MAC 2233 EXAM 3A **Dr. Rapalje** **NAME** _____
Sections 3.4 – 4.3

Show all work on separate paper. Turn in ALL worksheets.

1. Use implicit differentiation to find $\frac{dy}{dx}$: $x^2 - y^2 = 2x - 4y - 6$.

2. Find $\frac{dy}{dx}$ evaluated at x = 1 , y = - 4 : $y = 2xy + 4x$.

3. A cube of ice is melting so that each edge is decreasing at the rate of 2 inches per hour. Find how fast the volume of ice is decreasing at the moment when each edge is 10 inches long.

4. Find the value of $2500 if it is invested for 12 years at 8% interest per year compounded:

   a) annually b) quarterly

5. Find the value of $2500 if it is invested for 12 years at 8% interest per year compounded:

   a) daily (365 days) b) continuously

6. How much would you have to invest now at 8% interest per year compounded continuously in order to have $1,000,000 in 10 years?

7. a) $\log_4 64$ b) $\log_5 \frac{1}{5}$ c) $\ln(e^{12})$ d) $\ln \frac{1}{\sqrt{e}}$.

8. How long will it take $2500 invested for at 8% interest per year to double if the interest is

   a) compounded annually b) compounded continuously

9. The proportion of students who can remember an 8 digit number for t minutes was 0.9 – 0.2 ln t (for t>1). What proportion of the students remembered the number for three minutes?

10. Find the derivative of each function.

a) $f(x) = \ln\sqrt{(x^2+4)}$ b) $f(x) = x^2 \ln x$

11. Find the derivative of each function.

a) $f(x) = e^{(x^2+4)}$ b) $f(x) = \dfrac{e^{5x}}{x^3}$

12. A $10,000 automobile depreciates so that its rate value after t years is $V(t) = 10,000e^{-0.35t}$ dollars. Find the rate of change of its value

a) when it is new (t=0); b) after two years.

13. A bus stop shelter is built from four pieces of wood: two square end pieces, a rectangular back, and a rectangular roof. If the shelter is to have a volume of 1024 cubic feet, find the dimensions of the shelter that can be built with the least a mount of materials.

14. If the amount of drug in a person's blood after t hours is $f(t) = \dfrac{t}{t^2+9}$, when will the drug concentration be the greatest?

15. A peach tree will yield 100 pounds of peaches now, which will sell for 40 cents per pound. Each week that the farmer waits to harvest the crop will increase the yield by 10 pounds, but the selling price will decrease by 2 cents per pound. How long should the farmer wait to pick the fruit in order to maximize her revenue?

MAC 2233 EXAM 3A Solutions

1. $x^2 - y^2 = 2x - 4y - 6$

$2x - 2y\frac{dy}{dx} = 2 - 4\frac{dy}{dx}$

$4\frac{dy}{dx} - 2y\frac{dy}{dx} = 2 - 2x$

$\frac{dy}{dx}\cdot 2(2-y) = 2(1-x)$

$\frac{dy}{dx} = \frac{1-x}{2-y}$

2. $y = 2xy + 4x$

$\frac{dy}{dx} = 2x\frac{dy}{dx} + y\cdot 2 + 4$

$-2x\frac{dy}{dx} + \frac{dy}{dx} = 2y + 4$

$\frac{dy}{dx}(-2x+1) = 2y+4$

at $x=1,\ y=-4$

$\frac{dy}{dx} = \frac{2y+4}{-2x+1} = \frac{-4}{-1} = 4$

3. $V = x^3 \quad \frac{dx}{dt} = -2\ in/hr$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt} \quad x = 10\ in$

