MAC 2233 EXAM 2A Dr. Rapalje NAME _____
Sections 2.4 – 3.3

Show all work on this test or on separate paper. Turn in ALL worksheets.

1. If $f(x)=\frac{3}{4x^3}$, find f ′ (x) . (Simplify and express final answer without negative exponents!)

2. If $f(x)=\frac{x^2+1}{x^5-1}$, find f ′ (x) . (Give answer in factored form!)

3. A company's profit function is P(x) = 12x –1800 dollars.
   a) Find the average profit function AP(x) = P(x)/x.
   b) Find the marginal average profit function MAP(x).

4. If $f(x)=x^4-3x^3-8x+4$, find f ′ (x) , f ″ (x), f ‴ (x), $f^{(4)}(x)$.

5. If $f(x)=\frac{9}{\sqrt[3]{x}}$, find f ″ (x) and f ″ (3).

6. The distance a car travels in t hours is given by $s(t)=50t+\frac{100}{t+2}$.
   Find the velocity after 3 hours.

7. If $f(x)=(x^2-6x+3)^{10}$, find f ′ (x).

8. Find the second derivative of $f(x)=(x^2+3)^8$.
   (Extra Credit--factor completely.)

9a) What is a critical value of a function f?
 b) What is a point of inflection of a function f?

10. Find all critical values of $f(x)=(x^2-6x-7)^2$.

11. Given a function $f(x)=x^4+4x^3-8x^2+64$, find the first derivative, make a sign diagram for the derivative, plot all critical points, and sketch the graph.

12. Given that $f'(x) = 4x^3 - 12x^2$ and $f''(x) = 12x^2 - 24x$

a) make sign diagrams for the given derivatives (either as presented in the textbook or in class).
b) give all critical values for f(x).
c) give all points of inflection for f(x).
d) If f(0) = 0, f(2) = -16, and f(4) = -30, sketch the graph illustrating when the graph is increasing or decreasing, concave up or concave down.

13. Given the following sign diagram,

a) determine all critical values
b) determine all points of inflection
c) sketch the graph, indicating when the function is increasing/decreasing, concave up/down.

| x | -4 | -2 | 0 | 2 | 6 |
|---|---|---|---|---|---|
| f | 10 | 6 | 0 | 4 | -8 |
| f′ | ---------- | | Undef | ++0 ---------- | |
| f″ | ++++++ | 0---- | Undef | ----------0+++++ | |

14. Find the absolute extreme values of $f(x) = x - x^2$ on the interval [0,3].

15. A farmer has 600 yards of fence with which to enclose a rectangular area that borders a river. If no fence is required for the side along the river, find the dimensions of the largest possible rectangle and the maximum area that can be enclosed on three sides.

16. The cost to produce an automobile is $8000 each plus fixed costs of $20,000 per week. The selling price function for each car is p(x) = 22,000 - 70x, where p is the price (per car) at which exactly x cars will be sold.

a) Give the profit function P(x) for the sale of x automobiles per week.
b) Find the number of cars that must be sold per week to obtain maximum profit.
c) Find the maximum profit per week.

# MAC 2233 EXAM 2A Solutions

1. $f(x) = \frac{3}{4}x^{-3}$

$f'(x) = \frac{9}{4}x^{-4}$

(7) $= \frac{-9}{4x^4}$

2. $f(x) = \frac{x^2+1}{x^5-1}$

$f'(x) = \frac{(x^5-1)2x - (x^2+1)5x^4}{(x^5-1)^2}$

(7) $= \frac{2x^6 - 2x - 5x^6 - 5x^4}{(x^5-1)^2}$

$= \frac{-3x^6 - 5x^4 - 2x}{(x^5-1)^2}$

$= \frac{-x(3x^5+5x^3+2)}{(x^5-1)^2}$

3. $P(x) = 12x - 1800$

(3) a) $AP(x) = \frac{12x-1800}{x}$

(4) b) $MAP(x) = \frac{x(12) - (12x-1800)}{x^2}$

$= \frac{12x - 12x + 1800}{x^2}$

$= \frac{1800}{x^2}$

4. $f(x) = x^4 - 3x^3 - 8x + 4$

(8) $f'(x) = 4x^3 - 9x^2 - 8$

$f''(x) = 12x^2 - 18x$

$f'''(x) = 24x - 18$

$f^{(4)}(x) = 24$

5. $f(x) = \frac{9}{\sqrt[3]{x}} = 9x^{-1/3}$

(8) $f'(x) = -3x^{-4/3}$

$f''(x) = 4x^{-7/3}$

$f''(3) = 4 \cdot 3^{(-7/3)}$

$\approx .308$ or $\frac{4}{3^{7/3}}$ or $\frac{4}{9\sqrt[3]{3}}$ or $\frac{4\sqrt[3]{9}}{27}$

6. $s(t) = 50t + 100(t+2)^{-1}$

$v(t) = s'(t) = 50 - 100(t+2)^{-2} \cdot 1$

$v(3) = 50 - 100 \cdot 5^{-2}$

(7) $= 50 - 100 \cdot \frac{1}{25}$

$= 50 - 4 =$ 46 mph

7. $f(x) = (x^2-6x+3)^{10}$

$f'(x) = 10(x^2-6x+3)^9 \cdot (2x-6)$

(7) $= 20(x^2-6x+3)^9(x-3)$

8. $f(x) = (x^2+3)^8$

$f'(x) = 8(x^2+3)^7(2x)$

$= 16x(x^2+3)^7$

$f''(x) = 16[x \cdot 7(x^2+3)^6 \cdot 2x + (x^2+3)^7 \cdot 1]$

(8) $= 16[14x^2(x^2+3)^6 + (x^2+3)^7]$

$= 16(x^2+3)^6[14x^2 + x^2 + 3]$

$= 16(x^2+3)^6(15x^2+3)$

$= 48(x^2+3)^6(5x^2+1)$

9a) Critical value is a value of $x$ within the domain of $f$, where $f'(x) = 0$ or $f'(x)$ is undefined.

