p. 66 50. $(x + x\cos y)\,dy - (y + \sin y)\,dx = 0$

$x(1+\cos y)\,dy = (y+\sin y)\,dx$

$\frac{(1+\cos y)\,dy}{y+\sin y} = \frac{dx}{x}$

V.S.

$\ln(y+\sin y) = \ln x + \ln c$

$\boxed{cx = y + \sin y}$

51. $y' = 3x + 2y$ linear!

$y' - 2y = 3x$ I.F. $= e^{-2x}$

L.

ans. in book wrong.

$y e^{-2x} = \int 3x e^{-2x}\,dx$

$= -3\frac{e^{-2x}}{4}(2x+1) + c$

$\boxed{y = -\frac{3}{2}x - \frac{3}{4} + ce^{2x}}$ or $4y+6x+3 = ce^{2x}$

52. $y^2\,dx = (2xy + x^2)\,dy$ Homogeneous. $y = vx$

$dy =$

H.

I.F.

$v + xv' = \frac{dy}{dx} = \frac{y^2}{2xy+x^2} = \frac{v^2}{2v+1}$

$xv' = \frac{v^2 - 2v^2 - v}{2v+1} = -\frac{v^2+v}{2v+1}$

$\frac{2v+1\,dv}{v^2+v} = -\frac{dx}{x}$

$\ln(v^2+v) = -\ln x + \ln c$

$x(v^2+v) = c$ or $y^2 + xy = xc$

$\boxed{y^2 = x(c-y)}$

p. 66. 53. $\frac{dr}{d\theta} = \frac{r(1+\ln\theta)}{\theta(1+\ln r)}$ $\theta = e^2$ where $r = e$

$u = 1 + \ln r$
$du = \frac{1}{r}dr$

$$\frac{1+\ln r}{r}\,dr = \frac{(1+\ln\theta)\,d\theta}{\theta} = u\,du$$

V.S.

$$\tfrac{1}{2}(1+\ln r)^2 = \tfrac{1}{2}(1+\ln\theta)^2 + \frac{c}{2}$$

$$(1+\ln e)^2 = (1+\ln e^2)^2 + c\,,\quad 2^2 = 3^2 + C,\quad C = -5$$

$$(1+\ln r)^2 = (1+\ln\theta)^2 - 5 \quad \text{or} \quad \boxed{2\ln r + (\ln r)^2 = 2\ln\theta + (\ln\theta)^2 - 5}$$

54. $\frac{dU}{dt} = -a(U - 100at)$ $U(0) = 0$

L.

$u' + aU = 100at$ linear

$$I.F. = e^{at}$$

$$Ue^{at} = 100a\int te^{at}\,dt$$

$$= 100a\frac{e^{at}}{a^2}(at-1) + C \qquad 0 = \frac{100}{a}(-1) + c$$

$$U = 100t - \frac{100}{a} + \frac{100}{a}e^{-at} \qquad c = \frac{100}{a}$$

$$= 100t - \frac{100}{a}(1 - e^{-at})$$

$2u - 1$

55. $(uv - 2v)\,du + (u - u^2)\,dv = 0$ or $v(u-2)\,du = u(u-1)\,dv$

$u -$ V.S.

$$= \frac{(u-2)\,du}{u^2-u} = \frac{dv}{v} = \frac{u\,du}{u(u-1)} - \frac{2\,du}{u(u-1)} \quad a = -1\ b = 1$$

$$\ln v = \ln(u-1) - 2\left[-\frac{1}{-1}\cdot\ln\frac{u-1}{u}\right] + \ln C$$

$$= \ln(u-1) - \ln\left(\frac{u-1}{u}\right)^2 + \ln C$$

$$\frac{v}{u^2}(u-1) = C \qquad \boxed{v(u-1) = u^2C}$$

53. May write $\frac{1+\ln r}{r} dr$ as $\frac{dr}{r} + \frac{\ln r}{r} dr$

$$\int = \ln r + \frac{(\ln r)^2}{2} + c$$

55. $\frac{u-2}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$$(A+B) = 1$$

$$-A = -2$$

$$\boxed{\begin{array}{l} B = -1 \\ A = 2 \end{array}}$$

p.66. 56. $\frac{dI}{dt} + 3I = 10 \sin t \qquad I(0) = 0$

linear! $\quad I.F. = e^{3t}$

L.

