Intermediate Algebra Exam 2 Forms A, B Dr. Rapalje

INTERMEDIATE ALGEBRA EXAM 2 A* NAME _____
SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. Circle answers.
TURN IN ALL WORKSHEETS. CALCULATORS ARE PERMITTED ON THIS TEST.

$$X^3 - Y^3 = (X - Y)(X^2 + XY + Y^2)$$

$$X^3 + Y^3 = (X + Y)(X^2 - XY + Y^2)$$

In 1-6, factor completely:

1. $X^2 + X - 42$

2. $8X^3 - 27Y^6$

3. $X^3 - 2X^2 - 16X + 32$

4. $X^3 - 49X$

5. $X^4 - 8X^2 - 9$

6. $(X-3Y)^2 - 5(X-3Y) + 6$

In 7-9, solve for X:

7. $3X^2 + 14X - 5 = 0$

8. $X^3 = X^2 + 12X$

9. $Y = \frac{a - bX}{c}$

In 10-13, perform the indicated operations (add, subtract, multiply, or divide.) REDUCE ALL FRACTIONS COMPLETELY!

10. $\frac{XY - 3Y + 3X - 9}{X^2 - 6X + 9}$

11. $\frac{X^2 - X - 12}{2X^2 - 32} \times \frac{X^2 + 8X + 16}{3X^2 + 21X + 36}$

12. $\dfrac{6}{X^2 - 2X} - \dfrac{8}{X^2 - 4}$

13. $\dfrac{X}{X^2 - 10X + 25} + \dfrac{5}{X^2 - 6X + 5}$

14. Solve the equation for X:

$$\frac{X + 2}{X^2 - X - 6} - \frac{1}{X^2 - 5X + 6} = \frac{X}{X^2 - 4}$$

15. Y varies directly as the square of X and inversely as Z. If Y=18 when X=4 and Z=2, find Y when X=2 and Z=15.

16. Divide:

a) $\dfrac{9X^4 + 3X^2 - 4}{3X^2}$

b) $\dfrac{X^3 - 6X^2 + 10X - 4}{X - 4}$

In 17-20, simplify the complex fractions:

17. $\dfrac{\dfrac{1}{X} - \dfrac{1}{3}}{\dfrac{9}{X^2} - 1}$

18. $\dfrac{\dfrac{X - 1}{X^2 - 4}}{1 + \dfrac{1}{X - 2}}$

19. $\dfrac{2X^{-1} - (2Y)^{-1}}{2XY^{-1}}$

20. $(X^{-1} + Y^{-1})^{-1}$

Exam 2A* Solutions

1. $x^2+x-42 = (x+7)(x-6)$

2. $8x^3-27y^6 = (2x-3y^2)(4x^2+6xy^2+9y^4)$

3. $x^3-2x^2-16x+32 = x^2(x-2)-16(x-2) = (x-2)(x^2-16) = (x-2)(x-4)(x+4)$

4. $x^3-49x = x(x^2-49) = x(x-7)(x+7)$

5. $x^4-8x^2-9 = (x^2-9)(x^2+1) = (x-3)(x+3)(x^2+1)$

6. $(x-3y)^2-5(x-3y)+6 = [(x-3y)-2][(x-3y)-3] = (x-3y-2)(x-3y-3)$

7. $3x^2+14x-5=0$
$(3x-1)(x+5)=0$
$3x=1 \quad x=-5$
$x=\frac{1}{3}$

8. $x^3=x^2+12x$
$x^3-x^2-12x=0$
$x(x^2-x-12)=0$
$x(x-4)(x+3)=0$
$x=0 \quad x=4 \quad x=-3$

9. $y=\frac{a-bx}{c}$
$yc=a-bx$
$\frac{bx}{b}=\frac{a-yc}{b}$
$x=\frac{a-yc}{b}$

10. $\frac{xy-3y+3x-9}{x^2-6x+9} = \frac{y(x-3)+3(x-3)}{(x-3)^2} = \frac{(x-3)(y+3)}{(x-3)^2} = \frac{y+3}{x-3}$

