Curve Sketching Dr. Rapalje

EXAMPLE 1: $y = \dfrac{2(x^2-9)}{x^2-4}$ $y' = \dfrac{20x}{(x^2-4)^2}$ $y'' = \dfrac{-20(3x^2+4)}{(x^2-4)^3}$

$y=0$ at $x=\pm 3$

Asymptotes $x = \pm 2$

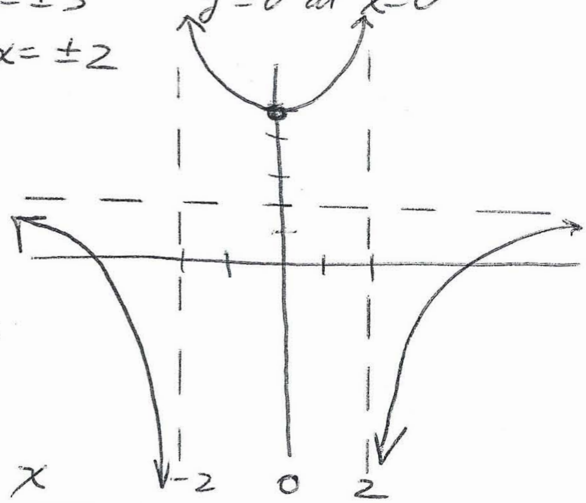
$y'=0$ at $x=0$

$\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to-\infty} f(x) = 2$

Horizontal Asymptote

$y=2$



x		-2		0		2	
f		$*$		$9/2$		$*$	
f'	$-$	$*$	$-$	0	$+$	$*$	$+$
f''	$-$	$*$	$+$	$+$	$+$	$*$	$-$
		Asymptote		Minimum		Asymptote	

EXAMPLE 2: $y = \dfrac{x^2-2x+4}{x-2}$ $y' = \dfrac{x(x-4)}{(x-2)^2}$ $y'' = \dfrac{8}{(x-2)^3}$

$y=0$ No x intercepts.

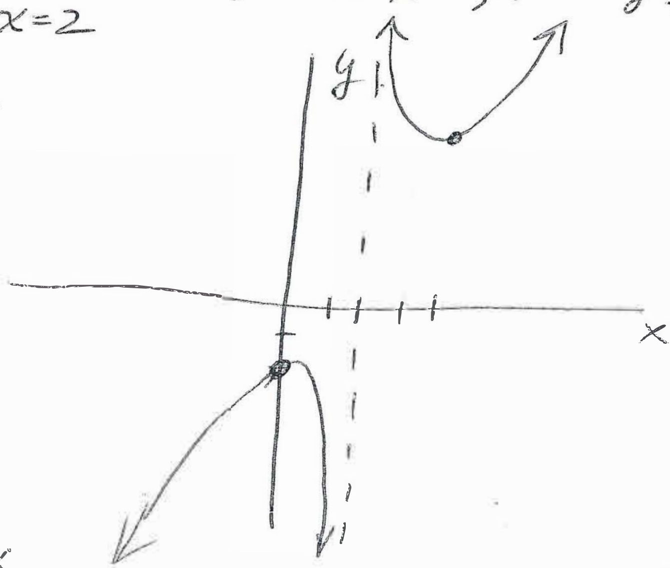
Asymp: $x=2$

$y'=0$ at $x=0, 4$

$y'' \neq 0$.

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$



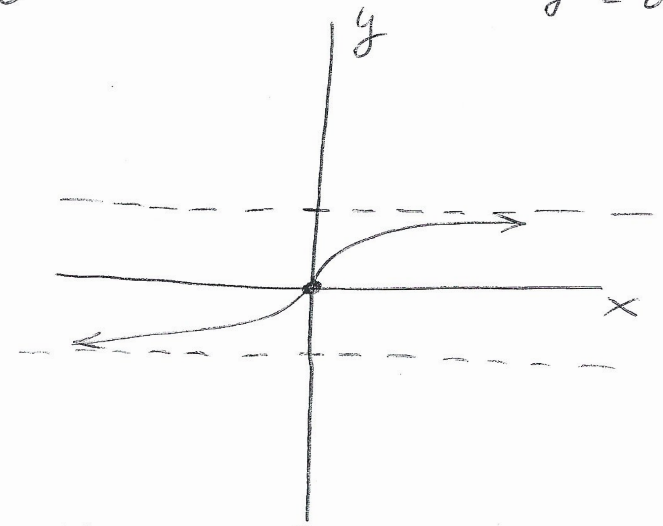
x		0		2		4	
f		-2		$*$		6	
f'	$+$	0	$-$	$*$	$-$	0	$+ + +$
f''	$-$	$- -$	$-$	$*$	$+$	$+$	$+ + +$
		Max		Asy		Min	

EXAMPLE 3. $y = \dfrac{x}{\sqrt{x^2+2}}$ $\quad y' = \dfrac{2}{(x^2+2)^{3/2}}$ $\quad y'' = -\dfrac{6x}{(x^2+2)^{5/2}}$

$y=0$ at $x=0$ $\qquad\qquad y''=0$ at $x=0$.

