TRIGONOMETRY EXAM 4 UCF RAPALJE

Calculators are allowed on this exam.

In 1-5, identify the type of problem (ASA, SSA, SAS, or SSS) and find all solutions. Give sides to nearest hundredth, angles to nearest tenth of degree.

(50) pts.

1. $\alpha = 42°10'$, $\gamma = 61°20'$, $b = 19.7$

2. $a = 47$, $c = 25$, $\beta = 15.3°$

3. $a = 51.8$, $b = 31.5$, $\beta = 33.7°$

4. $a = 17$, $b = 50$, $c = 45$

5. $b = 125$, $c = 100$, $\gamma = 67°$

In 6-8, find areas of the triangles (See # 12-14)

(12) pts

6. $\beta = 43.5°$, $\gamma = 77.3°$, $a = 16$

7. $a = 35$, $b = 12$, $c = 37$

8. $a = 35$, $b = 12$, $\gamma = 19.4°$

(7) pts 9. Use the law of tangents to find the angles given: $\alpha = 42.7°$, $b = 8.5$, $c = 12$

Use $\dfrac{a-b}{a+b} = \dfrac{\tan \frac{1}{2}(\alpha-\beta)}{\tan \frac{1}{2}(\alpha+\beta)}$ or some variation thereof.

(8) pts 10. The sides of a parallelogram are 25 cm. and 35 cm, and one angle is 36°. Find the lengths of its diagonals.

(8) pts 11. To find the height of a cliff, two sightings are made from points which are 500 meters apart, yet in line with the cliff. If the angles of elevation of the cliff from A and B are 15° and 22° respectively, find the height of the cliff.

In 12-14, prove the area formulas:

(15) pts

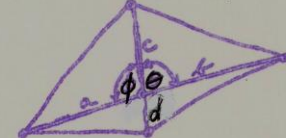
12. $A = \frac{1}{2} bc \sin \alpha$

13. $A = \dfrac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta}$

14. $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$

E.C. Prove that the area of any quadrilateral equals one half the product of the diagonals and the sine of either of the angles between the diagonals. [Hint: Use only the variables shown in the diagram!]

(5) pts



Trigonometry Exam 4 Solutions UCF R. Rapalje

1. $\alpha = 42^\circ 10'$, $\gamma = 61^\circ 20'$, $b = 19.7$ } ASA

$\beta = 180 - 103.5^\circ$

$\beta = 76.5^\circ$

$\frac{\sin 42.167^\circ}{a} = \frac{\sin 76.5^\circ}{19.7} = \frac{\sin 61.333^\circ}{c}$

$a = \frac{19.7 \sin 42.167^\circ}{\sin 76.5^\circ} = 13.60$

$c = \frac{19.7 \sin 61.333^\circ}{\sin 76.5^\circ} = 17.78$

2. $a = 47$, $c = 25$, $\beta = 15.3^\circ$ } SAS

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$= 47^2 + 25^2 - 47 \cdot 50 \cos 15.3^\circ$

$= 567.29$

$b = 23.82$

DO NOT USE LAW SINES TO FIND $\angle\alpha$. Why?

$\frac{\sin 15.3^\circ}{23.82} = \frac{\sin\gamma}{25}$

$\sin\gamma = \frac{25 \sin 15.3^\circ}{23.82} = .2769$

$\gamma = 16.1^\circ$ $\quad$ $15.3 + 16.1 = 31.4$

$\alpha = 180 - 31.4^\circ$

$\alpha = 148.6^\circ$

3. $a = 51.8$, $b = 31.5$, $\beta = 33.7^\circ$ } SSA Possibly 2 cases.

$\frac{\sin 33.7^\circ}{31.5} = \frac{\sin\alpha}{51.8}$

$\sin\alpha = \frac{51.8 \sin 33.7^\circ}{31.5}$

$= .9124$

$\alpha_1 = 65.8^\circ$ $\quad$ $\alpha_2 = 114.2^\circ$

$\alpha_1 + \beta = 99.5$ $\quad$ $\alpha_2 + \beta = 147.9$

$\gamma_1 = 80.5^\circ$ $\quad$ $\gamma_2 = 32.1^\circ$

$\frac{\sin 33.7^\circ}{31.5} = \frac{\sin 80.5^\circ}{c_1} = \frac{\sin 32.1^\circ}{c_2}$

$c_1 = \frac{31.5 \sin 80.5^\circ}{\sin 33.7^\circ}$ $\quad$ $c_2 = \frac{31.5 \sin 32.1^\circ}{\sin 33.7^\circ}$

$c_1 = 55.99$ $\quad$ $c_2 = 30.20$

4. $a = 17$, $b = 50$, $c = 45$ } SSS

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$

$= \frac{17^2 + 45^2 - 50^2}{2(17)(45)}$

$= -.121569$

$\beta = 97.0^\circ$

$\frac{\sin 97.0^\circ}{50} = \frac{\sin\alpha}{17} = \frac{\sin\gamma}{45}$

$\sin\alpha = \frac{17 \sin 97^\circ}{50} = .3375$

$\alpha = 19.7^\circ$

$\gamma = 180 - (97 + 19.7^\circ)$

$\gamma = 63.3^\circ$

5. $b = 125$, $c = 100$, $\gamma = 67^\circ$ } SSA Possibly two cases.

