MS 235 Differential Equations   Exam 1A   Dr. Rapalje

1. How many essential arbitrary constants has
(12 points) $c_1 \ln(c_2 x^{c_3}) + c_4 \ln c_5 x + c_6$? (Show simplification)

2. Give the order and degree of each of the following:
   a) $dy + (xy - \cos x)\,dx = 0$
   b) $\frac{d^2T}{dx^2} + \frac{dT}{dx} + T = 0$
   c) $y''' + xy'' + 2y(y')^2 + xy = 0$
(24 points) d) $\sqrt{r' + r^{-}} = \cos\theta$
   e) $y' + x = (y - xy')^{-3}$
   f) $\frac{d^2r}{d\theta^2} = \sqrt[3]{r + \left(\frac{dr}{d\theta}\right)^{[illegible]}}$

Solve the following differential equations, give form of
(12 each) answer without negative exponents or fractions:

3. $(2xy + 3x^2)\,dx + x^2\,dy = 0$

4. $y^2\,dx - x^3\,dy = 0$

5. $y\,dx + (3 + 3x - y)\,dy = 0$

6. $y' = 1 + y/x$

7. $yy'' = (y')^2$

(10 points) E.C. Solve by homogeneous method: $\frac{dy}{dx} = \frac{yx^2}{x^3 - y^3}$

Thought for today: I can do ALL things through Christ, who strengthens me. (Phil. 4:13)

MS 235 Exam 1A Solutions

Dr. Rapalje

1. $c_1 \ln(c_2 x^{c_3}) + c_4 \ln c_5 x + c_6$

$= c_1 \ln c_2 + c_1 \ln x^{c_3} + c_4 \ln c_5 + c_4 \ln x + c_6$

$= c_7 + c_8 \ln x + c_9 + c_4 \ln x + c_6$

$= c_{10} + c_{11} \ln x$. Two essential arb. const.

2a) $\frac{dy}{dx} = \cos x - xy$. Order 1, Degree 1

b) $\frac{d^2T}{dx^2} + \frac{dT}{dx} + T = 0$ Order 2, Degree 1

c) $y''' + xy' + 2y(y')^2 + xy = 0$ Order 3, Degree 1.

d) $\sqrt{r' + r} = \cos\theta$

$r' + r = \cos^2\theta$ Order 1, Degree 1.

e) $y' + x = (y - xy')^{-3}$

$(y' + x)(y - xy')^3 = 1$ Order 1, Degree 4.

$(y')^4 + \ldots = 1$

f) $\frac{d^2r}{d\theta^2} = \sqrt[3]{r + \left(\frac{dr}{d\theta}\right)^2}$ order 2, Degree 3

$\left(\frac{d^2r}{d\theta^2}\right)^3 = r + \left(\frac{dr}{d\theta}\right)^2$

3. $(2xy + 3x^2)dx + x^2 dy = 0$

$\frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2x$ Exact.

$U = \boxed{x^2 y + x^3 = c}$

4. $y^2 dx - x^3 dy = 0$ Variables separable.

$y^2 dx = x^3 dy$

$x^{-3} dx = y^{-2} dy$

$\frac{x^{-2}}{-2} = \frac{y^{-1}}{-1} + c_1$

$-\frac{1}{2x^2} = -\frac{1}{y} + c_1$

$\boxed{y = 2x^2 + cx^2 y}$

5. $y\,dx + (3 + 3x - y)dy = 0$ Integrating factor

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 3$

$\frac{\text{II} - \text{I}}{M} = \frac{3 - 1}{y} = \frac{2}{y}$

I.F. $= e^{\int \frac{2}{y} dy} = y^2$

$y^3 dx + (3y^2 + 3xy^2 - y^3)dy = 0$

$\frac{\partial M}{\partial y} = 3y^2 \qquad \frac{\partial N}{\partial x} = 3y^2$

$U = xy^3 + y^3 - \frac{y^4}{4} = c_1$

$\boxed{4xy^3 + 4y^3 - y^4 = c}$

6. $y' = 1 + y/x$ Homogeneous or Linear

$y' - \frac{1}{x} y = 1$

$P(x) = -\frac{1}{x} \quad Q(x) = 1$

I.F. $= e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$

$y \cdot \frac{1}{x} = \int 1 \cdot \frac{1}{x} dx$

$= \ln x + c$

$\boxed{y = x \ln x + cx}$

7. $yy'' = (y')^2$ Let $y' = v$

$y'' = v'$

$yv' = v^2$

$\frac{dv}{dx} = \frac{dv}{dy}\frac{dy}{dx} = \frac{dv}{dy} \cdot v$

$y \frac{dv}{dy} v = v^2$

$\frac{dv}{v} = \frac{dy}{y}$

$\ln v = \ln y + \ln c_1$

$v = c_1 y$

$y' = c_1 y$

$\frac{dy}{dx} = c_1 y$

$\frac{dy}{y} = c_1 dx$

$\ln y = c_1 x + c_2$

$y = e^{c_1 x + c_2}$

$y = e^{c_1 x} \cdot e^{c_2}$

$\boxed{y = c_3 e^{c_1 x}}$

E.C. $\frac{dy}{dx} = \frac{yx}{x^3 - y^3}$

let $y = vx$

$\frac{dy}{dx} = v + xv'$

$v = y/x$

$v^{-3} = x^3/y^3$

$v + xv' = \frac{vx^3}{x^3 - v^3x^3}$

$v + xv' = \frac{v}{1 - v^3}$

$xv' = \frac{v}{1-v^3} - \frac{v(1-v^3)}{(1-v^3)}$

$xv' = \frac{v - v + v^4}{1 - v^3}$

$x\frac{dv}{dx} = \frac{v^4}{1-v^3}$

$\frac{(1-v^3)dv}{v^4} = \frac{dx}{x}$

$\left(v^{-4} - \frac{1}{v}\right)dv = \frac{dx}{x}$

$\frac{v^{-3}}{-3} - \ln v = \ln x + \ln c$

$-\frac{1}{3}\frac{x^3}{y^3} = \ln\frac{y}{x} + \ln x + \ln c$

$-\frac{1}{3}\frac{x^3}{y^3} = \ln y \cancel{- \ln x} \cancel{+ \ln x} + \ln c$

$-x^3 = 3y^3 \ln cy$

$\boxed{3y^3 \ln cy + x^3 = 0}$

or $\underline{3y^3 \ln c} + 3y^3 \ln y + x^3 = 0$

or $3y^3 C_1 = 3y^3 \ln y + x^3$

or $x^3 + 3y^3 \ln y = C_2 y^3$