

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. Circle answers.
TURN IN ALL WORKSHEETS. CALCULATORS ARE REQUIRED ON THIS TEST.
EXPLAIN OR DESCRIBE CALCULATOR METHODS. SKETCH AND LABEL ALL GRAPHS!

1. Evaluate the determinants:

a)
$$\begin{vmatrix} 7 & 3 \\ -3 & -4 \end{vmatrix}$$

b)
$$\begin{vmatrix} 6 & -6 \\ -1 & -1 \end{vmatrix}$$

2. Evaluate the determinant by the method of your choice:

$$\begin{vmatrix} 5 & -2 & 8 \\ 3 & 4 & 0 \\ -9 & -6 & 3 \end{vmatrix}$$

3. Solve the systems of equations (use the method of your choice):

a)
$$\begin{aligned} -8x + 6y &= 32 \\ x &= 2y + 6 \end{aligned}$$

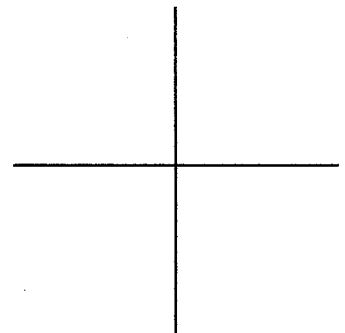
b)
$$\begin{aligned} 3y - 5x &= 10 \\ 10x &= 6y - 20 \end{aligned}$$

4. Solve by Cramer's Rule:

$$\begin{aligned} -3x + 7y &= 4 \\ 2x - 3y &= -6 \end{aligned}$$

5. Graph the intersection:

$$\begin{aligned} x + 3y &< 6 \\ x - 2y &\geq -4 \\ x &\leq 0 \end{aligned}$$



In 6 - 8, solve the systems of equations by method of your choice.
Show your work, sketch a graph, or explain what you did!

6. Solve the system:

$$\begin{aligned}8x + 3y + 2z &= 3 \\4x + 5y &= 7 \\2y - 3z &= -9\end{aligned}$$

7. Solve the system:

$$\begin{aligned}y &= x^2 - 2x - 5 \\y &= 3x + 1\end{aligned}$$

8. Solve the system:

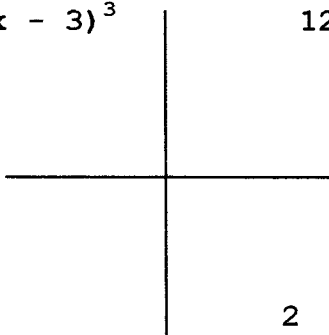
$$\begin{aligned}x^2 + 6xy + 9y^2 &= 25 \\x - 2y &= -10\end{aligned}$$

9. Find the remainder if $x^5 - 2x + 1$ is divided by $x + 2$.

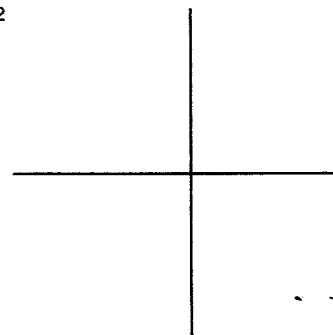
10. Find the equation whose roots are $x = 2$, $x = -5$, and $x = -1 \pm 3i$.

In 11 - 12, give the intercepts and sketch the graphs:

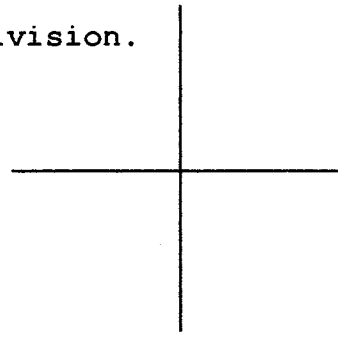
11. $y = x(x + 5)^2(x - 3)^3$



12. $y = x^3 - 15x^2$



13. Sketch the graph. Verify the roots by synthetic division.
 $y = x^4 - x^3 - 13x^2 + 25x - 12$



In 14 - 15, find all roots and multiplicities. Give radical form of irrational roots. (Use calculators and synthetic division to show your work!).

