# DIFFERENTIAL EQUATIONS — FINAL EXAM

Show all work on separate paper.

DO ANY 2 of the first 3 problems. Solve the d.e.

1. $xy' = x^3 + 2y$

2. $r^2 \sin\theta \, d\theta - (2r\cos\theta + 10)\, dr = 0$

3. $yy'' + (y')^2 = 0$

4. A 192-lb. weight has limiting velocity 16 ft/sec when falling in air, which provides air resistance force proportional to the instantaneous velocity. Find the equations for velocity and position at any time $t$.

5. a) If the growth of a population is proportional to the population $y$ at any time, show that $y = y_0 e^{kt}$, where $y_0$ is the initial population and $k$ is the constant of proportionality.

   b) If the initial population increases by 50% in $\frac{1}{2}$ hr, how long would it take the population to triple?

6. a) $\mathcal{L}^{-1}\left\{\dfrac{2s-10}{s^2-4s+20}\right\}$ b) $\mathcal{L}^{-1}\left\{\dfrac{10}{s(s^2-2s+5)}\right\}$

7. Solve by Laplace Transforms:

   $y'' + 9y = 20e^{-t}$ $\quad y(0) = 0 \quad y'(0) = 1$.

8. A spring with $k = 8$ lb/ft has a 64-lb. weight. The weight is given an upward velocity of 10 ft/sec from equilibrium position, and an applied external force of $F(t) = 16\cos 4t$ is applied. Assume down is positive and no damping. Find position and velocity at any time.

9. $L = 4$, $R = 20$, $C = .008$, $EMF = 500$. $I(0) = Q(0) = 0$. Find $I(t)$ and $Q(t)$. Find $I$ and $Q$ after a long time.

10. Solve $y'' + 16y = 5\sin x$ where $y'(0) = y(0) = 0$.

# DIFFERENTIAL EQUATIONS FINAL EXAM ANSWERS.

1. p. 66, # 32. $y = x^3 + cx^2$

2. p. 65, # 15. $r^2 \cos\phi + 10r = c$

3. p. 66, # 58. $y^2 = c_1 x + c_2$

4. p 79, # 7. $v = 16 - 16e^{-2t}$
$x = 16t + 8e^{-2t} - 8$

5A) p 109, # 7. 1.35 hr.

6a) p. 283, # 2f). $\frac{1}{2} e^{2t}(4\cos 4t - 3\sin 4t)$

A p. 283, # 3e). $2 - e^{t}(2\cos 2t - \sin 2t)$

7. p. 283, # 5c). $y = 2e^{-t} + \sin 3t - 2\cos 3t$

8. p. 237, # 2. $x = \frac{2}{3}\cos 2t - 5\sin 2t - \frac{2}{3}\cos 4t$
$v = -\frac{4}{3}\sin 2t - 10\cos 2t + \frac{8}{3}\sin 4t$

9. p. 244, # 1. $Q = 4 - 2e^{-5/2 t}(2\cos 5t + \sin 5t)$
$I = 25 e^{-\frac{5}{2}t} \sin 5t$ $\lim_{t\to\infty} Q = 4$ $\lim_{t\to\infty} I = 0$.

10. p. 194, # 2a). $y = \frac{1}{3}\sin x - \frac{1}{12}\sin 4x$