11. $\frac{(x-4)(x+3)}{2(x^2-16)} \times \frac{(x+4)(x+4)}{3(x^2+7x+12)} = \frac{(x-4)(x+3)}{2(x-4)(x+4)} \times \frac{(x+4)(x+4)}{3(x+3)(x+4)} = \frac{1}{6}$

12. $\frac{6}{x(x-2)} - \frac{8}{(x-2)(x+2)}$ LCD $= x(x-2)(x+2)$
$= \frac{6(x+2)}{x(x-2)(x+2)} - \frac{8(x)}{(x-2)(x+2)(x)}$
$= \frac{6x+12-8x}{x(x-2)(x+2)}$
$= \frac{-2x+12}{x(x-2)(x+2)}$

13. $\frac{x}{(x-5)^2} + \frac{5}{(x-1)(x-5)}$
$\frac{x(x-1)}{(x-5)^2(x-1)} + \frac{5(x-5)}{(x-1)(x-5)(x-5)}$
$= \frac{x^2-x+5x-25}{(x-5)^2(x-1)} = \frac{x^2+4x-25}{(x-5)^2(x-1)}$

14. $\frac{(x-3)(x-2)(x+2)}{(x-3)(x+2)} - \frac{(x-3)(x-2)(x+2)}{(x-3)(x-2)} = \frac{x}{x-2}$
$x^2-4-x-2=x^2-3x$
$x^2-x-6=x^2-3x$
$2x=6$
$x=3$ NoWay
Denom $\neq 0$.
NO SOLUTION

15. $y=\frac{kx^2}{z}$
$18=\frac{k\cdot 16}{2}$
$36=16k$
$k=\frac{36}{16}=\frac{9}{4}$
$y=\frac{\frac{9}{4}x^2}{2}=\frac{9x^2}{8}$
$\frac{9}{15}=\frac{3}{5}$

16a) $\frac{9x^4}{3x^2}+\frac{3x^2}{3x^2}-\frac{4}{3x^2} = 3x^2+1-\frac{4}{3x^2}$

b) $4\,|\,1 \;\; -6 \;\; 10 \;\; -8$
$\quad\quad\;\; 4 \;\; -8 \;\; 8$
$\quad\;\; 1 \;\; -2 \;\; 2 \;\; 0$
$x^2-2x+2+\frac{0}{x-4}$

17. $\frac{\left(\frac{1}{x}-\frac{1}{3}\right)}{\left(\frac{9}{x^2}-1\right)} = \left(\frac{1}{x}-\frac{1}{3}\right) \div \left(\frac{9}{x^2}-1\right)$
$= \frac{3-x}{3x} \div \frac{9-x^2}{x^2}$
$= \frac{3-x}{3x} \cdot \frac{x^2}{(3-x)(3+x)}$
$= \frac{x}{3(x+3)}$

18. $\frac{x-1}{x^2-4} \div \left(1+\frac{1}{x-2}\right)$
$= \frac{x-1}{(x-2)(x+2)} \div \frac{x-2+1}{x-2}$
$= \frac{x-1}{(x-2)(x+2)} \cdot \frac{x-2}{x-1} = \frac{1}{x+2}$

19. $\frac{\frac{2}{x}-\frac{1}{2y}}{\frac{2x}{y}} = \left(\frac{2}{x}-\frac{1}{2y}\right) \div \frac{2x}{y}$
$= \frac{4y-x}{2xy} \cdot \frac{y}{2x}$
$= \frac{4y-x}{4x^2}$

20. $(x^{-1}+y^{-1})^{-1}$
$= \left(\frac{1}{x}+\frac{1}{y}\right)^{-1}$
$= \left(\frac{y+x}{xy}\right)^{-1} = \frac{xy}{y+x}$

INTERMEDIATE ALGEBRA EXAM 2 B* NAME _____

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. Circle answers.