(8)

9b) Point of inflection is a point where $f''(x) = 0$ or undefined and the concavity changes.

10. $f(x) = (x^2-6x-7)^2$

(7) $f'(x) = 2(x^2-6x-7)^1(2x-6) = 0$

$2(x-7)(x+1)2(x-3) = 0$

$x = 7, x = -1, x = 3$

$f'(x) = 0$ and $f(x)$ is defined at these

11. $f(x) = x^4 + 4x^3 - 8x^2 + 64$

$f'(x) = 4x^3 + 12x^2 - 16x = 0$

(8) $4x(x^2+3x-4) = 0$

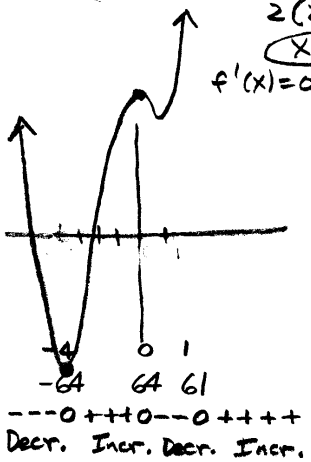
$4x(x+4)(x-1) = 0$

$x = 0 \quad x = -4 \quad x = 1$

Min at $(-4, -64)$

Rel Max at $(0, 64)$

Rel Min at $(1, 61)$



| x | | -4 | | 0 | | 1 | |
|---|---|---|---|---|---|---|---|
| f | | -64 | | 64 | | 61 | |
| f' | - - - | 0 | + + + | 0 | - - | 0 | + + + + |
| | Decr. | | Incr. | | Decr. | | Incr. |

12. $f'(x) = 4x^3 - 12x^2$

$= 4x^2(x-3)$

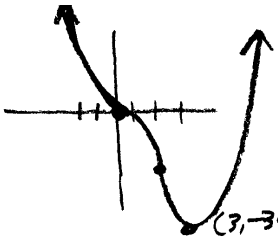
$x=0 \quad x=3$

(8) $f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

$x=0 \quad x=2$

d)




b) Critical values

$x=0, \; x=3$

c) Points of inflection

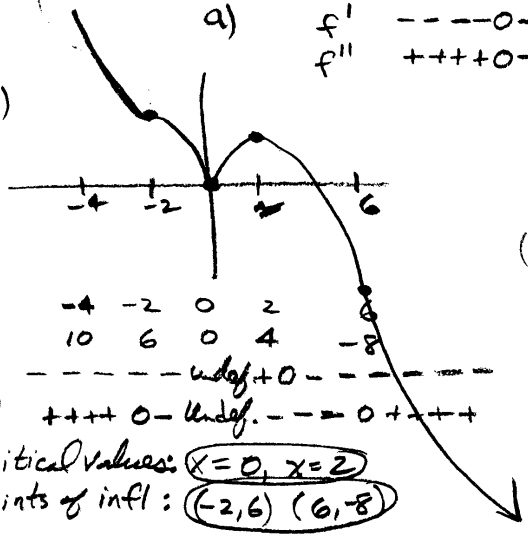
$x=0, \; x=2$

$(0,0) \quad (2,-16)$

| $x$ | | 0 | | 2 | | 3 |
|---|---|---|---|---|---|---|
| $f$ | | 0 | | -16 | | -27 |
| $f'$ | − − − | 0 | − − − | 0 | + + + | |
| $f''$ | + + + | 0 | − | 0 | + + + | |

13. c)

(8)




a)

| $x$ | -4 | -2 | 0 | 2 | 6 |
|---|---|---|---|---|---|
| $f$ | 10 | 6 | 0 | 4 | -8 |
| $f'$ | − − − − − | undef | + | 0 | − − − − − |
| $f''$ | + + + + | 0 | − | Undef. | − − − 0 + + + + |

a) Critical values: $x=0, \; x=2$

b) Points of infl: $(-2,6) \quad (6,-8)$

14. $f(x) = x - x^2 \quad [0,3]$

$f'(x) = 1 - 2x = 0$

(8) $-2x = -1$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

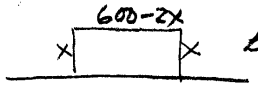
$f(0) = 0$

$f(3) = 3 - 9 = -6$

Absolute Max = $(\frac{1}{2}, \frac{1}{4})$ or $\frac{1}{4}$

Absolute Min = $(3,-6)$ or $-6$

15.




let $x$ = width

$600-2x$ = length

$A = LW$

(8) $A = x(600-2x)$

$A = 600x - 2x^2$

$A' = 600 - 4x = 0$

$600 = 4x$

$x = 150'$ width

$600 - 2x = 300'$ length

Max Area $= 150 \times 300$

$= 45,000$ sq ft.

16. cost $= 8000x + 20,000$

(for $x$ cars per week)

S.C.

$S.P = (22,000 - 70x) \cdot x$

$= 22,000x - 70x^2$

Profit $= 22,000x - 70x^2 - 8000x - 20,000$

$= -70x^2 + 14,000x - 20,000$

$(\text{Profit})' = -140x + 14,000 = 0$

$-140x = -140,000$

$x = 100$ cars/wk.

c) $P(x) = -70x^2 + 14,000x - 20,000$

$P(100) = \$680,000$