MISCELLANEOUS

p.65. 1. $(x^2+1)(y^3-1)\,dx = x^2y^2\,dy$

$1+\frac{1}{x^2} = \frac{x^2+1}{x^2}\,dx = \frac{y^2}{y^3-1}\,dy$ $\quad u = y^3-1$, $du = 3y^2$

V.S.

$$\boxed{x - \frac{1}{x} = \frac{1}{3}\ln(y^3-1) + c}$$

2. $(y^2+2xy)\,dx + (x^2+2xy)\,dy = 0$

Exact

$\frac{\partial M}{\partial y} = 2y+2x = \frac{\partial N}{\partial x}$ Exact.

$u_1 = y^2x + x^2y + f(y), u_2 = x^2y + xy^2 + f(x)$

$u = \boxed{y^2x + x^2y = C}$

3. $(x^2+2xy)\,dx + (y^2+2xy)\,dy = 0$

Hard

H.

Homogeneous. Let $y = vx$ $\quad y' = xv' + v$

$\frac{dy}{dx} = -\frac{x^2+2xy}{y^2+2xy}$

$\begin{array}{r} 2 \\ v+2\,\overline{)\,2v+1} \\ 2v+4 \\ -3 \end{array}$

$v + xv' = -\frac{x^2+2x^2v}{x^2v^2+2x^2v} = -\frac{1+2v}{v^2+2v}$

$x\frac{dv}{dx} = -\frac{1+2v+v^3+2v^2}{v^2+2v}$

$(v+1)(v^2+v+1)$

$\frac{(v^2+2v)\,dv}{v^3+2v^2+2v+1} = \frac{-dx}{x} = \frac{-dv}{v+1} + \frac{(2v+1)\,dv}{v^2+v+1}$

$\ln c - \ln x = -\ln(v+1) + \ln(v^2+v+1)$

$c = \frac{(v^2+v+1)x}{v+1} = \frac{(\frac{y^2}{x^2}+\frac{y}{x}+1)x}{\frac{y}{x}+1} = \frac{y^2+xy+x^2}{y+x}$

$\boxed{\cancel{x}\; y^2+xy+x^2 = c(y+x)}$

p.65. 4. $\frac{dy}{dx} + \frac{2y}{x} = x^2$

linear : I.F. $= e^{\int \frac{2}{x} dx} = x^2$

L.

$yx^2 = \int x^2 \cdot x^2 \, dx + c$

$= \frac{x^5}{5} + c$

$\boxed{5x^2y - x^5 = c}$

5. $(3-y)\,dx + 2x\,dy = 0$ $y(1) = 1.$

$\frac{dx}{x} = \frac{2\,dy}{y-3}$

$\ln x = 2\ln(y-3) + c$

V.S.

$\frac{x}{(y-3)^2} = c$ or $c(y-3)^2 = x$

$c(4) = 1$ $c = 1/4$

$\boxed{4x = (y-3)^2}$

6. $\frac{dy}{dx} + 2x = 2$

V.S.

$dy + (2x - 2)\,dx = 0$

$y + x^2 - 2x = c$ or $\boxed{y = c + 2x - x^2}$

p. 65. 7. $s^2 t \, ds + (t^2+4)\, dt = 0$

$$s^2 ds + \left(t + \frac{4}{t}\right) dt = 0$$

V. S.

$$\frac{s^3}{3} + \frac{t^2}{2} + 4 \ln t = \frac{c_1}{6}$$

$$\boxed{2s^3 + 3t^2 + 24 \ln t = c}$$

8. $2xyy' + x^2 + y^2 = 0$ Homogeneous.

$y = vx$

E.

H.

D.

or

$$-y' = \frac{x^2+y^2}{2xy} = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right)$$

$$-v - xv' = \frac{1}{2}\left(\frac{1}{v} + v\right) = \frac{1+v^2}{2v} \quad + \frac{2v^2}{v}$$

