

# Theorem of Pythagoras

from *Basic Algebra: One Step at a Time* © 2002

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Before introducing the Theorem of Pythagoras, it will be helpful to begin with some perfect square equations. Perfect square equations (see the first examples and the exercises that follow) can be solved by taking the square root of both sides of the equation. This is called the square root property of equations. When you use this property, you must include a " $\pm$ " (that is, "+" or "-") in order to obtain both solutions of the equation.

**Example 1.** Solve the equation  $x^2 = 16$ .

**Solution:** The solution is essentially to answer the question, "What number can be squared (multiplied times itself!) in order to get 16. There are actually two answers:  $x = 4$  and also  $x = -4$ . This answer may be also written  $x = \pm 4$ .

**Example 2.** Solve the equation  $x^2 = 5$ .

**Solution:** Unlike the first example, there is no whole number or integer that you can square in order to get 5. It is possible, however, to take the square root of both sides and write  $x = \pm\sqrt{5}$ . Using a calculator, you can give the decimal approximation which is  $x \approx \pm 2.236$  (round to nearest thousandth. Note: the wavy equal sign " $\approx$ " means "approximately equal.")

**Example 3.** Solve the equation  $x^2 + 12^2 = 15^2$ .

**Solution:**  $x^2 + 12^2 = 15^2$  You know that  $12^2 = 144$ , and  $15^2 = 225$   
 $x^2 + 144 = 225$  Subtract 144 from each side.

$$\begin{array}{r} - 144 \quad - 144 \\ \hline x^2 \quad = \quad 81 \end{array}$$

Because of the  $x^2$ , you have to have two answers: " $\pm$ ".

$$x = \pm 9.$$

**Example 4.** Solve the equation  $x^2 + 10^2 = 15^2$ .

**Solution:**  $x^2 + 10^2 = 15^2$  You know that  $10^2 = 100$ , and  $15^2 = 225$   
 $x^2 + 100 = 225$  Subtract 100 from each side.

$$\begin{array}{r} - 100 \quad - 100 \\ \hline x^2 \quad = \quad 125 \end{array}$$

Because there is no exact answer, use square root.

$$x = \pm\sqrt{125}$$

Don't forget the  $\pm$ . Round off to nearest thousandth.