$$Ie^{3t} = 10\int e^{3t} \sin t \, dt$$

$$= 10\, e^{3t}\left[\frac{3 \sin t - \cos t}{10}\right] + c$$

$$I = 3 \sin t - \cos t + c e^{-3t} \qquad 0 = 0 - 1 + c$$

$$\boxed{I = 3 \sin t - \cos t + e^{-3t}} \qquad c = 1$$

57. $\frac{ds}{dt} = \frac{1}{s+t+1} \qquad -dt + (s+t+1)\,ds = 0$

$\left(\frac{dt}{ds} = s+t+1 \text{ Linear!}\right) \qquad \frac{\partial M}{\partial s} = 0 \quad \frac{\partial N}{\partial t} = 1$

$$\frac{\frac{\partial N}{\partial t} - \frac{\partial M}{\partial s}}{m} = \frac{1}{-1}, \text{ so } e^{-s} \text{ is I.F.}$$

L.
I.F..
one way.

$$-e^{-s}dt + (s+t+1)e^{-s}ds = 0$$

$$u_1 = -te^{-s} + f(s)$$

$$\frac{\partial u}{\partial s} = te^{-s} + f'(s) \quad \text{so } f'(s) = (s+1)e^{-s}$$

$$f(s) = e^{-s}(-s-1) - e^{-s}$$

$$= -e^{-s}(s+2) + e^{c'}$$

$$u = -e^{-s}(t+s+2) = -e^{c'}$$

$$(t+s+2) = e^{c'+s}$$

$$\ln(t+s+2) = c' + s$$

$$\boxed{s - \ln(t+s+2) = c'}$$

57. $\frac{dt}{ds} - t = s+1$

I.F. = e^{-s}

$te^{-s} = \int (s+1)e^{-s}\,ds$

$= \int se^{-s}ds + \int e^{-s}\,ds$

$= -e^{-s}(s+1) - e^{-s} + e^{-c}$

$-t = s+1+1-e^{-c+s}$

$e^{-c+s} = -(t+s+2)$

$-c+s = \ln(t+s+2)$

$\boxed{s - \ln(t+s+2) = c}$

$$y^2 = c_1 x + c_2$$

$$2y\,y' = c_1 \qquad y' = \frac{c_1}{2y}$$

$$2\left[yy'' + (y')^2\right] = 0$$

$$\boxed{\begin{aligned} y &= c_2 e^{c_1 x} \\ y' &= c_1 c_2 e^{c_1 x} \\ y'' &= c_1^2 c_2 e^{c_1 x} \\ (y')^2 &= c_1^2 c_2^2 e^{2c_1 x} \\ yy'' &= c_1^2 c_2^2 e^{2c_1 x} \end{aligned}}$$

p.66. 58. Should be: $yy'' + (y')^2 = 0$

V.M.

let $y' = v$ $\qquad y\frac{dv}{dx} + (v)^2 = 0$

$$y\frac{dv}{dy}\frac{dy}{dx} + v^2 = 0$$

$$y\frac{dv}{dy}v + v^2 = 0$$

exact $\longrightarrow$ $y\,dv + v\,dy = 0$

$$u = vy = c_1 \quad \text{or} \quad y'y = \frac{c_1}{2}$$

$$y\,dy = \frac{c_1}{2}dx$$

$$\frac{y^2}{2} = \frac{c_1}{2}x + \frac{c_2}{2}$$

$$y^2 = 2c_1 x + c_2$$

(See across page) Problem is wrong!

p 67. 72. $yy'' = (y')^2$ $\quad y' = v$

V.M.

$y\frac{dv}{dx} = v^2$

$y\frac{dv}{dy}\frac{dy}{dx} = v^2$

$y\frac{dv}{dy}v = v^2$ so $y\,dv = v\,dy$

$\ln v = \ln y + \ln c_1$

$v = c_1 y$

$y' = c_1 y$

$\frac{dy}{y} = c_1\,dx$

$\ln y = c_1 x + \ln c_2$

$\frac{y}{c_2} = e^{c_1 x}$ or $\boxed{y = c_2 e^{c_1 x}}$

p. 66. 59. $x\sqrt{1-y^2} + yy'\sqrt{1-x^2} = 0$

$y\,dy\sqrt{1-x^2} + x\sqrt{1-y^2}\,dx = 0$

$\frac{du}{\sqrt{u}}$ V.S.

$\frac{y}{\sqrt{1-y^2}}dy + \frac{x\,dx}{\sqrt{1-x^2}} = 0$

E.

$\sqrt{1-y^2} + \sqrt{1-x^2} = c$

60. $y' + \cot x\,y = \cos x$ linear

I.F. $= e^{\int \cot x\,dx} = \sin x$

L.

