DIFFERENTIAL EQUATIONS EXAM 4A Dr. Rapalje

1. The general differential equation for oscillating springs is

$$\frac{W}{g}\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F(t),$$ where $F(t)$ is an applied external force.

a) What condition(s) cause(s) undamped oscillations?
b) What condition(s) cause(s) critical damping?
c) What condition(s) cause(s) damped oscillatory motion?
d) What condition(s) cause(s) overdamping?

Answer in terms of W, g, β, and k.

e) Which of the constants W, g, β, and k, if increased, would result in increased oscillations?

2. A 5 lb weight stretches a spring 6 inches. The weight is pulled down 3 inches below equilibrium and given an upward velocity of 6 ft/sec. Find the equation for position at any time t. (Assume down is positive.) Find amplitude, period, and frequency. (NO DAMPING)

3. A spring with spring constant 12.5, 4 lb. wt. and damping constant $\beta = 1.5$ is raised ½ ft. above equilibrium and given a downward velocity of 4 ft/sec. Find the equation for position at any time t. (Assume down is positive). Find the time varying amplitude and the quasi period.

4. A resistor of 50 ohms, an inductor of 2 henries, and a .005 farad capacitor are in series with an emf of 40 volts. Assuming that the initial charge is 4 coulombs and current is zero, find equations for charge $Q(t)$ and current $I(t)$.

5. An external force $F(t) = 10\sin\omega t$ produces resonance on a spring (NO DAMPING) with a weight of 8 lbs and a spring constant k. a) Write the differential equation for this problem. b) Find the spring constant k in terms of ω. c) How far does the weight stretch the spring in terms of ω? (Hooke's law) d) If $\omega = 4$, give the general equation of motion. (Need not solve for constants)

DIFFERENTIAL EQUATIONS EXAM 4 A Solutions Outline Dr. Rapalje

1. $\frac{W}{g}\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F(t)$

$\frac{W}{g}m^2 + \beta m + k = 0$ Aux. eq.

a) Discriminant $b^2 - 4ac = \beta^2 - 4\frac{W}{g}k$

No Damping if $\beta = 0$

b) Critical damping (real equal roots) if $\beta^2 - 4\frac{wk}{g} = 0$

c) Damped oscillatory (complex roots) if $\beta^2 - \frac{4wk}{g} < 0$

d) Overdamped (real roots) if $\beta^2 - \frac{4wk}{g} > 0$

e) oscillations increase with W + k, decrease with β and g.

2. $\frac{5}{32}\frac{d^2x}{dt^2} + 10x = 0$

$5 = k(\frac{1}{2})$, so $k = 10$

$x(0) = +\frac{1}{4}$

$v(0) = -6$

$x = \frac{1}{4}\cos 8t - \frac{3}{4}\sin 8t$

$= \frac{\sqrt{10}}{4}\sin(8t + \phi)$

$Amp = \frac{\sqrt{10}}{4}$ $T = \frac{2\pi}{8} = \frac{\pi}{4}$ $f = \frac{4}{\pi}$

3. $\frac{4}{32}\frac{d^2x}{dt^2} + 1.5\frac{dx}{dt} + 12.5x = 0$ $x(0) = -\frac{1}{2}$, $V(0) = 4$

$\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 100x = 0$

$x = e^{-6t}(c_1\sin 8t + c_2\cos 8t)$ $c_1 = \frac{1}{8}$, $c_2 = -\frac{1}{2}$

$x \;\; e^{-6t}\frac{\sqrt{13}}{4}\sin(8t + \phi)$

$A = \frac{\sqrt{13}}{4}e^{-6t}$ $T = \frac{2\pi}{8} = \frac{\pi}{4}$

4. $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E$

$2\frac{d^2Q}{dt^2} + 50\frac{dQ}{dt} + 200Q = 40$

$\frac{d^2Q}{dt^2} + 25\frac{dQ}{dt} + 100Q = 20$

$m = -5, -20$

$Q_c = c_1e^{-5t} + c_2e^{-20t}$ $Q_p = a$, $a = \frac{1}{5}$

$c_1 = \frac{76}{15}$ $c_2 = -\frac{19}{15}$

$Q = \frac{76}{15}e^{-5t} - \frac{19}{15}e^{-20t} + \frac{1}{5}$

$I = -\frac{76}{3}e^{-5t} + \frac{76}{3}e^{-20t}$

5. $\frac{8}{32}\frac{d^2x}{dt^2} + kx = 10\sin\omega t$

a) $\frac{d^2x}{dt^2} + 4kx = 40\sin\omega t$.

$x_c = c_1\sin 2\sqrt{k}t + c_2\cos 2\sqrt{k}t$

$x_p = a\sin\omega t + b\cos\omega t$

If resonance occurs, $2\sqrt{k} = \omega$

and $x_p = at\sin\omega t + bt\cos\omega t$.

b) $k = \frac{\omega^2}{4}$

c) $F = kx$

$x = \frac{F}{k}$

$= \frac{8}{\frac{\omega^2}{4}} = \frac{32}{\omega^2}$

d) $x = c_1\sin 4t + c_2\cos 4t + t(a\sin 4t + b\cos 4t)$

By the way, it turns out that

$x = c_1\sin 4t + c_2\cos 4t - 5t\cos 4t$