14. $x^3 + 2x^2 - 7x + 4 = 0$ 15. $x^4 - 3x^3 - 28x^2 + 66x - 36 = 0$

In 16 - 19, solve the inequalities. In the process you should find all roots and give final answer in interval notation. Sketch the graphs if you use calculator methods.

16. $|12 - 2x| \geq 10$ 17. $(x+2)^3 (x-4)^4 (x-1) > 0$

18. $\frac{12}{x} \leq \frac{x}{3}$ 19. $\frac{x(x-12)}{x-4} < 0$

COLLEGE ALGEBRA UG1 Solutions

1a) $\begin{vmatrix} 7 & 3 \\ -3 & -4 \end{vmatrix} = -28 - (-12) = -16$

2. $\begin{vmatrix} 5 & -2 & 8 \\ 3 & 4 & 0 \\ -9 & -6 & 3 \end{vmatrix} \text{ [MATRIX]} = 222$

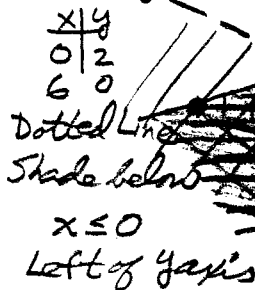
3a) $-8x + 6y = 32$
 $x - 2y = 6$
 2nd Simult $(-10, -8)$
 b) $-5x + 3y = 10$
 $10x - 6y = -20$
 Same Line

a) $\begin{vmatrix} 6 & -6 \\ -1 & -1 \end{vmatrix} = 6 - 6 = 0$

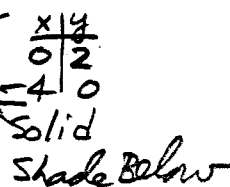
4. $-3x + 7y = 4$
 $2x - 3y = -6$

$x = \frac{\begin{vmatrix} 4 & 7 \\ -6 & -3 \end{vmatrix}}{\begin{vmatrix} -3 & 7 \\ 2 & -3 \end{vmatrix}} = \frac{-12 - 42}{9 - 14} = \frac{-54}{-5} = 10.8$
 $y = \frac{\begin{vmatrix} -3 & 4 \\ 2 & -6 \end{vmatrix}}{\begin{vmatrix} -3 & 7 \\ 2 & -3 \end{vmatrix}} = \frac{18 - 8}{9 - 14} = \frac{10}{-5} = -2$
 $(-6, -2)$

5. $x + 3y < 6$



$x - 2y > -4$



6. 2nd Simult $(-2, 3, 5)$

7. $y = x^2 - 2x - 5$
 $y = 3x + 1$

$x^2 - 2x - 5 = 3x + 1$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6 \quad x = -1$

$y = 3x + 1 \quad y = 3x + 1$

$= 19 \quad = -2$

$(6, 19) \quad (-1, -2)$

8. $x^2 + 6xy + 9y^2 = 25$

$x - 2y = -10$

$x = 2y - 10$

$(2y-10)^2 + 6y(2y-10) + 9y^2 = 25$

$4y^2 - 40y + 100 + 12y^2 - 60y + 9y^2 = 25$

$25y^2 - 100y + 75 = 0$

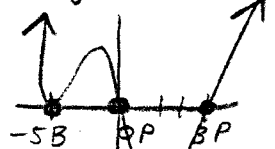
[2nd Poly] $\Rightarrow y = 3 \quad y = 1$

$x = 2y - 10 \quad x = 2y - 10$

$x = -4 \quad x = -8$

$(-4, 3) \quad (-8, 1)$

11. $y = x(x+5)(x-3)^2$



$(0, 0) \quad (-5, 0) \quad (3, 0)$

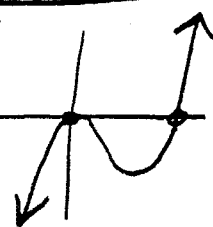
[2nd Poly]

