TRIGONOMETRY EXAM 3 UCF RAPALJE

Sections 3.4 to 3.7 (No CALCULATORS!) #1-18, 3 each; 19-26, 5 or 6 each

EXPRESS IN TERMS OF ONE FUNCTION OF ONE ANGLE, (OR CONSTANT).

1. $\cos 48^\circ \cos 35^\circ + \sin 48^\circ \sin 35^\circ$
2. $\sin 48^\circ \cos 35^\circ + \cos 48^\circ \sin 35^\circ$
3. $\dfrac{2\tan 48^\circ}{1-\tan^2 48^\circ}$
4. $1-\cos^2 48^\circ$
5. $1-2\sin^2 48^\circ$
6. $\sin^2 48^\circ + \cos^2 48^\circ$
7. $4\sin 4x \cos 4x$
8. $4\cos^2 4x - 4\sin^2 4x$
9. $4\cos^2 4x + 4\sin^2 4x$
10. $4\cos^2 4x - 2$
11. Express as a sum or difference: $5\cos x \sin 5x$
12. Express as a product: $\cos 5x - \cos x$

In 13–16, find the numbers:

13. $\text{Arcsin}\ \frac{1}{2} =$
14. $\text{Arcsin}\left(-\frac{1}{2}\right) =$
15. $\text{Arccos}\left(-\frac{1}{2}\right) =$
16. $\text{Arctan}\ \sqrt{3} =$
17. $\text{Arctan}(-1) =$
18. $\text{Arccos}\ 0 =$

In 17–, SHOW ALL WORK!

19. If $\cos\alpha = -\frac{3}{5}$ and $\cos\beta = -\frac{3}{5}$, where α is in QII, β in QIII, find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$ and the Quadrant of $\alpha+\beta$.
20. Show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.
21. Show that $\sin^4 t = \frac{3}{8} - \frac{1}{2}\cos 2t + \frac{1}{8}\cos 4t$.
22. Show that $\tan 3u = \dfrac{(3-\tan^2 u)\tan u}{1-3\tan^2 u}$
23. Simplify: $\tan(\arccos x)$
24. Simplify: $\sin\left[2\arccos\left(-\frac{3}{5}\right)\right]$
25. Find all x: $\sin x + \sin 3x = \sin 2x$
26. Find all x: $\sin 2x + 2\sin x - \cos x - 1 = 0$

TRIG EXAM 3 Solutions

1. $\cos 48^\circ \cos 35^\circ + \sin 48^\circ \sin 35^\circ$
$= \cos(48^\circ - 35^\circ) = \boxed{\cos 13^\circ}$

2. $\sin 48 \cos 35^\circ + \cos 48^\circ \sin 35^\circ$
$= \sin(48^\circ + 35^\circ) = \boxed{\sin 83^\circ}$

3. $\dfrac{2\tan 48^\circ}{1-\tan^2 48^\circ} = \tan 2(48^\circ) = \boxed{\tan 96^\circ}$

4. $1 - \cos^2 48^\circ = \boxed{\sin^2 48^\circ}$

5. $1 - 2\sin^2 48^\circ = \cos 2(48^\circ) = \boxed{\cos 96^\circ}$

6. $\sin^2 48^\circ + \cos^2 48^\circ = \boxed{1}$

7. $4 \sin 4x \cos 4x = 2(2 \sin 4x \cos 4x)$
$= \boxed{2 \sin 8x}$

8. $4\cos^2 4x - 4\sin^2 4x$
$= 4(\cos^2 4x - \sin^2 4x)$
$= \boxed{4 \cos 8x}$

9. $4\cos^2 4x + 4\sin^2 4x$
$= 4(\cos^2 4x + \sin^2 4x) = \boxed{4}$

10. $4\cos^2 4x - 2 = 2(2\cos^2 4x - 1)$
$= \boxed{2 \cos 8x}$

11. $5 \cos x \sin 5x = \dfrac{5}{2}(2 \cos x \sin 5x)$
$= \dfrac{5}{2}[\sin 6x - \sin(-4x)]$
$= \boxed{\dfrac{5}{2}(\sin 6x + \sin 4x)}$

12. $\cos 5x - \cos x = 2 \sin 3x \sin(-2x)$
$= \boxed{-2 \sin 3x \sin 2x}$

13. $\text{Arcsin}\ \frac{1}{2} = \boxed{\frac{\pi}{6} R}$

14. $\text{Arcsin}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6} R}$

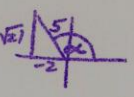
15. $\text{Arccos}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3} R}$

16. $\text{Arctan}\ \sqrt{3} = \boxed{\frac{\pi}{3} R}$

17. $\text{Arctan}(-1) = \boxed{-\frac{\pi}{4} R}$

18. $\text{Arccos}(0) = \boxed{\frac{\pi}{2} R}$

19. $\cos\alpha = -\frac{3}{5}$ QII $\quad \cos\beta = -\frac{3}{5}$ QIII



$\sin\alpha = \frac{\sqrt{21}}{5} \quad \sin\beta = -\frac{4}{5}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= \frac{\sqrt{21}}{5} \cdot \frac{-3}{5} + \frac{-2}{5} \cdot \frac{-4}{5}$
$= \boxed{\frac{8 - 3\sqrt{21}}{25}}$ (Negative)

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$= \frac{-2}{5} \cdot \frac{-3}{5} - \frac{\sqrt{21}}{5} \cdot \frac{-4}{5}$
$= \boxed{\frac{6 + 4\sqrt{21}}{25}}$ Positive. $\boxed{\alpha+\beta \text{ in QIV}}$

22. $\tan 3u = \tan(2u + u) = \dfrac{\tan 2u + \tan u}{1 - \tan 2u \tan u}$

$= \dfrac{\frac{2\tan u}{1-\tan^2 u} + \tan u}{1 - \frac{2\tan u}{1-\tan^2 u}\tan u}$

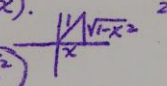
$= \dfrac{2\tan u + \tan u(1-\tan^2 u)}{(1-\tan^2 u) - 2\tan^2 u}$

$= \dfrac{3\tan u - \tan^3 u}{1 - 3\tan^2 u} = \boxed{\dfrac{(3-\tan^2 u)\tan u}{1-3\tan^2 u}}$

20. $\cos 4\theta = \cos 2(2\theta) = 2\cos^2(2\theta) - 1$
$= 2[2\cos^2\theta - 1]^2 - 1 = 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$
$= \boxed{8\cos^4\theta - 8\cos^2\theta + 1}$

21. $\sin^4 t = (\sin^2 t)^2 = \left(\dfrac{1-\cos 2t}{2}\right)^2$
$= \dfrac{1 - 2\cos 2t + \cos^2 2t}{4} = \frac{1}{4} - \frac{1}{2}\cos 2t + \frac{1}{4}\cos^2 2t$
$= \frac{1}{4} - \frac{1}{2}\cos 2t + \frac{1}{4}\left(\dfrac{1+\cos 4t}{2}\right)$
$= \frac{1}{4} - \frac{1}{2}\cos 2t + \frac{1}{8} + \frac{1}{8}\cos 4t$
$= \boxed{\frac{3}{8} - \frac{1}{2}\cos 2t + \frac{1}{8}\cos 4t}$

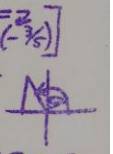
23. $\tan(\arccos x)$. $\quad y = \arccos x$



$\cos y = x$
$\tan y = \boxed{\dfrac{\sqrt{1-x^2}}{x}}$

24. $\sin\left[2\arccos\left(-\frac{3}{5}\right)\right]$ $\quad \theta = 2$

$\cos\theta = -\frac{3}{5}$ QIII



$\sin\theta = \frac{4}{5}$
$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2 \cdot \frac{4}{5} \cdot \frac{-3}{5}$
$= \boxed{-\frac{24}{25}}$

25. $\sin x + \sin 3x = \sin 2x$
$2\cos(-x)\sin 2x = \sin 2x$
$2\cos x \sin 2x - \sin 2x = 0$
$\sin 2x(2\cos x - 1) = 0$
$\sin 2x = 0 + k\pi \quad \cos x = \frac{1}{2}$
$x = \frac{k}{2}\pi \quad x = \frac{\pi}{3} + 2k\pi, \ \frac{5\pi}{3} + 2k\pi$

26. $2\sin x\cos x + 2\sin x - \cos x - 1 = 0$
$2\sin x(\cos x + 1) - 1(\cos x + 1) = 0$
$(2\sin x - 1)(\cos x + 1) = 0$
$\sin x = \frac{1}{2} \quad \cos x = -1$
$x = \frac{\pi}{6} + 2k\pi, \ \frac{5\pi}{6} + 2k\pi \quad x = \pi + 2k\pi$