$$-x\frac{dv}{dx} = \frac{1+3v^2}{2v}$$

$$-\frac{dx}{x} = \frac{2v \, dv}{1+3v^2} \qquad u = 1+3v^2 \quad du = 6v$$

$$-\ln x = \frac{1}{3}\ln(1+3v^2) - \frac{1}{3}\ln c$$

$$3 \ln x + \ln(1+3v^2) = \ln c$$

$$x^3(1+3v^2) = c$$

$$x(x^2+3y^2) = c$$

or ⑧. $2xy\,dy + x^2dx + y^2dx = 0$

exact→ $\left(y^2dx + 2xy\,dy\right) + x^2dx = 0$

$$y^2x + \frac{x^3}{3} = c$$

$$3xy^2 + x^3 = c.$$

$M = x^2 + y^2$

$\frac{\partial M}{\partial y} = 2y$

$N = 2xy$

$\frac{\partial N}{\partial x} = 2y$

p.65. 9. $\frac{dy}{dx} = \frac{2x^2 - ye^x}{e^x}$

$e^x\,dy + ye^x - 2x^2\,dx = 0$

$\frac{\partial N}{\partial x} = e^x = \frac{\partial M}{\partial y}$ Exact!

E.

L.

$u_2 = ye^x$

$u_1 = ye^x - \frac{2x^3}{3}.$ $\boxed{2x^3 - 3ye^x = c.}$

10. $x^2y' + xy = x+1$ linear.

$y' + \frac{1}{x}y = \frac{x+1}{x^2}$

I.F. $e^{\int \frac{1}{x}dx} = x$

L.

$yx = \int \frac{x+1}{x^2} \cdot \cancel{x}\,dx + c$

$yx = \int \left(1 + \frac{1}{x}\right) dx + c$

$\boxed{xy = x + \ln x}$

or

$\boxed{xy - x - \ln x = c}$

11. $\frac{dy}{dx} = \frac{y}{x} + \arctan y/x$ Homogeneous. $v = y/x$

H.

$v + xv' = v + \arctan v$

$\frac{dv}{\arctan v} = \frac{dx}{x}$

$\int \frac{1}{\arctan v}\,dv = \ln|x| + c.$, where $v = y/x$

p. 65. 12. $\frac{dy}{dx} = x + y$ or $y' - y = x$

Linear: $I.F. = e^{\int -1 dx} = e^{-x}$

L. $$ye^{-x} = \int x e^{-x} \, dx$$

$$\frac{y}{e^x} = -\frac{(x+1)}{e^x} + c.$$

13. $y' + xy = x^3$ linear!

$I.F. = e^{\int x dx} = e^{x^2/2}$

$$ye^{x^2/2} = \int e^{x^2/2} x^3 \, dx$$

let $\frac{x^2}{2} = u$, $du = x\,dx$

L. $$= \int e^u \, 2u \, du$$

$$= 2e^u(u-1) + c$$

$$= 2e^{x^2/2}\left(\frac{x^2}{2} - 1\right) + c$$

$$ye^{x^2/2} = x^2 e^{x^2/2} - 2e^{x^2/2} + c$$

$$\boxed{y = x^2 - 2 + ce^{-x^2/2}}$$

p. 65 14. $(3-x^2y)\,y' = xy^2+4$

$(xy^2+4)\,dx + (x^2y-3)\,dy = 0$

$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = 2xy$ Exact!

E.

$u_1 = \frac{x^2y^2}{2} + 4x + f(y) \qquad u_2 = \frac{x^2y^2}{2} - 3y + f(x)$

$\therefore \frac{x^2y^2}{2} + 4x - 3y = \frac{c}{2}$

$\boxed{x^2y^2 + 8x - 6y = c}$

15. $r^2 \sin\theta\, d\theta - (2r\cos\theta + 10)\,dr = 0$

$\frac{\partial M}{\partial r} = 2r\sin\theta \qquad \frac{\partial N}{\partial \theta} = 2r\sin\theta$ Exact!

E.

$u_1 = -r^2\cos\theta + f(r) \qquad u_2 = -r^2\cos\theta - 10r + f(\theta)$

$\therefore \boxed{r^2\cos\theta + 10r = c.}$

16. $y' = x^2 + 2y$ Linear

$y' - 2y = x^2 \qquad I.F. = e^{-2x}$

L.

