# Perimeter, Area, and Volume

**Dr. Robert J. Rapalje**
**Seminole Community College--Hunt Club Center**
**Tech Prep Integrations**
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So many of the applications from everyday life and the world of business involve concepts from geometry, especially those of perimeter, area, and volume. In architecture, construction, agriculture, interior design, and so many other arenas of business, these skills are essential.

**Perimeter** is simply the total distance around a geometric figure. It will always be given in **linear** (plain) **units** such as feet, inches, meters, centimeters, etc. To find the perimeter of a figure, you just add up the external sides of the figure. While perimeter is always expressed in linear units, **area** is always expressed in **square units**, such as square feet (sq. ft. or $ft^2$), square inches (sq. in. or $in^2$), square meters (sq. m. or $m^2$), square centimeters (sq. cm. or $cm^2$), etc. Perimeters and areas always involve two-dimensional figures, like squares, rectangles, circles, and triangles. Volumes always involve three-dimensional figures, like rectangular solids (boxes!), cylinders, cones, and spheres. The **volume** of a three-dimensional solid is always given in **cubic units**, such as cubic feet (cu. ft. or $ft^3$), cubic inches (cu. in. or $in^3$), cubic meters (cu. m. or $m^3$), or cubic centimeters (cc. or $cm^3$).
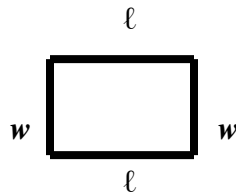
Before attempting to solve problems involving perimeters, areas, or volumes, a few basic formulas must be established. Also, in any given problem, you must be careful to have the same units for all sides being used (see Example 7).

## Perimeter and Area
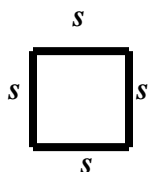
### Rectangle

$P = 2w + 2\ell$
$A = \ell\, w$

The perimeter consists of *two* widths and *two* lengths.
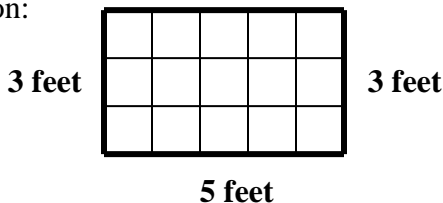
### Square

$P = 4s$
$A = s^2$

The area is simply the length times the width.

**EXAMPLE 1.**    Find the perimeter and the area of a rectangle that is 3 feet by 5 feet.

Solution:

| | |
|---|---|
| $P = 2w + 2\ell$ | $A = \ell\, w$ |
| $= 2(3ft) + 2(5ft)$ | $= (5\ ft) \times (3\ ft)$ |
| $= 6\ ft + 10\ ft$ | $= \mathbf{15\ sq.\ ft.}$ |
| $= \mathbf{16\ ft.}$ | |

5 feet

3 feet     3 feet

5 feet

**EXAMPLE 2.**   Find the perimeter and area of a 5-foot square.

Solution:

5 feet

5 feet    5 feet

5 feet

$$P = 4\,s$$
$$= 4\,(5\text{ ft})$$
$$= 20\text{ ft.}$$

$$A = s^2$$
$$= (5\text{ ft})^2$$
$$= 25\text{ sq. ft.}$$

**Triangle**

$\ell$
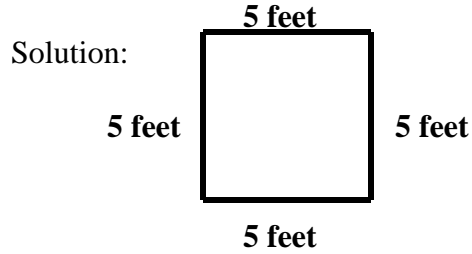
$w$   $h$=height

$\underline{b}$=base
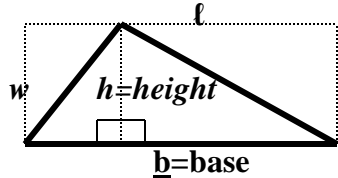
The perimeter of a triangle is simply the sum of the three sides.

It can be seen from the figure that the area of the triangle which has a **base b** and **height h** (b and h must be perpendicular) is actually half the area of the rectangle $\ell\,w$. Therefore,
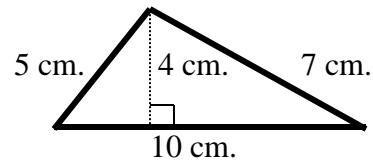
$$A = \frac{bh}{2} \quad \text{or} \quad A = \frac{1}{2}bh$$

**EXAMPLE 3.**   Find the perimeter and the area of the triangle that is shown below.

Solution:   **P = 10 + 5 + 7 = 22 cm.**

$$A = \frac{bh}{2}$$

$$A = \frac{10 \cdot 4}{2} = \frac{40}{2} = 20 \text{ sq. cm. or } 20 \text{ cm}^2$$

5 cm.   4 cm.   7 cm.

10 cm.