12. $y = x^3 - 15x^2$

$x^2(x-15)$

$x = 0 \quad x = 15$

$(0, 0) \quad (15, 0)$



13. $y = x^4 - x^3 - 13x^2 + 25x - 12$ [GRAPH] or [2nd Poly]

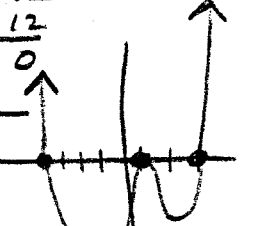
$-4 \mid \begin{array}{cccccc} 1 & -1 & -13 & 25 & -12 \\ \downarrow & -4 & 20 & -28 & 12 \end{array}$

$3 \mid \begin{array}{cccccc} 1 & -5 & 7 & -3 & 0 \\ \downarrow & 3 & -6 & 3 & 0 \end{array}$

$\begin{array}{cccccc} 1 & -2 & 1 & 0 & 0 \\ \downarrow & -2 & 3 & -2 & 0 \end{array}$

$x^2 - 2x + 1 = 0$
 $(x-1)^2 = 0$

Roots: $x = -4, 3, 1$ (mult 2)



14. $x^3 + 2x^2 - 7x + 4 = 0$

$-4 \mid \begin{array}{cccc} 1 & 2 & -7 & 4 \\ \downarrow & -4 & +8 & -4 \end{array}$

$\begin{array}{cccc} 1 & -2 & 1 & 0 \\ \downarrow & -2 & 4 & -2 \end{array}$

$x^2 - 2x + 1 = 0$
 $(x-1)^2 = 0$

$x = -4, x = 1$ (mult 2)

15. $x^4 - 3x^3 - 28x^2 + 66x - 36 = 0$

[2nd Poly] $\rightarrow x = 6, x = 1$

$6 \mid \begin{array}{cccccc} 1 & -3 & -28 & 66 & -36 \\ \downarrow & 6 & 18 & -60 & 36 \end{array}$

$\begin{array}{cccccc} 1 & 3 & -10 & 6 & 0 \\ \downarrow & 3 & 1 & 4 & -6 \end{array}$

$\begin{array}{cccccc} 1 & 4 & -6 & 0 & 0 \\ \downarrow & 4 & -2 & 8 & -4 \end{array}$

$x^2 + 4x - 6 = 0$
 $x^2 + 4x + 4 = 6 + 4$
 $(x+2)^2 = 10$

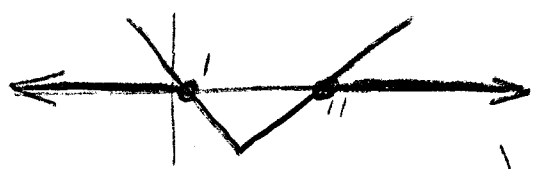
$x = -2 \pm \sqrt{10}, 6, 1$

16. $|12-2x| \geq 10$

$$12-2x = 10 \quad -2x = -2 \quad x = 1$$

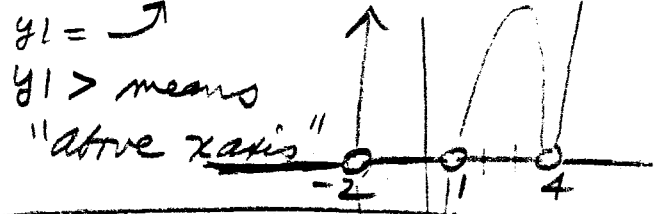
$$12-2x = -10 \quad -2x = -22 \quad x = 11$$

$$y1 = 2/3(12-2x) - 10 \geq 0$$



(Set window $x_{MAX} = 15$ or 20)
 $y1 \geq$ means "on or above x axis"
 $(-\infty, 1] \cup [11, \infty)$

