MS 229 Exam 5B   Chap 18: 1–12   Dr. Rapalje

1. Find the total differential (dz) of $z = x^3y + x^2y^2 + xy^3$

2. Find dy/dx by method of partial derivatives for $e^x \sin y + e^y \sin x = 1$

3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $x^2 + 3xy - 2y^2 + 3xz + z^2 = 0$.

4. If $z = x\cos y - y\cos x$, find $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial x \partial y}$.

5. Test for relative max, min, and saddle points: $f(x,y) = x^3 - 6xy + y^3$

6. Show that each of the following is exact, and find the function $f(x,y)$ of which it is the total differential:

   a) $(2x^3 + 3y)\,dx + (3x + y - 1)\,dy$

   b) $(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy$

7. Show that the integrand of $\int_C (2xy + z^3)\,dx + x^2\,dy + 3xz^2\,dz$ is an exact differential, then evaluate the integral along any path from $(1,-2,1)$ to $(3,1,4)$.

8. Evaluate $\int_C (x+y)\,dx + (2x-y)\,dy$ where C is the path from

   a) $(0,0)$ to $(2,0)$ and then from $(2,0)$ to $(2,1)$,

   b) $(0,0)$ to $(2,1)$ directly.

Thought for today: And he said unto me: My grace is sufficient for you; for my strength is made perfect in weakness.

2 Cor 12:9

MS 229 EXAM 5 solutions.

1. $z = x^3y + x^2y^2 + xy^3$

$\frac{\partial z}{\partial x} = 3x^2y + 2xy^2 + y^3$

$\frac{\partial z}{\partial y} = x^3 + 2x^2y + 3xy^2$

$dz = (3x^2y + 2xy^2 + y^3)\,dx + (x^3 + 2x^2y + 3xy^2)\,dy.$

2. $e^x \sin y + e^y \sin x = 1$

$\frac{dy}{dx} = \frac{-F_x}{F_y} = -\frac{e^x \sin y + e^y \cos x}{e^x \cos y + e^y \sin x}$

3. $x^2 + 3xy - 2y^2 + 3xz + z^2 = 0$

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x+3y+3z}{3x+2z}$

$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{3x-4y}{3x+2z}$

4. $z = x\cos y - y\cos x$

$\frac{\partial z}{\partial x} = \cos y + y\sin x$

$\frac{\partial^2 z}{\partial x^2} = y\cos x$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = -\sin y + \sin x$

$\frac{\partial z}{\partial y} = -x\sin y - \cos x$

$\frac{\partial^2 z}{\partial y^2} = -x\cos y$

5. $f(x,y) = x^3 - 6xy + y^3$

$f_1 = 3x^2 - 6y = 0; \quad f_2 = -6x + 3y^2 = 0$

$y = \frac{1}{2}x^2 \qquad -2x + \frac{1}{4}x^4 = 0$

$x(x^3 - 8) = 0$

$x = 0 \quad x = 2$

$y = 0 \quad y = 2$

$f_{1,1} = 6x = A$

$f_{1,2} = -6 = B$

$f_{2,2} = 6y = C$

At $(0,0)$ $AC - B^2 = -36 < 0$ saddle point.

At $(2,2)$ $AC - B^2 = 108 > 0$, $A > 0$ minimum.

6. a) $(2x^3 + 3y)\,dx + (3x + y - 1)\,dy$

$\frac{\partial P}{\partial y} = 3 \qquad \frac{\partial Q}{\partial x} = 3$ Exact

$f(x,y) = \frac{x^4}{2} + 3xy + \frac{y^2}{2} - y + c$

b) $(y^2 e^{xy^2} + 4x^3)\,dx + (2xy e^{xy^2} - 3y^2)\,dy$

$\frac{\partial P}{\partial y} = 2y e^{xy^2} + y^2 e^{xy^2}(2xy)$ Exact

$\frac{\partial Q}{\partial x} = 2y e^{xy^2} + 2xy e^{xy^2}(y^2)$

$f(x,y) = e^{xy^2} + x^4 - y^3 + c$

7. $\int (2xy + z^3)\,dx + x^2\,dy + 3xz^2\,dz$

$\frac{\partial P}{\partial y} = 2x = \frac{\partial Q}{\partial x}$

$\frac{\partial P}{\partial z} = 3z^2 = \frac{\partial R}{\partial x}$

$\frac{\partial Q}{\partial z} = 0 = \frac{\partial R}{\partial y}$

$\int_C = x^2y + z^3x \Big|_{(1,-2,1)}^{(3,1,4)}$

$= 9 + 64\cdot 3 - [-2 + 1]$

$= 192 + 10 = 202$

8 a) $\int (x+y)\,dx + (2x-y)\,dy \qquad C_1: (0,0)$ to $(2,0)$: $y = 0$, $dy = 0$

$= \int_{C_1} + \int_{C_2} \qquad C_2: (2,0)$ to $(2,1)$: $x = 2$, $dx = 0$

$= \int_0^2 x\,dx + 0 + \int_0^1 (4-y)\,dy$

$= \frac{x^2}{2}\Big|_0^2 + 4y - \frac{y^2}{2}\Big|_0^1 = 2 + 4 - \frac{1}{2} = 11/2$

b) $\int_{C_3} = \int_0^1 (2y+y)\,2\,dy + (4y-y)\,dy \qquad C_3: (0,0)$ to $(2,1)$: $y = \frac{1}{2}x$, $2y = x$, $2\,dy = dx$

$= \int_0^1 9y\,dy = \frac{9y^2}{2} = 9/2$