

TRIGONOMETRY EXAM 5 UCF (Rapa/je)

Show all work as necessary.  
Turn in all work sheets.

No CALCULATORS!

Express in the form  $a+bi$ :

1.  $(4+3i)(3-2i) =$

2.  $\frac{4+3i}{3-2i} =$

(5ea) 3.  $(4+3i)^3 =$

4.  $|3-2i| =$

5.  $\sqrt{-3}(3-\sqrt{-12}) =$

Solve for  $x$ ,  $x$  in the form  $a+bi$

(6ea) 6.  $5x^3 + 2x^2 + x = 0$

7.  $27x^4 + 21x^2 + 4 = 0$

(6) 8. Give a quadratic equation with roots  $-3-5i, -3+5i$ .

9. Prove that the sum of the roots of a quadratic equation

(8)  $ax^2 + bx + c = 0$  is  $-\frac{b}{a}$ , and that the product is  $\frac{c}{a}$ .

Express in trigonometric form:

(5ea) 10.  $-8i$

11.  $-3\sqrt{3} + 3i$

Use De Moivre's Theorem to express in the form  $a+bi$ :

12.  $(1+i)^{10}$

13.  $(1-\sqrt{3}i)^8$

(8ea) 14. Find all square roots of  $i$ .

15. Find all fifth roots of unity (trigonometric form is okay!)

16. Find all solutions in  $a+bi$  form:  $x^3 + 8i = 0$ .

TRIGONOMETRY EXAM 5 Solatims Rapalje

1.  $(4+3i)(3-2i) = 12 + 8i - 6i^2 = 12 + i + 6 = 18 + i$

2.  $\frac{4+3i(3+2i)}{(3-2i)(3+2i)} = \frac{12+17i-6}{9+4} = \frac{6+17i}{13} = \frac{6}{13} + \frac{17}{13}i$

3.  $(4+3i)^3 = (4+3i)^2(4+3i) = (16+24i-9)(4+3i) = (7+24i)(4+3i) = 28+117i-72 = -44+117i$

4.  $|3-2i| = \sqrt{3^2+(-2)^2} = \sqrt{13}$

5.  $\sqrt{3}(3-\sqrt{12}) = \sqrt{3}(3-2\sqrt{3}) = 3\sqrt{3} - 6i^2 = 6+3\sqrt{3}i$

6.  $5x^3+2x^2+x=0$   
 $x(5x^2+2x+1)=0$   
 $x=0$   $x = \frac{-2 \pm \sqrt{4-20}}{10} = \frac{-2 \pm 4i}{10}$   
 $x=0, x = -\frac{1}{5} \pm \frac{2i}{5}$

7.  $27x^4+21x^2+4=0$   
 $(9x^2+4)(3x^2+1)=0$   
 $x^2 = -\frac{4}{9}$   $x^2 = -\frac{1}{3}$   
 $x = \pm \frac{2}{3}i$   $x = \pm \frac{1}{\sqrt{3}}i$

8.  $x = -3-5i, x = -3+5i$   
 $(x+3+5i)(x+3-5i) = 0$   
 $(x+3)^2 - 25i^2 = 0$   
 $x^2+6x+9+25=0$   
 $x^2+6x+34=0$

9.  $ax^2+bx+c=0$   
 $x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$   $x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$   
 $x_1+x_2 = \frac{-b+\sqrt{b^2-4ac} - b - \sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$   
 $x_1 \cdot x_2 = \frac{(-b+\sqrt{b^2-4ac})(-b-\sqrt{b^2-4ac})}{2a \cdot 2a} = \frac{b^2 - (b^2-4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$

10.  $-8i$  on imaginary axis  
 $r=8, \theta = \frac{3\pi}{2}$   
 $8(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$

12.  $(1+i)^{10}$   $r = \sqrt{1^2+1^2} = \sqrt{2}$   
 $\theta = \frac{\pi}{4}$  (Q I) or  $45^\circ$   
 $z^{10} = (\sqrt{2})^{10} (\cos 45 \cdot 10 + i \sin 45 \cdot 10)$   
 $= 2^5 (\cos 450^\circ + i \sin 450^\circ)$   
 $= 32 (\cos 90^\circ + i \sin 90^\circ)$   
 $= 32i$

11.  $-3\sqrt{5}+3i$  in Q II  
 $r = \sqrt{(-3\sqrt{5})^2+3^2} = \sqrt{27+9} = 6$   
 $\tan \theta = \frac{3}{-3\sqrt{5}} = -\frac{1}{\sqrt{5}}$  (Q II)  
 $\theta = \frac{5\pi}{6}$   $6(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$

13.  $(1-\sqrt{3}i)^8$   $r = \sqrt{1^2+(-\sqrt{3})^2} = 2$   $\theta = 300^\circ$  or  $\frac{5\pi}{3}$   
 $z^8 = 2^8 (\cos 300 \cdot 8 + i \sin 300 \cdot 8)$   
 $= 256 (\cos 2400^\circ + i \sin 2400^\circ)$   
 $360 \mid \begin{matrix} 2400 \\ 2160 \\ 240 \end{matrix} = 256 (\cos 240^\circ + i \sin 240^\circ)$   
 $= 256 (-\frac{1}{2} + i(-\frac{\sqrt{3}}{2})) = 128 - 128\sqrt{3}i$

14.  $\sqrt{2}$   $\theta = 90^\circ, r = 1$   
 $z^{\frac{1}{2}} = 1^{\frac{1}{2}} (\cos \frac{90+2k\pi}{2} + i \sin \frac{90+2k\pi}{2})$   
 $k=0: z = \cos 45^\circ + i \sin 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$   
 $k=1: z = \cos 225^\circ + i \sin 225^\circ = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

16.  $x^3+8i=0$   
 $x = \sqrt[3]{-8i}$   $r=8$   $\theta = \frac{3\pi}{2}$   
 $x = 8^{\frac{1}{3}} (\cos \frac{\frac{3\pi}{2}+2k\pi}{3} + i \sin \frac{\frac{3\pi}{2}+2k\pi}{3})$   
 $k=0: x = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) = 2i$   
 $k=1: x = 2(\cos 210^\circ + i \sin 210^\circ) = 2(-\frac{\sqrt{3}}{2} - \frac{1}{2}i) = -\sqrt{3} - i$   
 $k=2: x = 2(\cos 330^\circ + i \sin 330^\circ) = 2(\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \sqrt{3} - i$

15.  $z^5=1$   $\theta=0, r=1$   
 $z^{\frac{1}{5}} = 1^{\frac{1}{5}} (\cos \frac{0+2k\pi}{5} + i \sin \frac{0+2k\pi}{5})$   
 $k=0: \cos 0 + i \sin 0 = 1$   
 $k=1: \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$   $72^\circ$   
 $k=2: \cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$   $144^\circ$   
 $k=3: \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$   $216^\circ$   
 $k=4: \cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$   $288^\circ$