MS 235 EXAM 2 A OPEN BOOK Dr. Rapalje

1. Find the orthogonal trajectories of the family $xy = C$. Graph the given and the orthogonal family.

2. Verify that the length of the normal line from P to the y axis is $\left|\frac{x\sqrt{1+(y')^2}}{y'}\right|^2$. (See p. 126–127).

3. An RL circuit has an emf of 5 volts, a resistance of 50 ohms, an inductance of 1 henry. If current is zero at $t=0$, find the current at any time t.

4. A 50 gallon tank initially contains 10 gallons of fresh water. At $t=0$ a brine solution containing 1 lb. salt per gallon begins to enter the tank at 4 gal per min, while the well stirred mixture leaves the tank at 2 gal per min. Find a) the time required for overflow to occur, and b) the amt salt in the tank at the moment of overflow.

5. A body moves in a straight line so that its velocity is numerically three more than twice the distance from a fixed point on the line. Find the equation of motion.

6. A cantilever beam of length L has a uniform weight of w lbs. per ft, and a concentrated load of 5 pounds at the free end. Find the equation of motion and the maximum displacement.

MS 235 EXAM 2A Solutions **Dr. Rapalje**

1. $xy = c$

$xy' + y = 0$

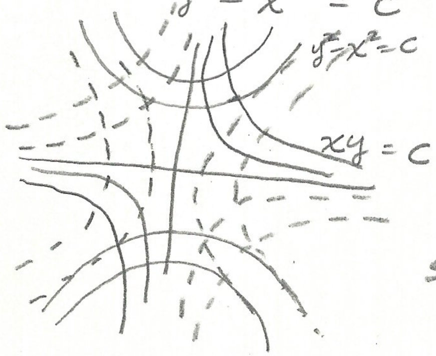
$y' = -y/x$

orthogonal $y' = \frac{x}{y}$

$\frac{dy}{dx} = \frac{x}{y}$

$y\,dy - x\,dx = 0$

$y^2 - x^2 = c$



Dotted graph $c<0$

Solid graph $c>0$

2. $y - y_0 = y'(x - x_0)$ = Eq. Tangent line.

$y - y_0 = -\frac{1}{y'}(x - x_0)$ = Eq Normal line.

y intercept $(0, Y)$: $\quad y - Y = -\frac{1}{y'}x$

$$Y = y + \frac{x}{y'}$$

Distance from $(0, Y)$ to (x, y)

$$= \sqrt{x^2 + (Y - y)^2}$$

$$= \sqrt{x^2 + \left(y + \frac{x}{y'} - y\right)^2} = \sqrt{x^2 + \frac{x^2}{(y')^2}}$$

$$= \frac{|x|\sqrt{(y')^2 + 1}}{|y'|}$$

3. $L\frac{dI}{dt} + RI = E$

$1\frac{dI}{dt} + 50I = 5$

Linear I.F. $e^{\int 50\,dt} = e^{50t}$

$Ie^{50t} = \int 5e^{50t}\,dt$

$Ie^{50t} = \frac{1}{10}e^{50t} + c$

$I = 0$ at $t = 0$, so

$0 = \frac{1}{10} + c, \quad c = -\frac{1}{10}$

$Ie^{50t} = \frac{1}{10}e^{50t} - \frac{1}{10}$

$I = \frac{1}{10} - \frac{1}{10}e^{-50t}$

4. Tank requires 40 gallons to fill, gains 2 gal/min.
So time required to fill tank is 20 min.
Let A = amt salt at time t.

$\frac{1\text{ lb}}{\text{gal}} \cdot \frac{4\text{ gal}}{\text{min}} = 4$ lb/min = rate entering.

$\frac{A\text{ lb.}}{10 + 2t} \cdot \frac{2\text{ gal}}{\text{gal. min}}$ = rate leaving.

$\frac{dA}{dt} = 4 - \frac{A}{5+t}$

$\frac{dA}{dt} + \frac{1}{5+t}A = 4$ Linear.

