

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. Circle answers.

TURN IN ALL WORKSHEETS. CALCULATORS ARE PERMITTED ON THIS TEST.

$$X^3 - Y^3 = (X - Y)(X^2 + XY + Y^2)$$

$$X^3 + Y^3 = (X + Y)(X^2 - XY + Y^2)$$

In 1 - 6, factor completely:

1. $6X^2 - 18X$

2. $X^2 - 16X - 36$

3. $(\text{Junk})^2 - 6(\text{Junk}) + 5$

4. $X^4Y^3 - 27X^{10}$

5. $X^3 - 4X^2 - 16X + 64$

6. $(X-3Y)^2 - 25$

In 7 - 8, solve for X:

7. $X^2 + 4X = 60$

8. $X^3 + 3X^2 + 2X = 0$

In 9-12, perform the indicated operations (add, subtract, multiply, or divide.) REDUCE ALL FRACTIONS COMPLETELY!

9.
$$\frac{X^4 - 16}{X^4 - 8X^2 + 16}$$

10.
$$\frac{X^3 - 8Y^3}{X + 2Y} \div \frac{X^2 + 2XY + 4Y^2}{X^2 - 4Y^2}$$

$$11. \frac{9}{X^2 - 9} - \frac{5}{3X + 9}$$

$$12. \frac{9X}{X^2 - X - 2} - \frac{5}{X^2 + 3X - 10}$$

13. Solve the equation for X:

$$\frac{X}{X - 4} - \frac{2}{X - 1} = \frac{12}{X^2 - 5X + 4}$$

14. Y varies directly as Z and inversely as the square of X.
If Y=8 when Z=4 and X=3, find Y when Z=3 and X=2.

15. Divide:

a)
$$\frac{20X^4 + 4X^2 - 12}{2X^2}$$

b)
$$\frac{X^3 + 2X^2 - 10X - 4}{X + 2}$$

In 16-19, simplify the complex fractions:

16.
$$\frac{1 - \frac{3}{X}}{\frac{X}{3} - \frac{3}{X}}$$

17.
$$\frac{\frac{X-2}{X} - \frac{X}{X-2}}{\frac{X-2}{X} + \frac{X}{X-2}}$$

18.
$$\frac{2X^{-1} - (2Y)^{-1}}{2XY^{-1}}$$

19.
$$(X^{-1} - Y^{-1})^{-2}$$

INT. ALG. EXAM 2 D* Solutions

1. $6x^2 - 18x = 6x(x-3)$

2. $x^2 - 16x - 36 = (x-18)(x+2)$

3. $(Junk)^2 - 6(Junk) + 5 = (Junk-5)(Junk-1)$

4. $x^4 y^3 - 27x^{10} = x^4(y^3 - 27x^6) = x^4(y-3x^2)(y^2+3xy+9x^2)$

5. $x^3 - 4x^2 - 16x + 64 = x^2(x-4) - 16(x-4) = (x-4)(x^2-16) = (x-4)(x-4)(x+4) = (x-4)^2(x+4)$

6. $(x-3y)^2 - 25 = (x-3y-5)(x-3y+5)$

7. $x^2 + 4x = 60 \Rightarrow x^2 + 4x - 60 = 0 \Rightarrow (x+10)(x-6) = 0 \Rightarrow x = -10, x = 6$

8. $x^3 + 3x^2 + 2x = x(x^2 + 3x + 2) = x(x+2)(x+1) = 0 \Rightarrow x = 0, x = -2, x = -1$

9. $\frac{x^4 - 16}{x^4 - 8x^2 + 16} = \frac{(x^2-4)(x^2+4)}{(x^2-4)(x^2-4)} = \frac{x^2+4}{x^2-4}$

10. $\frac{x^3 - 8y^3}{x+2y} \div \frac{x^2 + 2xy + 4y^2}{x^2 - 4y^2} = \frac{(x-2y)(x^2+2xy+4y^2)}{x+2y} \cdot \frac{(x-2y)(x+2y)}{(x^2+2xy+4y^2)} = (x-2y)^2$