$= 3(100\ in^2)\cdot(-2\ in/hr)$

$= -600\ in^3/hr$

4. $A = P(1+r)^n$

a) $n=12,\ r=.08$

$A = 2500(1.08)^{12}$

$= \$6295.43$

b) $n=48 \quad r=.02$

$A = 2500(1.02)^{48}$

$= \$6467.68$

5a) $A = P(1+r)^n$

$= 2500\left(1+\frac{.08}{365}\right)^{(365\cdot 12)}$

$= \$6528.55$

b) $A = Pe^{rt}$

$= 2500\cdot e^{(.08\cdot 12)}$

$= \$6529.24$

6. $A = Pe^{rt}$

$1000000 = Pe^{(.08\cdot 10)}$

$P = \frac{1000000}{e^{.8}}$

$= \$449,328.96$

7a) $\log_4 64 = x$

$4^x = 64$

$x = 3$

b) $\log_5 \frac{1}{5} = x$

$5^x = 1/5$

$x = -1$

c) $\ln(e^{12})$

$= 12$

d) $\ln\left(\frac{1}{\sqrt{e}}\right)$

$= \ln e^{-1/2} = -\frac{1}{2}$

8a) $A = P(1+r)^n$

$5000 = 2500(1+.08)^n$

$2 = 1.08^n$

$\ln 2 = \ln 1.08^n$

$\ln 2 = n \ln 1.08$

$n = \frac{\ln 2}{\ln 1.08} \approx 9.006\ yrs$

b) $A = Pe^{rt}$

$5000 = 2500e^{.08t}$

$2 = e^{.08t}$

$\ln 2 = \ln e^{.08t}$

$\ln 2 = .08t$

$t = \frac{\ln 2}{.08} \approx 8.664\ yrs$

9. $P(t) = 0.9 - 0.2\ln t$

$P(3) = 0.9 - 0.2\ln 3$

$= .680$ or 68.0%

10a) $f(x) = \ln\sqrt{x^2+4}$

$f(x) = \ln(x^2+4)^{1/2}$

$f(x) = \frac{1}{2}\ln(x^2+4)$

$f'(x) = \frac{1}{2}\frac{1}{x^2+4}\cdot 2x = \frac{x}{x^2+4}$

b) $f(x) = x^2\ln x$

$f'(x) = x^2\cdot\frac{1}{x} + \ln x\cdot 2x$

$= x + 2x\ln x$

11a) $f(x) = e^{x^2+4}$

$f'(x) = 2xe^{x^2+4}$

b) $f(x) = \frac{e^{5x}}{x^3}$

$f'(x) = \frac{x^3\cdot 5e^{5x} - e^{5x}\cdot 3x^2}{x^6}$

$= \frac{x^2e^{5x}(5x-3)}{x^6}$

$= \frac{e^{5x}(5x-3)}{x^4}$

12. $V(t) = 10{,}000e^{-0.35t}$

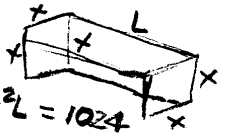
$\frac{dV}{dt} = 10000e^{-0.35t}(-.35)$

$= -3500e^{-0.35t}$

a) at $t=0$ $= -\$3500$ per year

b) at $t=2$ $= -\$1738.05$ per year

13.



$V = x^2L = 1024$

$L = \frac{1024}{x^2}$

$\text{Area} = 2x^2 + 2xL$

$= 2x^2 + 2x\left(\frac{1024}{x^2}\right)$

$= 2x^2 + 2048x^{-1}$

$A' = 4x - 2048x^{-2}$

$= 4x - \frac{2048}{x^2} = 0$

$4x = \frac{2048}{x^2}$

$4x^3 = 2048$

$x^3 = 512$

$x = 8\ ft.$

$L = \frac{1024}{64} = 16\ ft.$

14. $f(t) = \frac{t}{t^2+9}$

$f'(t) = \frac{(t^2+9)\cdot 1 - t\cdot 2t}{(t^2+9)^2}$

$= \frac{9-t^2}{(t^2+9)^2} = 0$

$t = 3\ hrs.$

15. Let x = no. weeks.

Rev = pounds × price per pd.

$= (100+10x)(.40-.02x)$

$= 40 - 2x + 4x - .2x^2$

$\frac{dR}{dt} = 2 - .4x = 0 \quad x = \frac{2}{.4} = 5\ wks$