**EXAMPLE 4.**   Find the perimeter and the area of the triangle that is shown below.

Solution:   **The height is unknown, so the Theorem of Pythagoras must be used. (See Theorem of Pythagoras for additional explanation.)**

$$15^2 + x^2 = 17^2$$
$$225 + x^2 = 289$$
$$x^2 = 64$$
$$x = 8 \text{ m.}$$

17 m.

$x$

15 m.

**Now,   P = 15 + 8 + 17 = 40 m.**

$$A = \frac{15 \cdot 8}{2} = \frac{120}{2} = 60 \text{ sq. m. or } 60 \text{ m}^2$$
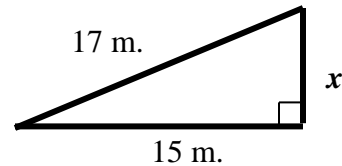
## Circle

The Greeks, especially a mathematician named Euclid, deserve credit for most of what we know about geometry in general, and circles and triangles in particular. In the days before cable television and video games, they worked math problems for recreation and education. One of the great discoveries regarding the circle was that the ratio of the distance across a circle (called the **circumference** of the circle) to the distance across the circle (called the **diameter** of the circle) was always about the same (called a **constant**). No matter how large or small the circle, the circumference divided by the diameter (**C/d**) always came out a little more than 3. In fact, it always came out about 3.14. This constant became the subject of much study and was named after one of the letters in their alphabet, the Greek letter $\pi$ **(pi)**.

So $\pi$ was defined as the ratio of the circumference to the diameter of a circle.

$$\pi = \frac{Circumference}{diameter} \qquad \text{or} \qquad \pi = \frac{C}{d}$$

It is then a simple algebraic step to multiply both sides of this equation by **d** to obtain

$$\pi d = C \qquad \text{or} \qquad C = \pi d.$$

Furthermore, since in a circle the diameter is twice the radius, **d = 2r**, so

$$C = \pi d$$
$$C = \pi(2r)$$
$$C = 2\pi r$$

This gives two formulas for finding the circumference (or perimeter) of a circle. After all, it is not possible in a circle to "add up the sides," as in other geometric figures.

It can also be shown that the area of a circle is given by the formula $A = \pi r^2$.

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$

$$A = \pi r^2$$

**EXAMPLE 5.**     Find the circumference and area of a circle whose radius is 20 cm.

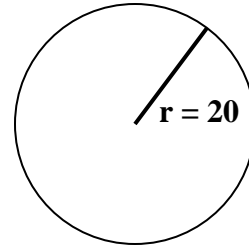Solution:     $C = 2\pi r$                    **$r = 20$ cm.**   (Using $\pi = 3.14$)
              $C = 2\pi(20$ cm.$)$
              $C = 40\pi$  or approximately **125.6 cm.**

              $A = \pi r^2$                    **$r = 20$ cm.**
              $A = \pi(20$ cm$)^2$
              $A = 400\,\pi$  or approximately **1256 cm.$^2$**

**EXAMPLE 6.**     Find the circumference and area of a circle whose diameter is 60 ft.

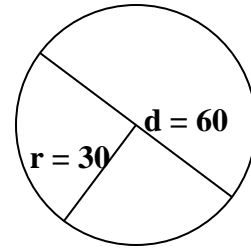Solution:     $C = \pi d$                    **$d = 60$ ft.**       (Using $\pi = 3.14$)
              $C = \pi(60$ ft.$)$
              $C = 60\pi$  or approximately **188.4 ft.**

              $A = \pi r^2$                    **$r = 30$ ft.**
              $A = \pi(30$ ft$)^2$
              $A = 900\,\pi$  or  approximately **2826 ft.$^2$**

**EXERCISES.**        **Find the perimeter and area of each of the following, using $\pi = 3.14$:**

1.     A rectangle whose length is 18 cm. and whose width is 10 cm.

2.     A rectangle whose length is 38 m. and whose width is 30 m.

3.     A square whose sides are each 12 m.

4.     A square whose sides are each 30 feet.

5.     A right triangle whose legs are 12 cm. and 5 cm.

6.     A right triangle whose hypotenuse is 15 in. and one of the legs 12 in.

7.     An equilateral triangle (all sides equal) whose sides are 20′, and whose height is 17.3′.

8.     An isosceles triangle (two sides equal), whose base is 10″, and whose height is 12.″
       (Hint:  See figure below.  First use the Theorem of Pythagoras to find the equal sides.)
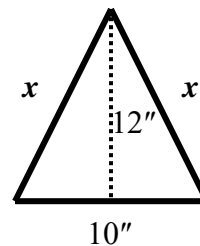
9.    An isosceles triangle whose base is 8 ft. and whose height is 3 ft.

10.   An isosceles triangle whose base is 12 ft. and whose height is 8 ft.

11.   An isosceles triangle whose equal sides are each 10 m. and whose base is 12 m.
      (Hint:  First use the Theorem of Pythagoras.)

12.   An isosceles triangle whose equal sides are each 13 feet and whose base is 10 ft.

13.   A circle whose radius is 10 cm.          14.   A circle whose diameter is 10 cm.

15.   A circle whose diameter is 6 m.          16.   A circle whose radius is 6 m.

17.   A circle whose radius is 25 m.           18.   A circle whose diameter is 25 m.


## VOLUMES

**Volume** may be defined as a measure of the inside capacity of a solid.  It must be measured in **cubic units.**  For example, a **cubic inch** is actually a cube that is one inch by one inch by one inch.  A **cubic meter** is a cube that is one meter by one meter by one meter.