$ye^{-2x} = \int e^{-2x}x^2\,dx$

$= \frac{e^{-2x}x^2}{-2} + \int x e^{-2x}\,dx$

$= -\frac{e^{-2x}x^2}{2} + \frac{e^{-2x}}{4}(-2x-1) + c$

$\boxed{y = -\frac{x^2}{2} - \frac{x}{2} - \frac{1}{4} + ce^{2x}}$

p.65. 17. $y' = \frac{2xy - y^4}{3x^2}$ or $(y^4 - 2xy)dx + 3x^2 dy = 0$

$$\frac{\partial M}{\partial y} = 4y^3 - 2x \quad \frac{\partial N}{\partial x} = 6x$$

I.F. is one variable.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{6x - 4y^3 + 2x}{y^4 - 2xy} = \frac{\overset{4(2x-y^3}{8x - 4y^3}}{y(y^3 - 2x)} = -\frac{4}{y}$$

$$\text{I.F.} = e^{\int -\frac{4}{y} dy} = y^{-4}$$

then $(1 - 2xy^{-3})dx + 3x^2 y^{-4}\, dy = 0$

$$u_1 = \cancel{x} - x^2 y^{-3} + f(y) \quad u_2 = -3x^2 y^{-3} + f(x)$$

$$\boxed{\therefore\ \cancel{x} - x^2 y^{-3} = c}$$

or $xy^3 = x^2 + cy^3$

18. $(x^2 + y^2)\, dx + 2y\, dy = 0 \quad y(0) = 2.$

I.F. one variable

$$\frac{\partial M}{\partial y} = 2y \quad \frac{\partial N}{\partial x} = 0$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = 1$$ Thus e^x is a I.F.

$$(x^2 + y^2)e^x\, dx + 2ye^x\, dy = 0$$

$$u_2 = y^2 e^x + f(x)$$

$\frac{\partial u}{\partial x} = y^2 e^x + f'(x)$, so $f'(x) = x^2 e^x$

$$f(x) = x^2 e^x - 2\int xe^x dx$$
$$= x^2 e^x - 2xe^x + 2e^x$$

$$u = e^x(y^2 + x^2 - 2x + 2) = c$$

$c = 6 \quad e^x(y^2 + x^2 - 2x + 2) = 6$

p.65. 19. $(x^2+y^2)\,dx + (2xy-3)\,dy = 0$

$$\frac{\partial M}{\partial y} = 2y = \frac{\partial N}{\partial x} \quad \text{Exact!}$$

E.

$$u = \frac{x^3}{3} + xy^2 - 3y = \frac{c}{3}$$

$$x^3 + 3xy^2 - 9y = c$$

20. $y'(2x+y^2) = y$

also ?? linear!

$$y\,dx - (2x+y^2)\,dy = 0$$

$$\frac{\partial M}{\partial y} = 1 \quad \frac{\partial N}{\partial x} = -2$$

L in x.

or I.F. – 1 var.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-3}{y} \quad \text{so } e^{\int -\frac{3}{y}dy} = y^{-3} = \text{I.F.}$$

$$\frac{1}{y^2}\,dx - \left(\frac{2x}{y^3} + \frac{1}{y}\right)dy = 0$$

$$u_1 = \frac{x}{y^2} \quad u_2 = \frac{x}{y^2} + \ln y$$

$$u = \frac{x}{y^2} + \ln y = c$$

$$\boxed{x = y^2 \ln y + cy^2}$$

p.66 21. $u^2 v \, du - (u^3 + v^3) dv = 0$

$$\frac{\partial M}{\partial v} = u^2 \quad \frac{\partial N}{\partial u} = -3u^2$$

IF. in 1 var.

$$\frac{\frac{\partial N}{\partial u} - \frac{\partial M}{\partial v}}{M} = \frac{-4u^2}{u^2 v} = -\frac{4}{v}, \text{ so } e^{-\int \frac{4}{v} dv} = v^{-4} = \text{IF.}$$

$$u^2 v^{-3} \, du - (u^3 v^{-4} + v^{-1}) dv = 0$$

$$U_1 = \frac{u^3}{3v^3} + f(v) \quad U_2 = \frac{u^3 v^{-3}}{3} - \ln v + f(u)$$

$$U = \frac{u^3}{3v^3} - \ln v = \frac{c}{3}$$

$$u^3 - v^3 \ln v = cv^3 \left(\text{or } u^3 v^{-3} - 3\ln v = c\right)$$

22. $(\tan y - \tan^2 y \cos x) dx - x \sec^2 y \, dy = 0$

IF. one var.

