TRIGONOMETRY FINAL EXAM

Show all work on separate paper.
Calculators are permitted.

1. If $\cos\theta = -.75$ and $\tan\theta < 0$, find $\sin\theta$ and $\sin 2\theta$; give exact form.
2. Give the amplitude, period, and graph of $y = -2\cos 3x$, $0 \le x \le 2\pi$.
3. Solve the right triangle, given $\gamma = 90°$, $\beta = 54.7°$, $a = 220$.

In 4–6, prove the identities:

4. $1 + \tan^2 x = \tan x \sec x \csc x$
5. $\cos^4 x - \sin^4 x = \cos 2x$
6. $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = 2\csc x$

In 7–9, find all values of x, $0 \le x \le 2\pi$ in radians, <u>exact form where possible</u>!

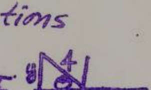
7. $2\cos^2 x - \cos x = 0$
8. $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$
9. $\cos 2x + 3\cos x + 2 = 0$
10. Give exact value: a) $\cos\left(\sin^{-1}\frac{4}{5}\right)$ b) $\cos\left(2\sin^{-1}\frac{4}{5}\right)$
11. In $\triangle ABC$, $c = 27$, $a = 15$, $b = 20$, find the angles
12. Solve the triangle: $\beta = 37°$, $c = 25$, $a = 29$.
13. Solve the triangle: $a = 320$, $b = 370$, $\alpha = 24.2°$.

$\Big\}$ Give <u>all</u> possible solutions, or say "No Solution."

14. Give $a + bi$ form: $\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^{12}$
15. Give $a + bi$ form for all cube roots of i. $\sqrt[3]{i}$

TRIG FINAL EXAM Solutions

1. $\cos\theta = -\frac{3}{4}$ QII, III.
$\tan\theta < 0$ QII, IV. } QII



$y^2 + 9 = 16$

$y^2 = 7$

$y = \sqrt{7}$

$\boxed{\sin\theta = \frac{\sqrt{7}}{4}}$

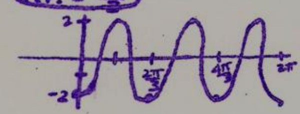
$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\frac{\sqrt{7}}{4}\cdot -\frac{3}{4} = -\frac{3\sqrt{7}}{8}$

2. $y = -2\cos 3x$

Amp = 2

Per. = $\frac{2\pi}{3}$

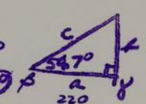


3. $a = 220$

$\gamma = 90°$

$\beta = 54.7°$

$\alpha = 35.3°$



$\tan 54.7° = \frac{b}{220}$

$b = 220\tan 54.7°$

$b = 310.72$

$\cos 54.7° = \frac{220}{c}$

$c = \frac{220}{\cos 54.7°} = 380.72$

4. $\cos 3x = \cos(2x + x)$

$= \cos 2x\cos x - \sin 2x\sin x$

$= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$

5. $\cos^4 x - \sin^4 x = \cos 2x$

LHS. $\cos^4 x - \sin^4 x$

$= (\cos^2 x - \sin^2 x)(\cos^2 x + \cos^2 x)$

$= (\cos 2x)(1)$

$= \cos 2x$ RHS.

6. $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

LHS $= \frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$

$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

$= \frac{2 + 2\cos x}{\sin x(1+\cos x)}$

$= \frac{2\cancel{(1+\cos x)}}{\sin x\cancel{(1+\cos x)}} = 2\csc x$ = RHS.

7. $2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$ $\quad$ $\cos x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

8. $\sin 2x(\csc 2x - 2) = 0$

$\sin 2x = 0$ $\quad$ $\csc 2x = 2$

$2x = 0 + k\pi$ $\quad$ $\sin 2x = \frac{1}{2}$

$x = \frac{k\pi}{2}$ $\quad$ $2x = \frac{\pi}{6} + 2k\pi$ or $\frac{5\pi}{6} + 2k\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ $\quad$ $x = \frac{\pi}{12} + k\pi, \frac{5\pi}{12} + k\pi$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

9. $\cos 2x + 3\cos x + 2 = 0$

$(2\cos^2 x - 1) + 3\cos x + 2 = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = -1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ $x = \pi$

10a) $\cos\left(\sin^{-1}\frac{4}{5}\right)$ $\quad$ $\theta = $

$\sin\theta = \frac{4}{5}$ QI.

Find $\cos\theta$.

$= \frac{3}{5}$

b) $\cos\left(2\sin^{-1}\frac{4}{5}\right)$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{4}{5}\right)^2$

$= 1 - \frac{32}{25} = -\frac{7}{25}$

11. (SSS) LAW of COSINE.

$a = 15, b = 20, c = 27$

(DO NOT USE LAW SINES TO FIND $\angle C$)

$\cos\angle C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{225 + 400 - 729}{2(15)(20)}$

$= \frac{-104}{600}$

$C = 100.0°$

$\frac{\sin 100}{27} = \frac{\sin B}{20}$

$\sin B = \frac{20\sin 100}{27}$

$B = 46.8°$

$A = 180 - (B + C)$

$A = 33.2°$

12. (SAS Law Cosines)

$\beta = 37°$, $c = 25$, $a = 29$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$= 625 + 841 - 50\cdot 29\cos 37°$ $\quad$ 1158.02

$= 307.98$

$b = 17.55$

$\frac{\sin 37}{17.55} = \frac{\sin C}{25}$

$\sin C = \frac{25\sin 37°}{17.55} = .8573$

$C = 59°$

$a = 180 - (37 + 59)$

$a = 84°$

(DO NOT USE LAW SINES TO FIND $\angle A$!!)

13. (SSA Law Sines) Possibly 2 cases.

$\frac{\sin 24.2°}{320} = \frac{\sin\beta}{370}$

$\sin\beta = .47397$

$\beta_1 = 28.3°$ or $\beta_2 = 180 - 28.3$

$\beta_2 = 151.7°$

$\beta_1 + \alpha = 52.5°$ $\quad$ $\beta_2 + \alpha = 175.9°$

$\gamma_1 = 180 - 52.5°$ $\quad$ $\gamma_2 = 180 - 175.9°$

$\gamma_1 = 127.5°$ $\quad$ $\gamma_2 = 4.1°$

$\frac{\sin 24.2}{320} = \frac{\sin\gamma_1}{c_1} = \frac{\sin\gamma_2}{c_2}$

$c_1 = \frac{320\sin 127.5°}{\sin 24.2°}$ $\quad$ $c_2 = \frac{320\sin 4.1°}{\sin 24.2°}$

$c_1 = 619.32$ $\quad$ $c_2 = 55.81$

14. $\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^{12} = z^{12}$

$r = 1$ $\quad$ $\theta = 135$

$z^{12} = 1^{12}(\cos 135\cdot 12° + i\sin(\ ))$

$= 1(-1 + 0i) = -1$

$\frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6}$

$\frac{\pi}{6} + \frac{4\pi}{3} = \frac{3\pi}{2}$

15. $i^{1/3}$ $\quad$ $r = 1, \theta = \frac{\pi}{2}$

$z^{1/3} = 1^{1/3}\left(\cos\left(\frac{\theta + 2k\pi}{3}\right) + i\sin\left(\frac{\theta + 2k\pi}{3}\right)\right)$

$k = 0$ $\quad$ $z^{1/3} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$k = 1$ $\quad$ $z^{1/3} = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$k = 2$ $\quad$ $z^{1/3} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = 0 - i$