$y\sin x = -\int \cos x(-\sin x\,dx) = -\frac{\cos^2 x}{2} + \frac{c}{2}$

$\boxed{2y\sin x + \cos^2 x = c}$

p. 66. 62. $xy' - 3y = x^4 e^{-x}$

$y' - \frac{3}{x} y = x^3 e^{-x}$ linear. I.F. $= e^{-3\int \frac{1}{x} dx} = x^{-3}$

L.

$y x^{-3} = \int x^3 e^{-x} \cdot x^{-3} \, dx$

$= \int e^{-x} \, dx = -e^{-x} + c$

$y = -x^3 e^{-x} + cx^3$ or $\boxed{y = cx^3 - x^3 e^{-x} \text{ or } y = x^3(c - e^{-x})}$

61. $y' = \left(\frac{y+3}{2x}\right)^2$ or $\frac{dy}{(y+3)^2} = \frac{dx}{4x^2}$

V.S.

$-(y+3)^{-1} = -\frac{1}{4} x^{-1} - c$

or $\boxed{(y+3)^{-1} = (4x)^{-1} + c}$

63. $y' = \sin x \tan y$

V.S. $\frac{dy}{\tan y} = \sin x \, dx$

$\ln|\sin y| = -\cos x + c$ or $\ln|\sin y| + \cos x = c$

64. $y' = \frac{x}{y} + \frac{y}{x}$ let $v = \frac{y}{x}$

$v + xv' = \frac{1}{v} + v$

H.

$v \, dv = \frac{dx}{x}$ so $\frac{v^2}{2} = \ln x + \frac{c}{2}$

$y^2 = 2x^2 \ln x + cx^2$

$\boxed{\text{or } y^2 = x^2(2 \ln x + c)}$

p. 66. 65. $x\,dy - y\,dx = 2x^2y^2\,dy$

$$\frac{x\,dy - y\,dx}{x^2} = 2y^2\,dy$$

D.

$$\frac{y}{x} = \frac{2}{3}y^3 - \frac{c}{3}$$

$$\boxed{2xy^3 - 3y = cx}$$

66. $xy' + y\ln x = y\ln y + y$ Homogeneous.

H.

$$xy' = y(\ln y - \ln x + 1)$$

$$y' = \frac{y}{x}\left(\ln\frac{y}{x} + 1\right) \qquad \text{let } \frac{y}{x} = v$$

$$v + xv' = v(\ln v + 1)$$

$$x\frac{dv}{dx} = v\ln v \quad \text{so} \quad \frac{dv}{v\ln v} = \frac{dx}{x}$$

$u = \ln v$
$du = \frac{1}{v}du$

$$\ln(\ln v) = \ln x + \ln c.$$

$$\ln v = cx$$

$$\ln|y| = cx - \ln|x|.$$

67. $y' = 2 - \frac{y}{x}$ linear!

L.

$$y' + \frac{1}{x}y = 2$$

$$\text{I.F.} = e^{\ln x} = x$$

$$yx = \int 2x\,dx + c$$

$$yx = x^2 - c$$

$$\boxed{x^2 - xy = c}$$

p.67 68. $xy'' + y' = 1$ let $v = y'$

$xv' + v = 1$ linear, also exact.

$x\,dv + (v-1)\,dx = 0$

E. $u = xv - x = c_1$

L. $v - 1 = \frac{c_1}{x}$

V.S. V.M., (after V.M., that is!) $dy = \left(1 + \frac{c_1}{x}\right)dx$

$$\boxed{y = x + c_1 \ln x + c_2}$$

69. $\frac{dI}{dt} = \frac{It^2}{t^3 - I^3}$ let $I = vt$

I.F. $v + tv' = \frac{vt^3}{t^3 - v^3t^3} = \frac{v}{1-v^3}$

$v - v^4 + (1-v^3)tv' = v$

H. then V.S. $(1-v^3)t\frac{dv}{dt} = v^4$ so $\frac{1-v^3}{v^4}dv = \frac{dt}{t}$

$\left(\frac{1}{v^4} - \frac{v^3}{v^4}\right)dv = \frac{dt}{t}$

$-\frac{1}{3v^3} - \ln v = \ln t - \frac{c}{3}$

$-\frac{1}{v^3} = \ln (vt)^3 - c$

$-\frac{t^3}{I^3} = \ln I^3 - c$

$t^3 I^{-3} + 3\ln I = c$

p67 70. $(e^y + x + 3)y' = 1$

$(e^y + 3\,dy) + x\,dy - dx = 0$

$(1 + 3e^{-y})dy + \frac{x\,dy - dx}{e^y} = 0$

$y - 3e^{-y} - xe^{-y} = c$

$xe^{-y} = y - 3e^{-y} + c$

$\boxed{x = ye^y - 3 + ce^y}$

Note:
also linear
in $\frac{dx}{dy}$

L.
D.

71. $\frac{dr}{d\theta} = e^\theta - 3r$; $r = 1$ at $\theta = 0$.

linear: $\frac{dr}{d\theta} + 3r = e^\theta$

L.

