+r^2$, $du=2r\,dr$, $\frac{du}{2}=r\,dr$

$= \int_0^{2\pi}\int_1^2 u^{1/2}\,\frac{du}{2}\,d\theta$

$= \int_0^{2\pi} \frac{1}{2}\,\frac{2}{3}u^{3/2}\Big|_1^2\,d\theta$

$= \frac{1}{3}\int_0^{2\pi} (2^{3/2}-1)\,d\theta = \frac{2\pi}{3}(2\sqrt{2}-1)$