$\lim\limits_{x\to\infty} f(x) = 1$

$\lim\limits_{x\to-\infty} f(x) = -1$



x			0		
f			0		
f'	+	+	+	+	+
f''	+	+	0	−	−
			Pt of Infl.		

EXAMPLE 4. $y = 2x^{5/3} - 5x^{4/3}$ $\quad y' = \frac{10}{3}x^{1/3}(x^{1/3}-2)$ $\quad y'' = \dfrac{10(x^{1/3}-1)}{9x^{2/3}}$

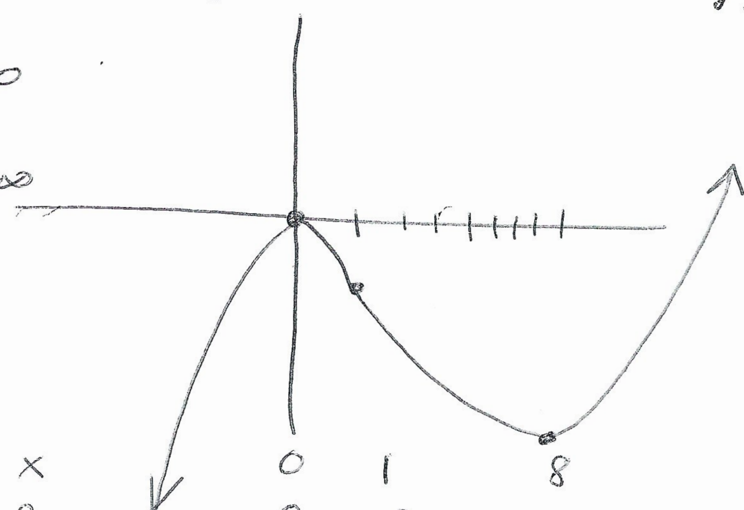
$= x^{4/3}(2x^{1/3}-5)$

$y=0$ at $x=0$ $\; x=\left(\frac{5}{2}\right)^3 = \frac{125}{8}$ $\qquad y'=0$ at $x=0, 8$ $\qquad y''=0$ at $x=1$

$y'' = *$ at $x=0$.

$\lim\limits_{x\to\infty} f(x) = \infty$

$\lim\limits_{x\to-\infty} f(x) = -\infty$



x		0		1		8	
f		0		−3		−16	
f'	+	0		−		0	+
f''	−	*	−	0	+	+	+
		Maximum		Pt of Inflection.		Minimum.	

EXAMPLE 5.

$$y = x^4 - 12x^3 + 48x^2 - 64x$$

also $y = x(x-4)^3$

$y = 0$ at $x = 0, 4$

$y' = 4(x-1)(x-4)^2$

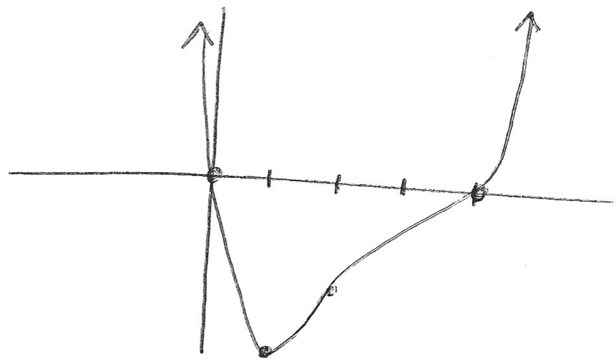
$y' = 0$ at $x = 1$ $x = 4$

$y'' = 12(x-4)(x-2)$

$y'' = 0$ at $x = 4$ $x = 2$

$\lim_{x \to \infty} = \infty$

$\lim_{x \to -\infty} = \infty$



x	0		1		2		4		
f	0		−27		−16		0		
f'	−		0		+		0		−
f''	+		+		0	−	0		+

EXAMPLE 6.

$$y = \frac{\cos x}{1 + \sin x}$$

$y = 0$ at $\frac{\pi}{2}$, $\cancel{\frac{3\pi}{2}}$

Asymptotes at

$x = \frac{3\pi}{2}, -\frac{\pi}{2}$

$y' = -\frac{1}{1+\sin x}$

$y' = *$ at $x = \frac{3\pi}{2}, -\frac{\pi}{2}$

y' always < 0

$y'' = \frac{\cos x}{(1+\sin x)^2}$

$y' = *$ at $x = \frac{3\pi}{2}, -\frac{\pi}{2}$