$\frac{\sin 67^\circ}{100} = \frac{\sin\beta}{125}$

$\sin\beta = \frac{125 \sin 67^\circ}{100} = 1.15$

No Solution

6. $\beta = 43.5^\circ$, $\gamma = 77.3^\circ$, $a = 16$ } ASA

$\alpha = 180 - 120.8$

$= 59.2^\circ$

$A = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$

$= \frac{16^2 \sin 43.5^\circ \sin 77.3^\circ}{2\sin 59.2^\circ}$

$= 100.07$ sq. units

7. $a = 35$, $s = 42$, $b = 12$, $c = 37$

$A = \sqrt{42 \cdot 7 \cdot 30 \cdot 5}$

$= 210$ sq units

8. $a = 35$, $b = 12$, $\gamma = 19.4^\circ$ } SAS

$A = \frac{1}{2}ab\sin\gamma$

$= \frac{1}{2} 35 \cdot 12 \sin 19.4^\circ$

$= 69.75$ sq units

9. $\frac{c-b}{c+b} = \frac{\tan\frac{1}{2}(\gamma-\beta)}{\tan\frac{1}{2}(\gamma+\beta)}$ $\quad$ $\gamma + \beta = 180 - 42.7^\circ = 137.3^\circ$

$\frac{12-8.5}{12+8.5} = \frac{3.5}{20.5} = \frac{\tan\frac{1}{2}(\gamma-\beta)}{\tan 68.65^\circ}$

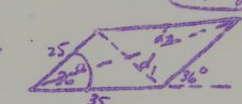
$\tan\frac{1}{2}(\gamma-\beta) = .436778$

$\frac{1}{2}\gamma - \frac{1}{2}\beta = 23.59$

$\frac{1}{2}\gamma + \frac{1}{2}\beta = 68.65$

$\gamma = 92.2^\circ$

$\beta = 137.3 - 92.2 = 45.1^\circ$

10. 

$d_1^2 = 35^2 + 25^2 - 2 \cdot 25 \cdot 35 \cos 36^\circ$

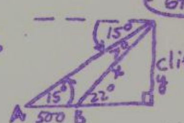
$= 435.2203$

$d_1 = 20.86$

$d_2^2 = 35^2 + 25^2 - 2 \cdot 25 \cdot 35 \cos 144^\circ$

$= 3265.7797$

$d_2 = 57.15$

11. 

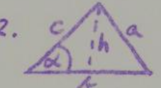
$\frac{\sin 7^\circ}{500} = \frac{\sin 15^\circ}{x}$

$x = \frac{500 \sin 15^\circ}{\sin 7^\circ} = 1061.871$

$\sin 22^\circ = \frac{y}{x}$

$y = x \sin 22^\circ$

$= 1061.871 \sin 22^\circ = 397.7$ meters

12. 

$\sin\alpha = \frac{h}{c}$

$h = c\sin\alpha$

$A = \frac{1}{2} b \cdot h = \frac{1}{2} bc\sin\alpha$

13. By law of sines, $\frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$c = \frac{b\sin\gamma}{\sin\beta}$

$A = \frac{1}{2} bc\sin\alpha$

$= \frac{1}{2} b\left(\frac{b\sin\gamma}{\sin\beta}\right)\sin\alpha$

$= \frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta}$

14. $A = \frac{1}{2} bc\sin\alpha$

$A^2 = \frac{1}{4} b^2c^2(1 - \cos^2\alpha)$

$= b^2c^2 \frac{1+\cos\alpha}{2} \cdot \frac{1-\cos\alpha}{2}$

$= b^2c^2 \left[\frac{1}{2}\left(1 + \frac{b^2+c^2-a^2}{2bc}\right)\right]\left[\frac{1}{2}\left(1 - \frac{b^2+c^2-a^2}{2bc}\right)\right]$

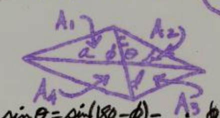
$= b^2c^2 \left[\frac{b^2+2bc+c^2-a^2}{4bc}\right]\left[\frac{a^2-(b^2-2bc+c^2)}{4bc}\right]$

$= \frac{(b+c)^2 - a^2}{4} \cdot \frac{a^2 - (b-c)^2}{4}$

$= \frac{b+c+a}{2} \cdot \frac{b+c-a}{2} \cdot \frac{a-b+c}{2} \cdot \frac{a+b-c}{2}$

Let $s = \frac{a+b+c}{2}$, then $s - a = \frac{a+b+c-2a}{2} = \frac{b+c-a}{2}$, etc.

$A = \sqrt{s \cdot (s-a)(s-b)(s-c)}$

EC. 

$\sin\theta = \sin(180 - \phi) = \sin\phi$

$A_1 = \frac{1}{2} ac\sin\phi$

$A_2 = \frac{1}{2} bc\sin\theta$

$A_3 = \frac{1}{2} bd\sin\phi$

$A_4 = \frac{1}{2} ad\sin\theta$

$A = \frac{1}{2} ac\sin\phi + \frac{1}{2} bc\sin\phi + \frac{1}{2} bd\sin\phi + \frac{1}{2} ad\sin\phi$

$= \frac{1}{2}\sin\phi[ac + bc + bd + ad]$

$= \frac{1}{2}\sin\phi[c(a+b) + d(a+b)]$

$= \frac{1}{2}\sin\phi(a+b)(c+d)$