17. $(x+2)^3(x-4)(x-1) > 0$



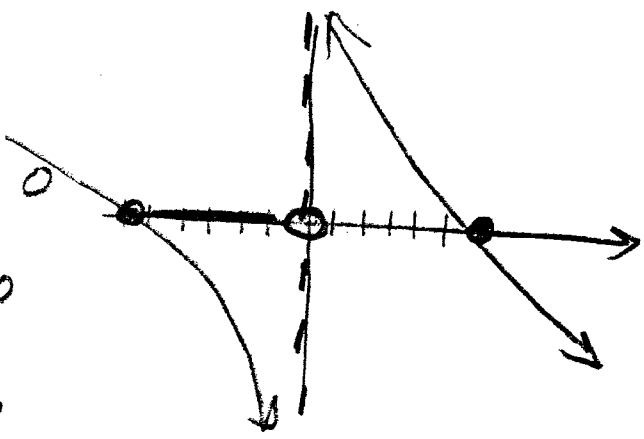
$y1 >$ means "above x axis"
 $(-\infty, -2) \cup (1, 4) \cup (4, \infty)$

18. $\frac{12}{x} \leq \frac{x}{3}$

$$\frac{12}{x} - \frac{x}{3} \leq 0$$

$$\frac{36 - x^2}{3x} \leq 0$$

$$y1 = \frac{36 - x^2}{3x} \leq 0$$



$y1 \leq$ means "on or below x axis."

Roots $x = 6, -6$

Asymptotes $x = 0$

$[-6, 0) \cup [6, \infty)$

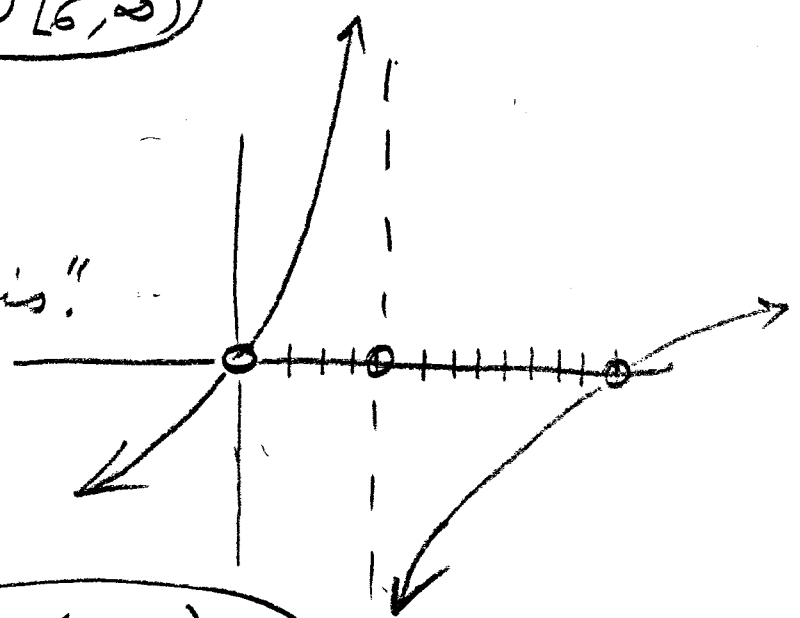
19. $\frac{x(x-12)}{x-4} < 0$

$y1 < 0$ means "below x axis."

Roots $x = 0, x = 12$

Asymptotes $x = 4$

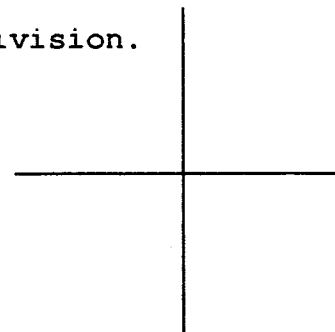
(Extend window by setting WINDOW $x_{MAX} = 15$ or 20)



$(-\infty, 0) \cup (4, 12)$

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