$$x \approx \pm 11.180.$$

**EXERCISES.** Solve the following perfect square equations. In some, a calculator is needed!

1.  $x^2 = 9$   
 $x = \pm$  \_\_\_\_\_

2.  $x^2 = 25$   
 $x =$  \_\_\_\_\_

3.  $x^2 = 49$   
 $x =$  \_\_\_\_\_

4.  $x^2 = 169$   
 $x =$  \_\_\_\_\_

5.  $x^2 = 81$   
 $x =$  \_\_\_\_\_

6.  $x^2 = 36$   
 $x =$  \_\_\_\_\_

7.  $x^2 = 144$   
 $x =$  \_\_\_\_\_

8.  $x^2 = 121$   
 $x =$  \_\_\_\_\_

9.  $x^2 = 6$   
 $x =$  \_\_\_\_\_

10.  $x^2 = 30$   
 $x =$  \_\_\_\_\_

11.  $x^2 = 200$   
 $x =$  \_\_\_\_\_

12.  $x^2 = 120$   
 $x =$  \_\_\_\_\_

13.  $6^2 + 8^2 = x^2$

14.  $x^2 + 5^2 = 13^2$

15.  $15^2 + x^2 = 17^2$

16.  $x^2 = 3^2 + 4^2$

17.  $5^2 + 6^2 = x^2$

18.  $x^2 + 10^2 = 13^2$

19.  $13^2 + x^2 = 17^2$

20.  $x^2 = 12^2 + 9^2$

21.  $40^2 + 42^2 = x^2$

22.  $x^2 + 24^2 = 25^2$

23.  $70^2 + x^2 = 74^2$

24.  $x^2 = 13^2 + 84^2$

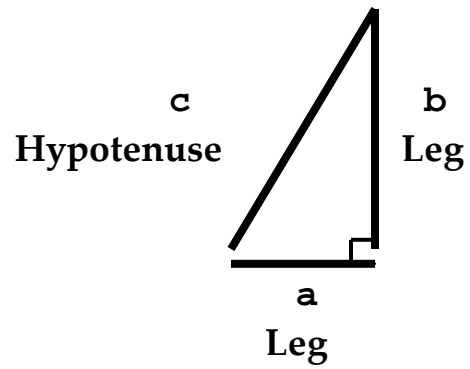
The **Theorem of Pythagoras** is one of the most important formulas in all of mathematics. Although this theorem was known to the Babylonians 1000 years earlier, the credit for the first proof was given to the Greek mathematician **Pythagoras**, 6th century B.C. The **Theorem of Pythagoras** deals specifically with **right triangles**. In a right triangle, the two sides that are mutually perpendicular are called **legs**, and the third side, always opposite the right angle, and always the longest side, is called the **hypotenuse** of the triangle. According to the **Theorem of Pythagoras**, if “**a**” and “**b**” are **legs**, and “**c**” is the **hypotenuse**, then  $a^2 + b^2 = c^2$ .

Given any two sides of a right triangle, the **Theorem of Pythagoras** can be used to find the third side. The first step is to identify which side is the hypotenuse.

### THEOREM OF PYTHAGORAS

In any right triangle, where "a" and "b" are legs, and "c" is the hypotenuse,

$$a^2 + b^2 = c^2.$$



**Example 5.**

To find the distance across a swamp without getting your feet wet, you can measure a distance of 3 miles, make a 90 degree turn, and measure off a distance of 4 miles, forming a right triangle and going around the swamp as shown in the figure. Find the distance across the swamp.

**Solution:**

Let  $x$  = unknown distance across the swamp (hypotenuse).

Equation:  $3^2 + 4^2 = x^2$

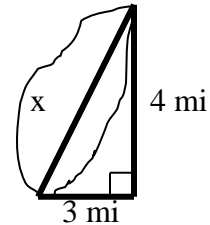
$$9 + 16 = x^2$$

$$x^2 = 25$$

$$x = \pm 5 \text{ miles}$$

Answers:  $x = -5$  is meaningless

$x = 5$  miles is distance across swamp



**Example 6.**

Suppose the sides on the swamp problem (see Example 5) are changed so that the longer leg of the triangle is in the swamp, with the hypotenuse of the right triangle 13 miles, and the shorter leg 5 miles, as shown in the figure below. Find the distance across this swamp.

**Solution:**

Let  $x$  = unknown distance across the swamp (the other leg).

Equation:  $5^2 + x^2 = 13^2$

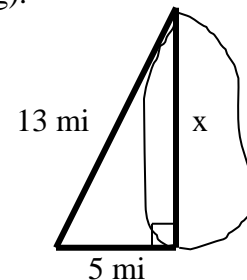
$$25 + x^2 = 169$$

$$x^2 = 144$$

$$x = \pm 12 \text{ miles}$$

Answers:  $x = -12$  is meaningless

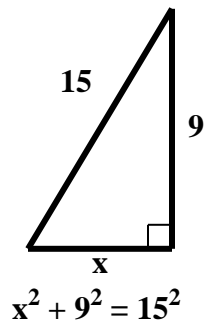
$x = 12$  miles is the distance across swamp



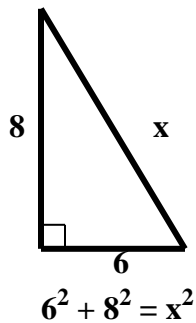
**EXERCISES.**

Find the missing side of each triangle. (Solve for x.)

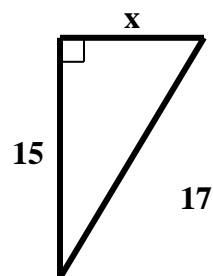
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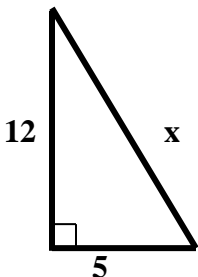
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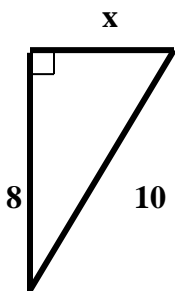
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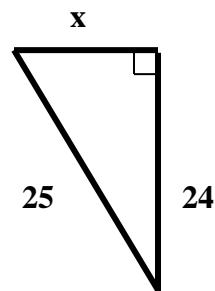
28.