$$\frac{\partial M}{\partial y} = \sec^2 y - 2\tan y \sec^2 y \cos x \quad \frac{\partial N}{\partial x} = -\sec^2 y$$

$$= \sec^2 y (1 - 2\tan y \cos x)$$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{\sec^2 y(-2 + 2\tan y \cos x)}{\tan y (1 - \tan y \cos x)} = -\frac{2\sec^2 y}{\tan y}$$

$$= -\frac{2}{\sin y \cos y} = -4 \csc 2y$$

$$\text{I.F.} = e^{-2\int \csc 2y \, 2dy} = e^{-2 \ln \tan y} = \frac{\cos^2 y}{\sin^2 y} \text{ or } \cot^2 y$$

22. (con't)

$$(\cot y - \cos x)\,dx - (x \csc^2 y)\,dy = 0$$

$$u_1 = x \cot y - \sin x + f(y)$$

$$\frac{\partial u_1}{\partial y} = -x \csc^2 y + f'(y), \text{ so } f'(y) = 0.$$

$$u = \boxed{x \cot y - \sin x = c}$$

p.66 23. $\frac{dy}{dx} = \frac{x+2y}{y-2x}$ or $(x+2y)\,dx + (2x-y)\,dy = 0$

Exact.
Homogeneous.

$x\,dx - y\,dy + 2(y\,dx + x\,dy) = 0$

D.

$\frac{x^2}{2} - \frac{y^2}{2} + 2(xy) = \frac{c}{2}$

$\boxed{x^2 + 4xy - y^2 = c}$

24. $y' \sin x = y \cos x + \sin^2 x$

L.

I.F. of $\frac{1}{\sin^2 x}$, if we do not use linear method. linear I.F. = $1/\sin x$

$y' - (\cot x)y = \sin x$

$I.F. = e^{-\int \cot x\,dx} = e^{\ln \csc x} = \csc x$

$\cdot y \csc x = \int 1\,dx + c$

$y = (x + c)\sin x$

H. or D.

25. $(x^2 - y^2)\,dx + 2xy\,dy = 0$

also homogeneous.

$\frac{2xy^2\,dy - xy^2\,dx}{x^2} + dx = 0$

$\frac{y^2}{x} + x = c$

I.F.

$\boxed{y^2 + x^2 = cx}$

E. 26. $(2x^2 - ye^x)\,dx - e^x\,dy$. Exact.

$\frac{\partial M}{\partial y} = -e^x = \frac{\partial N}{\partial x}$

$u = \frac{2x^3}{3} - ye^x = \frac{c}{3}$

$\boxed{2x^3 - 3ye^x = c}$

p.66 27. $(x+y)y' = 1$ (Note: also linear in x!)

$$dx - (x+y)\,dy = 0$$

$$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = -1$$

L.

I.F.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = -1 \quad \text{so} \quad e^{-\int dy} = e^{-y}$$

$$e^{-y}dx - (x+y)e^{-y}\,dy = 0$$

$$u_1 = xe^{-y} + f(y)$$

$$\frac{\partial u_1}{\partial y} = -xe^{-y} + f'(y), \quad \text{so} \quad f'(y) = -ye^{-y}$$

$$f(y) = e^{-y}(-y-1)$$

$$u = -xe^{-y} - ye^{-y} - e^{-y} = -c$$

$$\boxed{x + y + 1 = ce^{y}}$$

28. $(x + 2y)\,dx + x\,dy = 0$

D.

I.F.

or $x^2dx + 2xy\,dy + x^2dy = 0$

$$\frac{x^3}{3} + x^2y = \frac{c}{3}, \quad \text{or} \quad \boxed{x^3 + 3x^2y = c}$$

29. $\sin y\,dx + (x\cos y - y)\,dy = 0$

D.

E.

$$\sin y\,dx + x\cos y\,dy - y\,dy = 0$$

$$x\sin y - \frac{y^2}{2} = \frac{c}{2}$$

$$\boxed{2x\sin y - y^2 = c}$$

p. 66 30. $y' = e^{y/x} + \frac{y}{x}$ Homogeneous $v = \frac{y}{x}$

$v + xv' = e^{v} + v$

$e^{-v}dv = \frac{dx}{x}$

H. $-e^{-v} = \ln x + c$

$\boxed{\ln x + e^{-y/x} = c}$

31. $\sin x \cos y \, dx + \cos x \sin y \, dy = 0.$

E. $\frac{\partial M}{\partial y} = -\sin x \sin y = \frac{\partial N}{\partial x}$, so exact!

$u = -\cos x \cos y = -c$, so $\boxed{\cos x \cos y = c}$

32. $xy' = x^3 + 2y$ linear.