TURN IN ALL WORKSHEETS. CALCULATORS ARE RECOMMENDED ON THIS TEST.

$$X^3 - Y^3 = (X - Y)(X^2 + XY + Y^2)$$

$$X^3 + Y^3 = (X + Y)(X^2 - XY + Y^2)$$

In 1-6, factor completely:

1. $X^3 - 7X^2 + 6X$ 2. $X^4 - 81$ 3. $(X+2Y)^2 - 2(X+2Y) - 8$

4. $X^3 - 3X^2 - 25X + 75$ 5. $16X^4 + 250X$ 6. $X^2+2XY+Y^2 + 5X+5Y + 4$

In 7-9, solve for X:

7. $X^2 - X = 12$ 8. $(X + 2)(2X + 3) = 10$ 9. $\frac{aX + b}{4} = X + 2$

In 10-12, perform the indicated operations (add, subtract, multiply, or divide.) REDUCE ALL FRACTIONS COMPLETELY!

10. $\frac{X^3 - 8}{X^3 - 2X^2 + 4X - 8}$ 11. $\frac{16X^2 - 9Y^2}{16X^2 - 24XY + 9Y^2} \div \frac{8X^2 + 10XY + 3Y^2}{8X^2 - 6XY}$

12. $\dfrac{4}{X^2 - 16} - \dfrac{5}{2X + 8}$

13. $\dfrac{X - 2}{X^2 - 10X + 25} + \dfrac{2}{X^2 - 6X + 5}$

14. Solve the equation for X:

$$\frac{2}{X + 1} - \frac{3}{X^2 - X - 2} = -\frac{2}{X - 2}$$

15. Y varies directly as X and inversely as the square of Z. If Y=27 when X=12 and Z=2, find Y when X=4 and Z=9.

16. Divide:

a) $\dfrac{20X^4 - 4X^2 + 12}{4X^2}$

b) $\dfrac{X^3 - 6X^2 + 10X - 4}{X - 4}$

In 17-20, simplify the complex fractions:

17. $\dfrac{\frac{2}{X} - \frac{3}{Y}}{2 - \frac{5}{XY}}$

18. $\dfrac{\frac{6X}{X+3} + 3}{4 - \frac{12}{X+4}}$

19. $\dfrac{X^{-1} + Y^{-1}}{X^{-2} - Y^{-2}}$

20. $(X^{-1} - Y^{-1})^{-1}$

EXAM 2B* Solutions

1. x^3-7x^2+6x
$= x(x^2-7x+6)$
$= x(x-6)(x-1)$

2. x^4-81
$=(x^2-9)(x^2+9)$
$=(x-3)(x+3)(x^2+9)$

3. $(x+2y)^2-2(x+2y)-8$
$=[(x+2y)-4][(x+2y)+2]$
$=(x+2y-4)(x+2y+2)$

4. $x^3-3x^2-25x+75$
$=x^2(x-3)-25(x-3)$
$=(x-3)(x^2-25)$
$=(x-3)(x-5)(x+5)$

5. $16x^4+250x$
$=2x(8x^3+125)$
$=2x(2x+5)(4x^2-10x+25)$

6. $x^2+2xy+y^2+5x+5y+4$
$=(x+y)^2+5(x+y)+4$
$=[(x+y)+4][(x+y)+1]$
$=(x+y+4)(x+y+1)$

7. $x^2-x=12$
$x^2-x-12=0$
$(x-4)(x+3)=0$
$x=4 \quad x=-3$

8. $(x+2)(2x+3)=10$
$2x^2+7x+6=10$
$2x^2+7x-4=0$
$(2x-1)(x+4)=0$
$x=1/2 \quad x=-4$

9. $\frac{ax+b}{4}=\frac{x+2}{1}$
$ax+b=4x+8$
$ax-4x=8-b$
$x(a-4)=8-b$
$x=\frac{8-b}{a-4}$ (or $\frac{b-8}{4-a}$)