29.



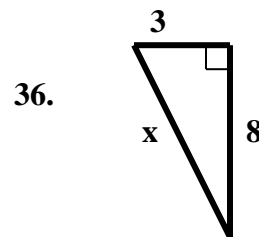
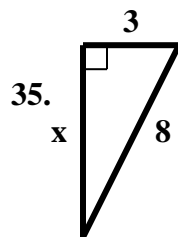
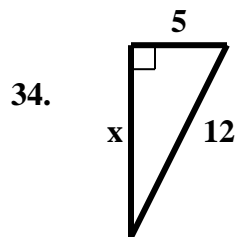
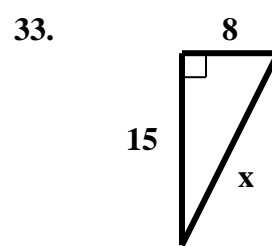
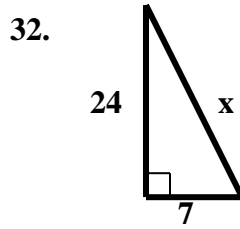
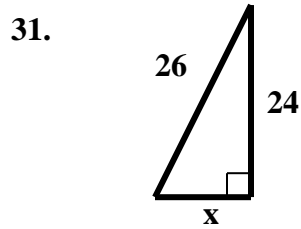
30.



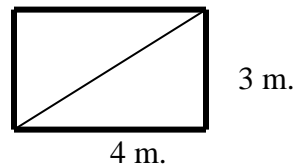
Did you notice that in the swamp examples and the triangle problems so far, all of the sides came out even? Do you think in all such problems, in which you are given two sides of a triangle and asked to find the third side, that the answers come out whole numbers as these did? The truth is that, like the very first examples and exercises of this section, they do not always come out even, and in fact there are really “special” triangles that are like this. Of course, those who make up the exercises (and test questions!) are well aware of these “special” triangles that come out even, and consequently exercises in homework and on tests are frequently (usually?) “rigged” to work

out. Perhaps it would be helpful to let you in on these special numbers. They are called **Pythagorean Triples**. Although there are infinitely many such special triangles, only a few have numbers that are small enough to be useful. The two most common triples were used in the swamp examples: **3,4,5** and **5,12,13**. You may also encounter **8,15,17** or **7,24,25**. In addition, any multiple of these numbers is also a Pythagorean Triple, such as **6,8,10** or **9,12,15**, which are multiples of 3,4,5, and **10,24,26** and **15,36,39** which are multiples of 5,12,13. Nevertheless, in real life, you can't expect things to come out even very often!

**EXERCISES.** Find the missing side of each triangle. (Find  $x$ .) For answers that do not come out even, use a calculator and round to nearest hundredth. Watch for special triangles.



37. Notice in the figure at the right that the diagonal of a rectangle divides the rectangle into two triangles. Use this to find the diagonal if the width is 3 m. and the length is 4 m.



38. Find the diagonal of a rectangle whose width is 6 ft. and whose length is 8 ft.

39. Find the diagonal of a rectangle whose width is 12 cm. and whose length is 16 cm.

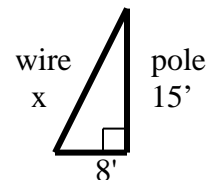
40. Find the length of a rectangle whose width is 8 ft. and whose diagonal is 17 ft.

41. Find the width of a rectangle whose diagonal is 25 cm. and length is 24 cm.

42. Find the width of a rectangle whose diagonal is 29 cm. and whose length is 21 cm.

43. Find the diagonal of a rectangle whose width is 13 cm. and whose length is 84 cm.

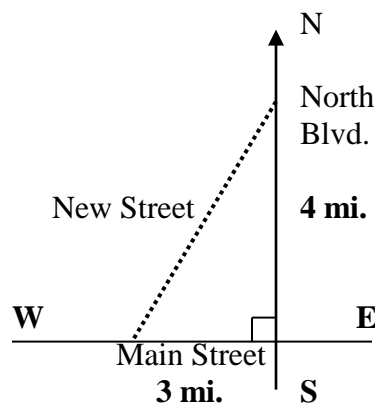
44. A guy wire to the top of a 15 foot pole reaches the ground 8 feet from the base of the pole. How long is the wire?