$y' - \frac{2}{x}y = x^2$

I.F. $= e^{\int \frac{2}{x} dx} = \frac{1}{x^2}$

L.

$\frac{y}{x^2} = \int x^2 \cdot \frac{1}{x^2} \, dx + c$

$= x + c$ so $\boxed{y = x^3 + cx^2}$

33. $(3xy^2 + 2) \, dx + 2x^2 y \, dy = 0$

D. $3x^2y^2dx + 2x^3y\,dy + 2x\,dx = 0$

I.F. $\boxed{x^3y^2 + x^2 = c}$

p. 66 34. $(2y^2 - x)\,dy + y\,dx = 0$

$2y^2 dy - \left[x\,dy - y\,dx\right] = 0$

D.

$2dy - \left[\frac{x\,dy - y\,dx}{y^2}\right] = 0$

$\boxed{2y + \frac{x}{y} = c}$

$2y^2 + x = cy$

35. $y'' = y' + 2x$ $y' = v$

$v' = v + 2x$ $y'' = v'$

L.

$v' - v = 2x$ $I.F. = e^{-\int dx} = e^{-x}$

$ve^{-x} = \int 2xe^{-x}\,dx = -2\left[e^{-x}(x+1)\right] + c_1$

$v = -2(x+1) + c_1e^{x}$

$\boxed{y = -x^2 - 2x + c_1e^{x} + c_2}$

also variables separable

36. $(1+y)\frac{dy}{dx} = x\sqrt{y}$ let $u = \sqrt{y}$ $du = \frac{1}{2\sqrt{y}}dy$

$(1+u^2)2u\,du = xu\,dx$ $2u^2 = y$ $dy = 2u\,du$

$2(1+u^2)\,du = x\,dx$ $2u\,du = dy.$

V.S.

$2u + \frac{2u^3}{3} = \frac{x^2}{2} + c$

$\boxed{2y^{1/2} + \frac{2}{3}y^{3/2} = \frac{x^2}{2} + c}$

p.66. 37. $\tan x \sin y \, dx + 3 \, dy = 0$

$\tan x \, dx + 3 \csc y \, dy = 0$

V.S.

or $\begin{cases} \ln \sec x + 3 \ln \tan \frac{y}{2} = \ln C \\ -\ln \cos x + 3 \ln(\csc y - \cot y) = \ln c \end{cases}$

$\sec x (\csc y - \cot y)^3 = c$

$\frac{1}{\cos x}\left(\frac{1}{\csc y + \cot y}\right)^3 = c$ or $\cos x (\csc y + \cot y)^3 = c$

38. $x \, dy - y \, dx = x \cos\left(\frac{y}{x}\right) dx$ Homogeneous. $y = vx$

H.

$dy - \frac{y}{x} dx = \cos\left(\frac{y}{x}\right) dx$

$dy = (v + \cos v)d$

$v + xv' = \frac{dy}{dx} = v + \cos v$

$xv' = \cos v$ so $\frac{dv}{\cos v} = \frac{dx}{x}$

$\ln(\sec v + \tan v) = \ln x + \ln c.$

$\boxed{\sec v + \tan v = cx.}$

39. $\frac{ds}{dt} = \sqrt{\frac{1-t}{1-s}}$, $s = 0$ when $t = 1$

V.S.

$\sqrt{1-s} \, ds = \sqrt{1-t} \, dt$

$-\frac{2}{3}(1-s)^{3/2} = -\frac{2}{3}(1-t)^{3/2} + \frac{2}{3}c$, $c = -1$

$(1-s)^{3/2} - (1-t)^{3/2} = 1$

p.66 40. $(2y + 3x)\,dx + x\,dy = 0$

I.F. $3x^2\,dx + 2xy\,dx + x^2\,dy = 0$

D. L. $x^3 + x^2y = c$

V.S. 41. $x^2y\,dx + (1+x^3)\,dy = 0$

$u = 1+x^3$
$du = 3x^2$

$$\frac{x^2}{1+x^3}\,dx + \frac{dy}{y} = 0$$

$$\frac{1}{3}\ln(1+x^3) + \ln|y| = c$$

$y(1+x^3)^{1/3} = c$ or $\boxed{y^3(1+x^3) = c}$

42. $(\sin y - x)\,y' = 2x + y \quad y(1) = \frac{\pi}{2}$

(exact also)

$(2x+y)\,dx + (x - \sin y)\,dy = 0$

$2x\,dx + y\,dx + x\,dy - \sin y\,dy = 0.$

E.