I.F. $e^{\int \frac{1}{5+t}dt} = 5 + t$

$A(5+t) = \int (5+t)4\,dt$

$A(5+t) = 20t + 2t^2 + c$

At $t = 0$, $A = 0$, so $c = 0$.

At $t = 20$, $25A = 400 + 800$

$A = \frac{1200}{25} = \boxed{48 \text{ min.}}$

5. $V = 2X + 3$

$\frac{dx}{dt} = 2X + 3$

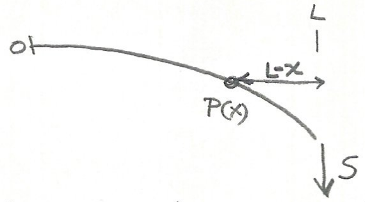
$\frac{dx}{2X+3} = dt$

$\frac{1}{2}\ln|2X + 3| = t + c.$

$2X + 3 = Ae^{2t}$

$X = A_1e^{2t} - 3/2$

6.



$$m(x) = S(L-x) + w(L-x)\frac{(L-x)}{2}$$

$$EI\,y'' = SL - Sx + \frac{wL^2}{2} - wxL + \frac{wx^2}{2}$$

$$EI\,y' = SLx - \frac{Sx^2}{2} + \frac{wL^2x}{2} - \frac{wx^2L}{2} + \frac{wx^3}{6} + C_1$$

$y'(0) = 0$ so $C_1 = 0$.

$$EI\,y = \frac{SLx^2}{2} - \frac{Sx^3}{6} + \frac{wL^2x^2}{4} - \frac{wx^3L}{6} + \frac{wx^4}{24} + C_2$$

$y(0) = 0$ so $C_2 = 0$.

$$y = \frac{S}{6EI}\left(3Lx^2 - x^3\right) + \frac{W}{24EI}\left(6L^2x^2 - 4x^3L + x^4\right)$$

y_{max} is at $x = L$

$$y_{max} = \frac{S}{6EI}\left(2L^3\right) + \frac{W}{24EI}\left(3L^4\right)$$

$$= \frac{1}{EI}\left(\frac{SL^3}{3} + \frac{WL^4}{8}\right) \text{ or } \frac{8SL^3 + 3WL^4}{24EI}.$$

E.C. $m = 10$ slugs

$V_0 = 20$ ft/sec.

$R = -8V^2$

$ma = mg - 8V^2$

$$10\frac{dV}{dt} = 320 - 8V^2$$

$$\frac{dV}{40 - V^2} = \frac{8\,dt}{10}$$

$2\sqrt{10}$

$$\frac{1}{2\cdot 2\sqrt{10}} \log\frac{2\sqrt{10}+V}{2\sqrt{10}-V} = \frac{4}{5}t + C$$

$$\log\frac{2\sqrt{10}+V}{2\sqrt{10}-V} = \frac{16\sqrt{10}}{5}t + C_1$$

$$\frac{2\sqrt{10}+V}{2\sqrt{10}-V} = Ae^{\frac{16\sqrt{10}}{5}t}$$

$$t=0,\ V=20, \text{ so } A = \frac{2\sqrt{10}+20}{2\sqrt{10}-20} = \frac{\cancel{2}(\sqrt{10}+10)}{\cancel{2}(\sqrt{10}-10)}$$

$$2\sqrt{10} + V = 2\sqrt{10}Ae^{\frac{16\sqrt{10}}{5}t} - Ae^{\frac{16\sqrt{10}}{5}t}V$$

$$V + Ae^{\frac{16\sqrt{10}}{5}t}V = 2\sqrt{10}Ae^{\frac{16\sqrt{10}}{5}t} - 2\sqrt{10}$$

$$V = \frac{2\sqrt{10}\left(Ae^{\frac{16\sqrt{10}}{5}t} - 1\right)}{\left(Ae^{\frac{16\sqrt{10}}{5}t} + 1\right)}$$

where $A = \dfrac{\sqrt{10}+10}{\sqrt{10}-10}$ or $\dfrac{11+2\sqrt{10}}{-9}$

$$A = -1.92$$

$$V = \frac{+12.14e^{10.11t} + 6.32}{+1.92e^{10.11t} - 1}$$