45. A guy wire to the top of a 35 foot pole reaches the ground 18 feet from the base of the pole. How long is the wire?
46. A guy wire to the top of a pole is 35 feet long. It reaches the ground 18 feet from the base of the pole. How tall is the pole?
47. A guy wire to the top of a pole is 73 feet long. It reaches the ground 48 feet from the base of the pole. How tall is the pole?

**NOTE:** Example 7 and Exercises 48-50 are taken from The CLAST Study Guide for the Mathematics Subtest (1994) and the CLAST Item Specifications Sample Problems (1990), State of Florida, Department of State.

**Example 7.** The city commission wants to construct a new street connecting Main Street and North Boulevard as shown in the diagram. Construction cost has been estimated at \$100 per linear foot. What is the estimated cost for constructing the new street?



**Solution:** Let  $x$  = length the new street in miles.

By Theorem of Pythagoras,

$$3^2 + 4^2 = x^2$$

$$25 = x^2$$

$$x = (5 \text{ miles}) \times 5280 = 26,400 \text{ feet}$$

$$\text{Cost} = 26,400 \times \$100 \text{ per foot} = \mathbf{\$2,640,000}$$

## EXERCISES.

48. A radio station is going to construct a 12 foot tower for a new antenna on top of a tall building. The tower will be supported by three cables, each attached to the top of the tower and to points on the roof which are 5 feet from the base of the tower. What is the total length of these cables?
49. A tent is being set up for a group of people. A 12-foot pole is to be placed in the center. Four pieces of heavy-duty rope are to be attached to the top of the center pole and also attached to points on the ground that are 9 feet from the base of the pole. What is the total length of the four pieces of rope?
50. The owner of a rectangular piece of land 12 yards in length and 9 yards in width wants to divide it into two parts. He plans to join two opposite corners with a fence as shown in the diagram below. The cost of the fence will be approximately \$40 per linear foot. What is the estimated cost for the fence needed by the owner?

## ANSWERS TO EXERCISES

1.  $\pm 3$ ; 2.  $\pm 5$ ; 3.  $\pm 7$ ; 4.  $\pm 13$ ; 5.  $\pm 9$ ; 6.  $\pm 6$ ; 7.  $\pm 12$ ; 8.  $\pm 11$ ; 9.  $\pm\sqrt{6}$ ,  $\pm 2.45$ ;  
10.  $\pm\sqrt{30}$ ,  $\pm 5.48$ ; 11.  $\pm\sqrt{200}$ ,  $\pm 14.14$ ; 12.  $\pm\sqrt{120}$ ,  $\pm 10.95$ ; 13.  $\pm 10$ ;  
14.  $\pm 12$ ; 15.  $\pm 8$ ; 16.  $\pm 5$ ; 17.  $\pm\sqrt{61}$ ,  $\pm 7.81$ ; 18.  $\pm\sqrt{69}$ ,  $\pm 8.31$ ;  
19.  $\pm\sqrt{120}$ ,  $\pm 10.95$ ; 20.  $\pm 15$ ; 21.  $\pm 58$ ; 22.  $\pm 7$ ; 23.  $\pm 24$ ; 24.  $\pm 85$ ; 25. 12;  
26. 10; 27. 8; 28. 13; 29. 6; 30. 7; 31. 10; 32. 25; 33. 17; 34.  $\sqrt{119}$ , 10.91;  
35.  $\sqrt{55}$ , 7.42; 36.  $\sqrt{73}$ , 8.54; 37. 5 m; 38. 10 ft; 39. 20 cm; 40. 15 ft; 41. 7 cm;  
42. 20 cm; 43. 85 cm; 44. 17 ft; 45. 39.36 ft; 46. 30.02 ft; 47. 55 ft; 48. 39 ft;  
49. 60 ft; 50. \$1800.