11. $\frac{9}{(x-3)(x+3)} - \frac{5}{3(x+3)(x-3)} = \frac{27 - 5x + 15}{3(x-3)(x+3)} = \frac{42 - 5x}{3(x-3)(x+3)}$

12. $\frac{9x}{(x-2)(x+1)} - \frac{5}{(x+5)(x-2)} = \frac{9x(x+5)}{(x-2)(x+1)(x+5)} - \frac{5(x+1)}{(x+5)(x-2)(x+1)} = \frac{9x^2 + 45x - 5x - 5}{(x-2)(x+1)(x+5)} = \frac{9x^2 + 40x - 5}{(x-2)(x+1)(x+5)}$

13. $\frac{(x-4)(x-1)}{x-4} - \frac{(x-4)(x-1)}{x-1} = \frac{(x-4)(x-1)}{(x-4)(x-1)} \Rightarrow x^2 - x - 2x + 8 = 12 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x-4)(x+1) = 0 \Rightarrow x = 4, x = -1$
Reject $x = -1$

14. $y = \frac{8z}{x^2} \Rightarrow 8 = \frac{8(4)}{9} \Rightarrow 72 = 4z \Rightarrow z = 18$

15a) $\frac{20x^4}{2x^2} + \frac{4x^2}{2x^2} - \frac{12}{2x^2} = 10x^2 + 2 - \frac{6}{x^2}$

16. $\frac{1 - \frac{3}{x}}{\frac{x}{3} - \frac{3}{x}} = \left(1 - \frac{3}{x}\right) \div \left(\frac{x}{3} - \frac{3}{x}\right) = \frac{x-3}{x} \div \frac{x^2-9}{3x} = \frac{x-3}{x} \cdot \frac{3x}{(x-3)(x+3)} = \frac{3}{x+3}$

$y = \frac{18z}{x^2} \Rightarrow y = \frac{9 \cdot 18 \cdot 3}{4^2} = \frac{27}{2}$

17. $\frac{x^3 + 2x^2 - 10x - 4}{x+2}$

$$\begin{array}{r} -2 \overline{) 1 \ 2 \ -10 \ -4} \\ \underline{\downarrow -2 \ \ 0 \ 20} \\ 1 \ 0 \ -10 \ 16 \end{array}$$

$$x^2 - 10 + \frac{16}{x+2}$$

17. $\left(\frac{x-2}{x} - \frac{x}{x-2}\right) \div \left(\frac{x-2}{x} + \frac{x}{x-2}\right) = \frac{x^2 - 4x + 4 - x^2}{x(x-2)} \div \frac{x^2 - 4x + 4 + x^2}{x(x-2)} = \frac{-4x+4}{x(x-2)} \cdot \frac{x(x-2)}{2x^2 - 4x + 4} = \frac{-4(x-1)}{2(x^2 - 2x + 2)} = \frac{-2(x-1)}{x^2 - 2x + 2}$

18. $\frac{2x^{-1} - (2y)^{-1}}{2xy^{-1}} = \frac{\frac{2}{x} - \frac{1}{2y}}{\frac{2x}{y}} = \left(\frac{2}{x} - \frac{1}{2y}\right) \div \frac{2x}{y}$

19. $(x^{-1} - y^{-1})^{-2} = \left(\frac{1}{x} - \frac{1}{y}\right)^{-2} = \left(\frac{y-x}{xy}\right)^{-2} = \left(\frac{xy}{y-x}\right)^2 = \frac{y^2 x^2}{(y-x)^2}$

$= \left(\frac{2}{x} - \frac{1}{2y}\right) \div \frac{2x}{y} = \frac{4y - x}{2xy} \cdot \frac{y}{2x} = \frac{4y - x}{4x^2}$