TRIGONOMETRY EXAM 2 UCF RAPALJE

Show all work as necessary on separate paper. NO CALCULATORS! Turn in all work sheets. You may keep this test and a copy of your answers.

Omit one problem, or do all, the worst one to be omitted for you.

1. If $\cos t = -\frac{\sqrt{3}}{2}$ and $\csc t = 2$, find the $\sin t$, $\tan t$, $\cot t$, and $\sec t$.

2. If $\tan t = -3/4$ and $\cos t > 0$, find $\sin t$, $\cos t$, $\cot t$, $\sec t$, and $\csc t$.

3. Express $\tan t$ in terms of $\sin t$. (Must be "$\sin t$" only!)

In 4-10, verify the trig identities by classroom approved techniques:

4. $\cot t + \tan t = \csc t \sec t$

5. $\dfrac{1+\cos t}{\sin t} + \dfrac{\sin t}{1+\cos t} = 2\csc t$

6. $\dfrac{1+\sec\beta}{\tan\beta + \sin\beta} = \csc\beta$

7. $\cos^4\theta + \sin^2\theta = \sin^4\theta + \cos^2\theta$

8. $\sec y \csc y + \cot y = \tan y + 2\cos y \csc y$

9. $\cos^4 w + 1 - \sin^4 w = 2\cos^2 w$

10. $\log \cot x = \log \cos x - \log \sin x$

In 11-13, solve the equations by appropriate methods:

11. $4\sin^2 x - 3 = 0$, for all x

12. $2\cos^2 2x + \cos 2x = 0$, for all x, and also $0 \le x < 2\pi$.

13. $\tan\theta + \sec\theta = 1$, for $0 \le \theta < 2\pi$.

14. Use the sum of angles formula to find the exact value of $\cos\frac{11\pi}{12} = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) =$ __________. (Rationalize denominators)

15. Express in terms of one function of one angle:
   a) $\sin 36^\circ \cos 4^\circ - \cos 36^\circ \sin 4^\circ =$
   b) $\cos 36^\circ \cos 4^\circ - \sin 36^\circ \sin 4^\circ =$

16. If $\cos\alpha = -4/5$ and $\tan\beta = \frac{5}{12}$, α in QII, β in QIII, find $\sin(\alpha+\beta)$ $\cos(\alpha+\beta)$ and $\tan(\alpha+\beta)$. In what quadrant is $\alpha+\beta$?

17. Prove: $\cos(u+v)\cdot\cos(u-v) = \cos^2 u - \sin^2 v$.

TRIGONOMETRY EXAM 2 Solutions Rapalje

1. $\cos t = -\frac{\sqrt{3}}{2}$, $\sin t = \frac{1}{2}$

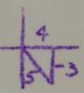
$t = \frac{\pi}{6}$ in Q II

$\tan t = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

$\cot t = -\sqrt{3}$ $\sec t = -\frac{2}{\sqrt{3}}$

2. $\tan t = -\frac{3}{4}$

$\cos t > 0$ Q IV



$\sin t = -3/5$

$\cos t = 4/5$

$\cot t = -4/3$

$\sec t = 5/4$

$\csc t = -5/3$

3. $\tan t = \frac{\sin t}{\cos t}$ $\quad \cos^2 t = 1 - \sin^2 t$

$= \frac{\sin t}{\pm\sqrt{1-\sin^2 t}}$ $\quad \cos t = \pm\sqrt{1-\sin^2 t}$

4. $\cot t + \tan t = \csc t \sec t$

$LHS = \frac{\cos t}{\sin t} + \frac{\sin t}{\cos t} = \frac{\cos^2 t + \sin^2 t}{\sin t \cos t}$

$= \frac{1}{\sin t \cos t}$

$= \csc t \sec t = RHS.$

5. $\frac{1+\cos t}{\sin t} + \frac{\sin t}{1+\cos t} = 2\csc t$

$LHS = \frac{(1+\cos t)^2 + \sin^2 t}{\sin t\,(1+\cos t)}$

$= \frac{1 + 2\cos t + \overbrace{(\cos^2 t + \sin^2 t)}^{=1}}{\sin t\,(1+\cos t)}$

$= \frac{2 + 2\cos t}{\sin t\,(1+\cos t)}$

$= \frac{2\cancel{(1+\cos t)}}{\sin t\,\cancel{(1+\cos t)}} = 2\csc t = RHS.$

6. $\frac{1+\sec\beta}{\tan\beta + \sin\beta} = \csc\beta$

$RHS. = \frac{\csc\beta}{1} \cdot \frac{(\tan\beta + \sin\beta)}{(\tan\beta + \sin\beta)}$

$= \frac{\frac{1}{\cancel{\sin\beta}} \cdot \frac{\cancel{\sin\beta}}{\cos\beta} + \frac{1}{\cancel{\sin\beta}} \cdot \cancel{\sin\beta}}{\tan\beta + \sin\beta}$

$= \frac{\sec\beta + 1}{\tan\beta + \sin\beta} = LHS$

7. $\cos^4\theta + \sin^2\theta = \sin^4\theta + \cos^2\theta$

$LHS. (1-\sin^2\theta)^2 + \sin^2\theta$

$= 1 - 2\sin^2\theta + \sin^4\theta + \sin^2\theta$

$= 1 - \sin^2\theta + \sin^4\theta.$

$= \cos^2\theta + \sin^4\theta = RHS$

8. $\sec y \csc y + \cot y = \tan y + 2\cos y \csc y.$

$LHS. = \frac{1}{\cos y} \cdot \frac{1}{\sin y} + \frac{\cos y}{\sin y} \cdot \frac{(\cos y)}{(\cos y)}$

$= \frac{1 + \cos^2 y}{\cos y \sin y}$

$= \frac{\sin^2 y + \cos^2 y + \cos^2 y}{\cos y \sin y}$

$= \frac{\sin^2 y + 2\cos^2 y}{\cos y \sin y}$

$= \frac{\sin^2 y}{\cos y \sin y} + \frac{2\cos^2 y}{\cos y \sin y}$

$= \frac{\sin y}{\cos y} + \frac{2\cos y}{\sin y}$

$= \tan y + 2\cos y \csc y = RHS.$

9. $\cos^4 w + 1 - \sin^4 w = 2\cos^2 w$

$LHS = \cos^4 w - \sin^4 w + 1$

$= (\cos^2 w - \sin^2 w)(\cos^2 w + \sin^2 w) + 1$

$= \cos^2 w - \sin^2 w + 1$

$= \cos^2 w + 1 - \sin^2 w$

$= \cos^2 w + \cos^2 w = 2\cos^2 w = RHS.$

10. $\log \cot x = \log \cos x - \log \sin x$

$LHS\ \log \cot x = \log \frac{\cos x}{\sin x}$

$= \log \cos x - \log \sin x = RHS.$

Property of logarithms

$\log \frac{M}{N} = \log M - \log N.$

11. $4\sin^2 x - 3 = 0$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$ All Quadrants.

$x = \frac{\pi}{3} + 2k\pi,\ \frac{2\pi}{3} + 2k\pi$

$\frac{4\pi}{3} + 2k\pi,\ \frac{5\pi}{3} + 2k\pi$

$= \frac{\pi}{3} + k\pi,\ \frac{2\pi}{3} + k\pi$

12. $2\cos^2 2x + \cos 2x = 0$

$\cos 2x\,(2\cos 2x + 1) = 0$

$\cos 2x = 0 \quad \cos 2x = -\frac{1}{2}$

$2x = \frac{\pi}{2} + k\pi \quad 2x = \frac{2\pi}{3} + 2k\pi$

or $\frac{4\pi}{3} + 2k\pi$

$x = \frac{\pi}{4} + \frac{k\pi}{2} \quad x = \frac{\pi}{3} + k\pi$

or $\frac{2\pi}{3} + k\pi$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

13. $\tan\theta + \sec\theta = 1$

$\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = 1, \ (\cos\theta \neq 0)$

$\sin\theta + 1 = \cos\theta$

$\cos\theta - \sin\theta = 1$

Sq both sides: $\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 1$

$-2\sin\theta\cos\theta = 0$

$\sin\theta = 0$ or $\cancel{\cos\theta = 0}$ Reject

$\theta = k\pi = 0, \pi$

π check: $0 + 1 = 1$

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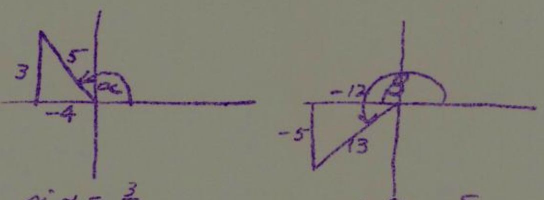
14. $$\cos\frac{11\pi}{12} = \cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$$
$$= -\frac{1}{2}\,\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{-\sqrt{2}-\sqrt{6}}{4} \text{ or } -\frac{(\sqrt{2}+\sqrt{6})}{4}$$

15 a) $\sin 36^\circ \cos 4^\circ - \cos 36^\circ \sin 4^\circ = \sin(36^\circ - 4^\circ) = \sin 32^\circ$

b) $\cos 36^\circ \cos 4^\circ - \sin 36^\circ \sin 4^\circ = \cos(36^\circ + 4^\circ) = \cos 40^\circ$

16.



$\sin\alpha = \frac{3}{5}$ $\qquad$ $\sin\beta = -\frac{5}{13}$

$\cos\alpha = -\frac{4}{5}$ $\qquad$ $\cos\beta = -\frac{12}{13}$

$$\left.\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\
&= \frac{3}{5}\cdot\frac{-12}{13} + \frac{-4}{5}\cdot\frac{-5}{13} = \\
&= \frac{-36+20}{65} = \frac{-16}{65} \\
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
&= -\frac{4}{5}\cdot\frac{-12}{13} - \frac{3}{5}\cdot\frac{-5}{13} \\
&= \frac{48+15}{65} = \frac{63}{65} \\
\tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{-16}{63}
\end{aligned}\right\} (\alpha+\beta) \text{ in QIV.}$$

17. $\cos(u+v)\cos(u-v) = \cos^2 u - \sin^2 v$

$$\text{LHS} = \cos(u+v)\cos(u-v) = (\cos u\cos v - \sin u \sin v)(\cos u \cos v + \sin u \sin v)$$
$$= \cos^2 u \cos^2 v - \sin^2 u \sin^2 v$$
$$= \cos^2 u(1-\sin^2 v) - (1-\cos^2 u)\sin^2 v$$
$$= \cos^2 u - \cancel{\sin^2 v \cos^2 u} - \sin^2 v + \cancel{\cos^2 u \sin^2 v}$$
$$= \cos^2 u - \sin^2 v = \text{RHS}.$$

ASSIGNMENT

3.5 p. 137: 1-9 odds, 11-30 all, 31-39 odd, 40.