10. $\frac{(x-2)(x^2+2x+4)}{x^2(x-2)+4(x-2)}$
$=\frac{(x-2)(x^2+2x+4)}{(x-2)(x^2+4)}$
$=\frac{x^2+2x+4}{x^2+4}$

11. $\frac{(4x-3y)(4x+3y)}{(4x-3y)(4x-3y)} \cdot \frac{2x(4x-3y)}{(4x+3y)(2x+y)}$
$=\frac{2x}{2x+y}$

12. $\frac{4}{(x-4)(x+4)}-\frac{5}{2(x+4)}$
$=\frac{4\cdot 2}{(x-4)(x+4)\cdot 2}-\frac{5(x-4)}{2(x+4)(x-4)}$
$=\frac{8-5x+20}{2(x-4)(x+4)}$
$=\frac{28-5x}{2(x-4)(x+4)}$

13. $\frac{x-2}{(x-5)^2}+\frac{2}{(x-5)(x-1)}$
$=\frac{(x-2)(x-1)}{(x-5)^2(x-1)}+\frac{2}{(x-5)(x-1)}\cdot\frac{(x-5)}{(x-5)}$
$=\frac{x^2-3x+2+2x-10}{(x-5)^2(x-1)}$
$=\frac{x^2-x-8}{(x-5)^2(x-1)}$

14. $\frac{(2x+1)(x-2)}{2x+1}-\frac{3}{(x-2)(x+1)}=\frac{-2}{x-2}$
$2(x-2)-3=-2(x+1)$
$2x-4-3=-2x-2$
$4x=5$
$x=5/4$

15. $y=\frac{kx}{z^2} \quad y=\frac{9x}{z^2}$
$27=\frac{k\cdot 12}{4} \quad y=\frac{9\cdot 4}{81}$
$27=3k \quad y=\frac{4}{9}$
$k=9$

16a) $\frac{20x^4-4x^2+12}{4x^2}$
$=\frac{20x^4}{4x^2}-\frac{4x^2}{4x^2}+\frac{12}{4x^2}$
$=5x^2-1+\frac{3}{x^2}$

b)
$$\begin{array}{c|cccc} 4 & 1 & -6 & 10 & -4 \\ & & \downarrow 4 & -8 & 8 \\ \hline & 1 & -2 & 2 & 4 \end{array}$$
$x^2-2x+2+\frac{4}{x-4}$

17. $\frac{xy\left(\frac{2}{x}-\frac{3}{y}\right)}{xy\left(2-\frac{5}{xy}\right)}$
$=\frac{2y-3x}{2xy-5}$

18. $\frac{\left(\frac{6x}{x+3}+3\right)}{\left(4-\frac{12}{x+4}\right)}=\left(\frac{6x}{x+3}+3\right)\div\left(4-\frac{12}{x+4}\right)$
$=\frac{6x+3(x+3)}{x+3}\div\frac{4(x+4)-12}{x+4}$
$=\frac{6x+3x+9}{x+3}\div\frac{4x+16-12}{x+4}$
$=\frac{9x+9}{x+3}\div\frac{4x+4}{x+4}$
$=\frac{9(x+1)}{x+3}\cdot\frac{x+4}{4(x+1)}=\frac{9(x+4)}{4(x+3)}$

19. $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}=\left(\frac{1}{x}+\frac{1}{y}\right)\div\left(\frac{1}{x^2}-\frac{1}{y^2}\right)$
$=\frac{y+x}{xy}\div\frac{y^2-x^2}{x^2y^2}$
$=\frac{y+x}{xy}\cdot\frac{x^2y^2}{(y-x)(y+x)}$
$=\frac{xy}{y-x}$

20. $(x^{-1}-y^{-1})^{-1}$
$=\left(\frac{1}{x}-\frac{1}{y}\right)^{-1}$
$=\left(\frac{y-x}{xy}\right)^{-1}=\frac{xy}{y-x}$

Dr. Robert J. Rapalje

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE