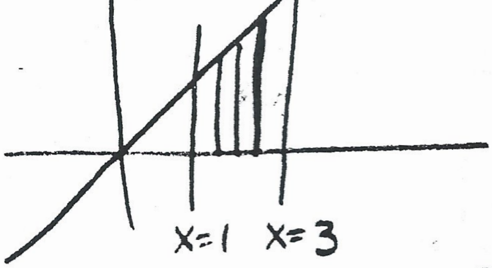
6.




$$\Delta x = \frac{b-a}{n} = \frac{2}{n}$$

$$x_0 = 1$$

$$x_1 = 1 + \Delta x = 1 + \frac{2}{n}$$

$$x_2 = 1 + 2\Delta x = 1 + 2 \cdot \frac{2}{n}$$

$$x_3 = 1 + 3\Delta x = 1 + 3 \cdot \frac{2}{n}$$

$$x_4 = 1 + 4\Delta x = 1 + 4 \cdot \frac{2}{n}$$

$$x_i = 1 + i\Delta x = 1 + i\frac{2}{n}$$

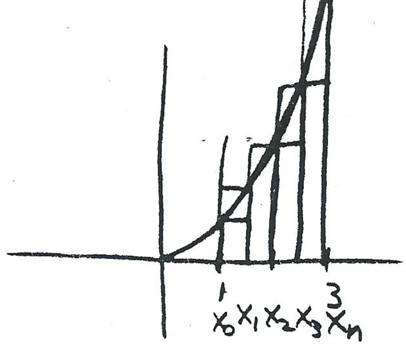
$$x_n = 1 + n\Delta x = 1 + n \cdot \frac{2}{n} = 1 + 2 = 3$$

$$A_n = \sum_{i=0}^{n-1} f(x_i)\Delta x \qquad S_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \sum_{i=0}^{n-1}\left(1 + \frac{2i}{n}\right)\left(\frac{2}{n}\right) \qquad = \sum_{i=1}^{n}\left(1 + \frac{2i}{n}\right)\frac{2}{n}$$

t $j = i+1$ $$= \sum_{j=0}^{n}\left[1 + \frac{2}{n}(j-1)\right]\frac{2}{n}$$

13. $y = 2x^2 \quad [1, 3]$




$$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x$$
$$= 1 + \frac{2i}{n}$$

$$f(x_i) = 2\left(1 + \frac{2i}{n}\right)^2$$

$$A = \sum f(x_i)\Delta x$$

$$= \sum 2\left(1 + \frac{2i}{n}\right)^2 \frac{2}{n}$$

$$= \frac{4}{n}\sum\left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right)$$

$$A = \lim_{n\to\infty}\left(\frac{4}{n}\sum_{i=1}^{n} 1 + \frac{16}{n^2}\sum_{i=1}^{n} i + \frac{16}{n^3}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty}\left[\frac{4}{n}\cdot n + \frac{16}{n^2}\cdot\frac{n(n+1)}{2} + \frac{16}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[4 + 8\left(\frac{n+1}{n}\right) + \frac{8}{3}\,\frac{(n+1)(2n+1)}{n^2}\right]$$

$$= 4 + 8 + \frac{16}{3} = \frac{12}{3} + \frac{24}{3} + \frac{16}{3} = \frac{52}{3}$$

For increasing functions

$$A_{max} = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$A_{min} = \sum_{i=0}^{n-1} f(x_i)\Delta x$$

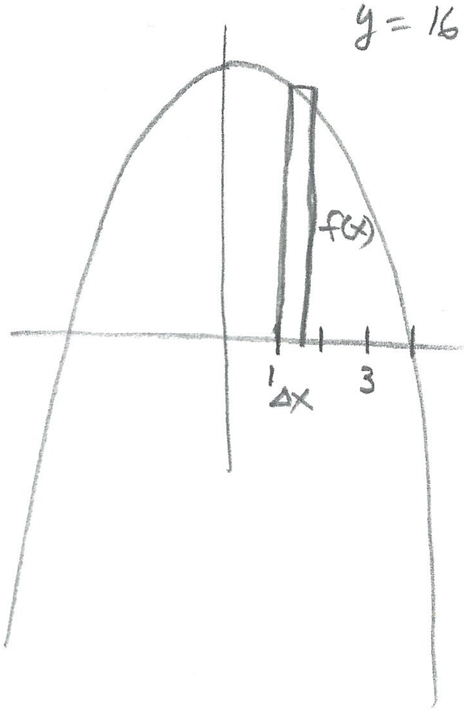
$$A_{exact} = \lim_{n\to\infty}\sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$\text{or} = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

For decreasing $f(x)$, $A_{max}$ and $A_{min}$ are reversed p263.

51.

$y = 16 - x^2$




$$A = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$x_i = 1 + i\Delta x = 1 + \frac{2i}{n}$$

$$A = \sum_{i=1}^{n} \left(16 - x_i^2\right)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[16 - \left(1 + \frac{2i}{n}\right)^2\right]\frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[16 - 1 - \frac{4i}{n} - \frac{4i^2}{n^2}\right]\frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[15 - \frac{4i}{n} - \frac{4i^2}{n^2}\right]\frac{2}{n}$$

$$\frac{30}{n}\sum_{i=1}^{n} 1 - \frac{8}{n^2}\sum i - \frac{8}{n^3}\sum i^2$$

$$\frac{30}{\cancel{n}}\,\cancel{n} - \frac{8}{n^2}\cdot\frac{n(n+1)}{2} - \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6}$$

$$30 - 4 - 8\cdot\frac{1}{3}$$

$$26 - \frac{8}{3} = 23\tfrac{1}{3}$$

$$\frac{78-8}{3} = \frac{70}{3}$$