$I.F. = e^{3\theta}$

$re^{3\theta} = \int e^{4\theta}\,d\theta = \frac{1}{4}e^{4\theta} + c$

$1 = \frac{1}{4} + c$; $c = 3/4$

$\boxed{4re^{3\theta} - e^{4\theta} = 3}$

$\boxed{r \; r = \frac{e^\theta}{4} + \frac{3e^{-3\theta}}{4}}$

72. See problem 58 $yy'' = (y')^2$
Already worked four pages ago!

p.67 73. $x^4 y''' + 1 = 0$

$$y''' = -x^{-4}$$

integration

$$y'' = \frac{+x^{-3}}{3} + k_1$$

$$y' = \frac{-x^{-2}}{6} + k_1 x + k_2$$

I.

$$y = \frac{1}{6x} + \frac{k_1 x^2}{2} + k_2 x + k_3$$

or $\boxed{y = \frac{1}{6x} + c_1 + c_2 x + c_3 x^2}$

$\boxed{\text{or } 6xy = 1 + c_1 x + c_2 x^2 + c_3 x^3}$

$\boxed{\text{Hard}}$ 74. $\frac{dy}{dx} = \frac{x+3y}{x-3y}$ let $y = vx$

Homogeneous.

H.

$$v + xv' = \frac{x+3vx}{x-3vx} = \frac{1+3v}{1-3v}$$

$$x\frac{dv}{dx} = \frac{1+3v-v+3v^2}{1-3v} = \frac{1+2v+3v^2}{1-3v}$$

$$\frac{1-3v}{1+2v+3v^2}dv = \frac{dx}{x} = \frac{dv}{1+2v+3v^2} - \frac{3v\,dv}{1+2v+3v^2}$$

$$\ln x = \frac{2}{\sqrt{8}}\tan^{-1}\frac{6v+2}{\sqrt{8}} - \cancel{3}\left[\frac{1}{2\cdot\cancel{3}}\ln(1+2v+3v^2) - \frac{2}{2\cdot\cancel{3}}\int\frac{dv}{1+2v+3v^2}\right]$$

$$= (1+1)\frac{1\,\sqrt{2}}{\sqrt{2}\sqrt{2}}\tan^{-1}\frac{3v+1}{\sqrt{2}} - \frac{1}{2}\ln|1+2v+3v^2| + c.$$

$$2\ln x = 2\sqrt{2}\tan^{-1}\frac{3y+x}{\sqrt{2}x} - \ln\left|\frac{x^2+2xy+3y^2}{x^2}\right| + c$$

$$\ln(x^2+2xy+3y^2) = 2\sqrt{2}\tan^{-1}\left(\frac{3y+x}{\sqrt{2}x}\right) + c.$$

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Hard

75. $y' \cos x = y - \sin 2x \qquad \sin 2x = 2 \sin x \cos x$

linear $\quad y' - (\sec x) y = -2 \sin x \quad$ Linear!

L.

$$\text{I.F.} = e^{-\int \sec x \, dx} = e^{-\ln(\sec x + \tan x) dx} = \frac{1}{\sec x + \tan x}$$

$$= \boxed{\sec x - \tan x \text{ is easier}} \quad \frac{1}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos x}{1 + \sin x}$$

$$\frac{y}{\sec x + \tan x} = \int \frac{-2 \sin x}{\sec x + \tan x} dx = \int \frac{-2 \sin x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$\overset{''}{y(\sec x - \tan x)} = -2 \int \frac{\sin x \cos x}{1 + \sin x} dx = -2 \int \frac{u \, du}{1+u}$$

$$= -2 \left[u - \ln(1+u) \right] + c$$

$$y (\sec x - \tan x) = -2 \sin x + 2 \ln (1 + \sin x) + c$$

76. $e^{2x-y} dx + \overset{e^{-(2x}}{e^{y-2x}} dy = 0$

Variable separable.

V.S.

$$e^{2x} \cdot e^{-y} dx + e^{y} \cdot e^{-2x} dy = 0$$

$$e^{4x} dx + e^{2y} dy = 0$$

$$\frac{e^{4x}}{4} + \frac{e^{2y}}{2} = \frac{c}{4} \quad \text{or} \quad e^{4x} + 2e^{2y} = c$$

77. $r^3 \frac{dr}{d\theta} = \sqrt{a^8 - r^8}$

V.S.

$$\frac{4 r^3}{4 \sqrt{(a^4)^2 - (r^4)^2}} dr = d\theta$$

$$\frac{1}{4} \sin^{-1}\left(\frac{r^4}{a^4}\right) = \theta + \frac{c}{4}$$

$$\sin^{-1}\left(\frac{r^4}{a^4}\right) = 4\theta + C \quad \text{or} \quad \sin(4\theta + C) = \left(\frac{r}{a}\right)^4$$

$$\text{or} \left(r^4 = a^4 \sin(4\theta + C) \right)$$

76, 77, 79, 81

p.67 78. $(2x^2 - ye^x)\,dx - e^x\,dy = 0$

D. E.

Differential and exact

$2x^2\,dx - e^x(y\,dx + dy) = 0$

$\frac{2x^3}{3} - ye^x = \frac{c}{3}$

also note: Exact.

or $\boxed{2x^3 - 3ye^x = c}$

79. $x\,dy + 2y\,dx - x\cos x\,dx = 0$

$x^2\,dy + 2xy\,dx - \overset{u}{x^2}\,\overset{dv}{\cos x\,dx} = 0$

D.

$x^2y - \left[x^2\sin x - 2\int \overset{u}{x}\,\overset{dv}{\sin x\,dx}\right] = c$

use handbook for integration

$x^2y - \left[x^2\sin x + 2\left(-x\cos x + \int\cos x\,dx\right)\right] = c$

$x^2y - x^2\sin x - 2x\cos x + 2\sin x = c$

$x^2y = 2x\cos x + (x^2-2)\sin x + c$

80. $\sqrt{1+x^3}\,\frac{dy}{dx} = x^2(y+1)$

V.S.

$\frac{dy}{y+1} = \frac{3x^2\,dx}{3\sqrt{1+x^3}} = \frac{du}{3\sqrt{u}}$

$\ln|y+1| = \frac{2}{3}\sqrt{1+x^3} + \frac{c}{3}$

$3\ln|y+1| = 2\sqrt{1+x^3} + c$

p67. 81. $(3y^2+4xy)dx + (2xy+x^2)dy = 0$

Homogeneous

$$\frac{dy}{dx} = -\frac{(3y^2+4xy)}{(2xy+x^2)} \qquad \text{Let } y = vx$$

I.F.

$$v + x\frac{dv}{dx} = -\frac{3v^2x^2+4vx^2}{2x^2v+x^2} = -\frac{3v^2+4v}{2v+1}$$

$$x\frac{dv}{dx} = \frac{-3v^2-4v-2v^2-v}{2v+1} = \frac{-5v^2-5v}{2v+1} = 5(2v$$

$$\frac{2v+1}{-5(v^2+v)}dv = \frac{dx}{x}$$

$$-\frac{1}{5}\ln|v^2+v| = \ln x - \frac{1}{5}\ln c$$

$$x^5(v^2+v) = c$$

$$x^5\left(\frac{y^2}{x^2}+\frac{y}{x}\right) = c$$

$$\boxed{x^3y^2 + x^4y = c}$$

60.5

p.67 82. $y' = y(x+y)$

$$\frac{dx}{dy} = \frac{1}{y(x+y)}$$

$(xy+y^2)dx - dy = 0$

Not exact, Not linear, Not Homogeneous. Not variable separable, not integ factor.

Need a trick: Let $v = y^{-1}$

$y' = xy + y^2$ $\qquad v' = -y^{-2}y'$

$y^{-2}y' = y^{-1}x + 1$

$0 = -y^{-2}y' + y^{-1}x + 1$

$0 = v' + vx + 1$

$v' + vx = -1$

Linear in v. $IF = e^{\frac{x^2}{2}}$

$$ve^{x^2/2} = \int -e^{x^2/2}dx + c$$

$$y^{-1} = v = -e^{-x^2/2}\int e^{x^2/2}dx + ce^{-x^2/2}$$

p.67. 83. $y' = x^2 + xy$

L.

$$y' - xy = x^2$$

$$I.F. = e^{\int -x\,dx} = e^{-\frac{x^2}{2}}$$

$$ye^{-\frac{x^2}{2}} = \int x^2 e^{-\frac{x^2}{2}}\,dx + c$$

$$y = e^{\frac{x^2}{2}} \int x^2 e^{-\frac{x^2}{2}}\,dx + ce^{\frac{x^2}{2}}$$

84. $\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} = 4(1-r)$ $\quad u = 15, \ \frac{du}{dr} = 0$ at $r = 1$

V.M.

then L.

let $V = \frac{du}{dr}$, $\frac{dV}{dr} = \frac{d^2u}{dr^2}$

$$\frac{dV}{dr} + \frac{1}{r}V = 4(1-r)$$

$$I.F. = e^{\int \frac{1}{r}dr} = e^{\ln r} = r$$

$$V \cdot r = \int 4(1-r) \cdot r\,dr$$

$$= \int 4r - 4r^2\,dr$$

$$V \cdot r = 2r^2 - \frac{4r^3}{3} + c_1 \qquad 0 = 2 - \frac{4}{3} + c_1$$

$$c_1 = -\tfrac{2}{3}$$

$$\frac{du}{dr} = V = 2r - \frac{4r^2}{3} - \frac{2}{3}r^{-1}$$

$$u = r^2 - \frac{4r^3}{9} - \frac{2}{3}\ln r + c_2$$

$$15 = 1 - \frac{4}{9} - 0 + c_2 \qquad c_2 = 14\tfrac{4}{9} = \frac{130}{9}$$

$$\boxed{U = \frac{130}{9} + r^2 - \frac{4}{9}r^3 - \frac{2}{3}\ln r}$$

$$(\sec x + \tan x)(\sec x + \tan x)$$

$$= \sec^2 x - \tan^2 x = \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$

p. 67

85. $\frac{dy}{dx} = 1 - (x-y)^2 \quad y(0) = 1$

Let $v = x - y$

$$\frac{dv}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

$$\cancel{1} - \frac{dv}{dx} = \cancel{1} - v^2$$

$$\frac{dv}{dx} = v^2$$

$$\frac{dv}{v^2} = dx$$

$$\frac{v^{-1}}{-1} = x + c$$

$$\frac{1}{y-x} = x + c$$

$$\frac{1}{1-0} = 0 + c \quad c = 1$$

$$\boxed{1 = (x+1)(y-x)}$$

86. $\dfrac{dy}{dx} = \dfrac{e^{x-y}}{y}$

V.S.

$$\frac{dy}{dx} = \frac{e^x}{ye^y}$$

$u=y \quad dv=e^y dy$

$du=dy \quad v=e^y$

$$\int ye^y\,dy = \int e^x\,dx$$

$$ye^y - \int e^y dy = e^x + c_1$$

$$ye^y - e^y = e^x + c_1$$

$$\boxed{e^x + e^y - ye^y = C}$$

or $\boxed{e^y(1-y) + e^x = c}$

or $\boxed{(y-1)e^y - e^x = c}$

$$\frac{1-3V}{(1+3V)(1-V)} = \frac{A}{1+3V} + \frac{B}{1-V}$$

$A + B = 1$

$-A + 3B = -3$

$4B = -2$

$B = -1/2$

$A = 3/2$

p.67 84. $y' = \frac{2}{x+2y-3}$ $\qquad \frac{dv}{dx} = 1 + 2\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2}{v} \qquad \frac{1}{2}\left(-1 + \frac{dv}{dx}\right) = \frac{dy}{dx}$$

$$\frac{1}{2}\left(-1 + \frac{dv}{dx}\right) = \frac{2}{v}$$

$$-1 + \frac{dv}{dx} = \frac{4}{v}$$

$$\frac{dv}{dx} = \frac{4+v}{v}$$

$$\begin{array}{r|l} & 1 \\ v+4 & v \\ & v+4 \\ \hline & -4 \end{array}$$

$$\frac{v\,dv}{4+v} = dx$$

$$\left(1 - \frac{4}{v+4}\right) dv = dx$$

$$v - 4\ln(v+4) = x + c$$

$$\cancel{x} + 2y - 3 - 4\ln|x+2y+1| = \cancel{x} + c$$

$$2y - 3 = \ln(x+2y+1)^4 + \ln c_1$$

ANS Does not agree.

$$\boxed{c_1(x+2y+1)^4 = e^{2y-3}}$$