DIFFERENTIAL EQUATIONS EXAM 2 B Dr. Rapalje

1. A paratrooper and parachute weigh 200 lb. At the instant the parachute opens, he is travelling downward at 30 ft/sec. If the air resistance varies directly as the instantaneous velocity, and the air resistance is 80 lb. when the velocity is 20 ft/sec. Determine the velocity and position at any time, and find the limiting velocity.

2. An emf of $E = 100e^{-5t}$ is in series with a 20 ohm resistor and a .01 farad capacitor. If $Q=0$ at $t=0$, find the charge and current at any time. Find the maximum charge and the time when it occurs.

3. Find the family of orthogonal trajectories to $x^2 = cy + y^2$. Find the particular member which passes through $(3,-1)$.

4. A tank has 100 gallons of salt water with 2 lb. of salt per gallon. A solution with 3 lb. of salt per gallon enters at 2 gal per minute, and the mixture well stirred leaves at the same rate. When will 150 lb. of salt be in the tank?

5. A beam of length L and of negligible weight has a uniform weight of 12 lb/ft, and a concentrated load of 48 lb. at the center.
Find the equation of the elastic curve and the maximum deflection.

Diff Eq. Exam 2B Solutions — Dr. Rapalje

1. $R = \beta V$

$80 = \beta \cdot 20 \quad \beta = 4$

down = positive

$ma = \frac{W}{g}\frac{dV}{dt} = W - \beta V$

$\frac{200}{32}\frac{dV}{dt} = 200 - 4V$

$\frac{dV}{dt} + \frac{16}{25}V = 32$

I.F. $= e^{\int \frac{16}{25}dt}$; $Ve^{16/25\,t} = \int 32e^{16/25\,t}\,dt$

$Ve^{16/25\,t} = 50e^{16/25\,t} + C$

$V(0) = 50 \qquad 50 = 50 + C$ so $C = 0$

$Ve^{16/25\,t} = 50e^{16/25\,t}$

$V = 50$ (Constant velocity) Sorry about that

$x = 50t + C$

$x(0) = 0$ so $C = 0$

$x = 50t$

3. $x^2 = cy + y^2 \qquad c = \frac{x^2 - y^2}{y}$

$2x = cy' + 2yy'$

$y' = \frac{2x}{c + 2y} = \frac{2x}{\frac{x^2 - y^2 + 2y^2}{y}}$

$= \frac{2xy}{x^2 + y^2}$

$y'_{\perp} = -\frac{x^2 + y^2}{2xy} = \frac{dy}{dx}$

$(x^2 + y^2)\,dx + 2xy\,dy = 0$

$\frac{\partial M}{\partial y} = 2y = \frac{\partial N}{\partial x} =$ Exact (Praise the Lord!)

$\frac{x^3}{3} + xy^2 = C_1$ or $x^3 + 3xy^2 = C_2$

Through $(3,-1)$ $\quad 27 + 3\cdot 3\cdot(-1)^2 = C_2$

$C_2 = 36 \qquad x^3 + 3xy^2 = 36$

2. $E = 100e^{-5t}$

$R = 20$ Ohm

$C = .01$ Farad

$R\frac{dQ}{dt} + \frac{Q}{C} = E$

$20\frac{dQ}{dt} + 100Q = 100e^{-5t}$

$\frac{dQ}{dt} + 5Q = 5e^{-5t}$

I.F. $= e^{5t}$; $Qe^{5t} = \int 5e^{-5t}e^{5t}\,dt$

$Qe^{5t} = 5t + C$

$Q(0) = 0 \qquad 0 = 0 + C \qquad C = 0.$

$Q = 5te^{-5t}$

$I = \frac{dQ}{dt} = -5Q + 5e^{-5t}$

$= -25te^{-5t} + 5e^{-5t} = 5e^{-5t}(1 - 5t)$

Q max at $I = \frac{dQ}{dt} = 0 \Rightarrow 1 - 5t = 0$

$t = \frac{1}{5}$ sec.

$Q\max = 5\left(\frac{1}{5}\right)e^{-5\left(\frac{1}{5}\right)} = e^{-1} = .368$ coul.

4. Let A = amt salt at any time t (in lbs.)

$\frac{dA}{dt} = \text{lb/min} = 3\frac{\text{lb}}{\text{gal}}\,2\frac{\text{gal}}{\text{min}} - \frac{A\text{ lb}}{100\text{ gal}}\cdot\frac{2\text{ gal}}{\text{min}}$

$\frac{dA}{dt} + \frac{1}{50}A = 6 \qquad$ I.F. $= e^{-1/50\,t}$

or $\frac{dA}{dt} = \frac{300 - A}{50}$ or Variables Separable.

$\frac{dA}{300 - A} = \frac{dt}{50}$ so $-\ln|300 - A| = \frac{1}{50}t + C$

$\ln|300 - A| = -\frac{1}{50}t + C_1$

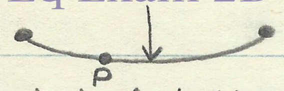
$300 - A = C_3e^{-\frac{1}{50}t}$

$A(0) = 200$ lb. so $C_3 = 100$

$A = 300 - 100e^{-\frac{t}{50}}$

If $A = 150$, $100e^{-t/50} = 150$ Sorry about that again

5.



Length L

$w = 12$ lb/ft

$S = 48$ lb. at center

moment about left

$$m(x) = (-24 - 6L)x + 12x \cdot \frac{x}{2} \quad \text{for } 0 \le x \le \frac{L}{2}$$

$$EI\,y'' = (-24 - 6L)x + 6x^2$$

$$y' = \frac{1}{EI}\left[(-12 - 3L)x^2 + 2x^3 + C_1\right] \qquad y'\left(\frac{L}{2}\right) = 0$$

$$0 = (-12 - 3L)\frac{L^2}{4} + \frac{L^3}{4} + C_1 \qquad C_1 = \frac{12L^2 + 3L^3 - L^3}{4} = \frac{12L^2 + 2L^3}{4}$$

$$y' = \frac{1}{EI}\left[(-12 - 3L)x^2 + 2x^3 + 3L^2 + \frac{L^3}{2}\right] \qquad C_1 = 3L^2 + \frac{1}{2}L^3$$

$$y = \frac{1}{EI}\left[\frac{(-12 - 3L)x^3}{3} + \frac{x^4}{2} + 3L^2x + \frac{L^3}{2}x + C_2\right] \qquad y(0) = 0.$$

$$C_2 = 0.$$

$$y = \frac{1}{EI}\left[-4x^3 - Lx^3 + \frac{x^4}{2} + 3L^2x + \frac{L^3x}{2}\right]$$

$$= \frac{1}{2EI}\left[x^4 - 8x^3 - 2Lx^3 + 6L^2x + L^3x\right]$$

$$y_{max} \text{ at } \frac{L}{2} = \frac{1}{2EI}\left[\frac{L^4}{16} - \frac{8L^3}{8} - \frac{2L^4}{8} + \frac{6L^3}{2} + \frac{L^4}{2}\right]$$

$$y\left(\frac{L}{2}\right) = \frac{1}{32EI}\left[L^4 - 16L^3 - 4L^4 + 48L^3 + 8L^4\right]$$

$$= \frac{1}{32EI}\left[5L^4 + 32L^3\right] \quad \text{or} \quad \frac{L^3}{32EI}(5L + 32)$$