D.

$x^2 + xy + \cos y = c$

$1 + \frac{\pi}{2} + 0 = c, \quad c = \frac{2+\pi}{2}$

$$\boxed{2x^2 + 2xy + 2\cos y = 2+\pi}$$

p.57 43. $\frac{dN}{dt} = -\alpha N \qquad N = N_0$ at $t = 0$

V.S.

L.

$\frac{dN}{N} = -\alpha\,dt$, so $\ln|N| = -\alpha t + c$, $c = \ln N_0$

$\ln|N| + \alpha t = \ln N_0$

or $\ln\left|\frac{N}{N_0}\right| = -\alpha t$

Familiar? No

$$\boxed{N = N_0 e^{-\alpha t}}$$

p. 66. 44. $\frac{dy}{dx} = \frac{y(x+y)}{x(x-y)}$ Homogeneous $y = vx$

$$v + xv' = v\left(\frac{1+v}{1-v}\right)$$

$$xv' = v\left[\left(\frac{1+v}{1-v}\right) - 1\right] = v\left(\frac{1+v-1+v}{1-v}\right) = v\left(\frac{2v}{1-v}\right)$$

H.

$$\frac{1-v}{2v^2}dv = \frac{dx}{x} \quad \text{or} \quad \frac{1}{2v^2}dv - \frac{1}{2v}dv = \frac{1}{x}dx$$

$$-\frac{1}{2v} - \frac{1}{2}\ln v = \ln x + \ln c$$

$$-\frac{1}{v} - \ln v = 2\ln x + \ln c$$

$$-\frac{1}{v} = \ln cvx^2$$

$$-\frac{x}{y} = \ln cxy$$

so $cxy = e^{-x/y}$ or

$\boxed{xye^{x/y} = c_1}$ where $c = \frac{1}{c_1}$.

45. $\frac{dI}{dt} + I = e^t$

(linear!) I.F. $= e^t$

L.

$$Ie^t = \int e^{2t}\,dt = \frac{1}{2}e^{2t} + c$$

$$\boxed{I = \frac{1}{2}e^t + ce^{-t}}$$

$y(1)=2$

p.66. 46. $xy' + y = x^2$ Linear!

$(x^2-y)\,dx - x\,dy = 0$ Exact

L. $u = \frac{x^3}{3} - xy = \frac{c}{3}$ or $\boxed{x^3 - 3xy = c}$

E. $y(1)=2 \Rightarrow c = 1-6 = -5$

$x^3 - 3xy = -5$

$\boxed{\text{or } x^3 + 5x = 3xy}$

47. $x\,dy - y\,dx = x^2 y\,dy$

D. $\frac{x\,dy - y\,dx}{x^2} = y\,dy$

I.F. $\frac{y}{x} = \frac{y^2}{2} + \frac{c}{2}$, or $\boxed{xy^2 - 2y = cx}$

48. $\frac{dq}{dp} = \frac{p}{q} e^{p^2 - q^2}$ Homogeneous?

H. $\frac{q\,dq}{p\,dp} = \frac{e^{p^2}}{e^{q^2}}$ Let $u = p^2$ $v = q^2$

V.S. $du = 2p\,dp$ $dv = 2q\,dv.$

$\frac{dv}{du} = e^{u^2 - v^2}$ or $\frac{dv}{du} = \frac{e^{u^2}}{e^{v^2}}$ or $e^{v^2}\,dv = e^{u^2}\,du$

$e^v - e^u = c$

$\boxed{e^{p^2} - e^{q^2} = c.}$

49. $(3y\cos x + 2)y' = y^2 \sin x;\ y(0) = -4.$

$y^2 \sin x\,dx - 3y\cos x\,dy - 2\,dy = 0$

D. $\underbrace{y^3 \sin x\,dx - 3y^2 \cos x\,dy} - 2y\,dy = 0$

I.F. $-y^3 \cos x - y^2 = c$

$64 - 16 = 48 = c$

$\boxed{y^2 + y^3 \cos x + 48 = 0}$