**EXAMPLE 7.**  Find the volume of a cube that is 5 inches long, 4 inches wide, and 3 inches high.
             (See figure below.)

Solution:      Remember that a cubic inch is a cube that
               is one inch by one inch by one inch.  There
               are 4 x 5 = 20 such cubes in the top layer,
               another 20 cubes in the middle layer, and
               20 cubes in the bottom layer, for a total of
               60 **cubic inches**.  This example illustrates
               the formula for the volume of a box, which is

$$\textbf{Box:  V = LWH}$$

In general, the volume of any solid that is the same from bottom to top (called a **prism**) is the **area of the base (A)** times the **height (H)**.

$$\textbf{Prism:  V = A•H}$$

A special case of this is a **cylinder**.  If the base is a circle of radius **r** and the height is **h**, then the area of the base is **π r $^2$**, and the volume of the cylinder is given by

$$\textbf{Cylinder: } V = \pi\, r^{\,2}\, h$$

**EXAMPLE  8.**   Find the volume of a cylindrical can whose height is 10 inches and whose base has a diameter of 10 inches.

Solution:          **V = π r $^2$ h**, where r = 5 inches and h = 10 inches.
                        **V = 3.14 • (5 in) $^2$ • (10 in)  = 785 cubic inches**.

Two other important solids that are not prisms are the sphere and the cone.  Their formulas will be given without detailed explanation.

$$\textbf{Cone: } V = \frac{1}{3}\pi r^2 h \qquad \textbf{Sphere: } V = \frac{4}{3}\pi r^3$$

**EXERCISES.**   Find the volumes of each of the following.

19.     A rectangular box which is 6 m. by 4 m. by 3 m.

20.     A rectangular box which is 9 ft. by 5 ft. by 4 ft.

21.     A cube whose edge is (whose sides are) 5 inches.

22.     A cube whose edge is 30 cm.

23.     A circular cylinder whose height is 5 meters and whose base radius is 10 meters.

24.     A circular cylinder whose height is 10 meters and whose base radius is 5 meters.

25.     A circular cylinder whose height is 10 meters and whose base is of diameter 4 meters.

26.     A circular cylinder whose height is 10 meters and whose base is of diameter 5 meters.

27.     A sphere of radius 10 centimeters.

28.     A sphere of diameter 5 centimeters.

29.     A cone whose base is of radius 10 inches and whose height is 10 inches.

30.     A cone whose base is of diameter 10 inches and whose height is 12 inches.

## CONVERSIONS

Sometimes it is necessary to convert from one unit to another unit.  When converting units of measure, you must know the basic conversion numbers.  For the examples and exercises here, you will need to know that there are 12 inches in 1 foot, 3 feet in 1 yard, 5280 feet in 1 mile, 100 centimeters in 1 meter, 1000 meters in 1 kilometer, 1000 millimeters in 1 meter, and 10 millimeters in 1 centimeter. (See <u>Metric System</u> if you need help with these metric units.)  Also, it may be helpful to know that when converting from larger units to smaller units (like feet to inches), you multiply times the conversion number.  Because you are converting to smaller units, there will be more of them, so you multiply!  When converting from smaller units to larger units (like inches to feet), you divide by the conversion number.  Because you are converting to larger units, there will be less of them, so you divide!

For example, suppose you want to convert 24 inches to feet (smaller to larger). Because there are 12 inches in a foot, you must divide 24 by 12 to obtain 2 feet. If you want to convert 24 feet to inches, then you must multiply 24 times 12 to obtain 288 inches.

**EXAMPLE 9.** How many square feet are in one square yard?

Solution: The critical question: How many feet are in 1 yard?
[Answer: It depends upon how many people are in the yard!]



1 sq yd = 3ft by 3 ft = **9 sq. ft.**
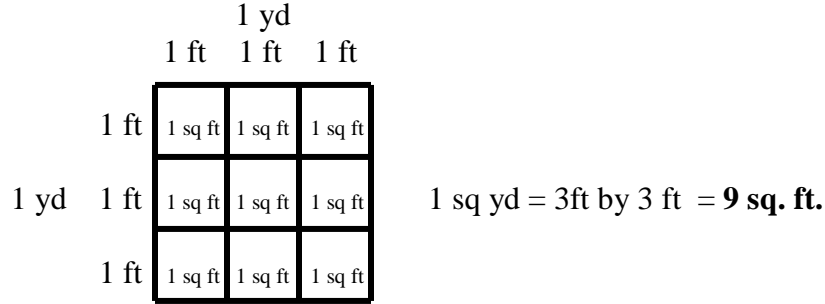
**EXAMPLE 10a)** How many square inches are in 1 square foot?
**b)** How many square centimeters are in 1 square meter?
**c)** If there are 6 giggles in a gaggle, how many square giggles are in 1 square gaggle?

Solution: **a)** 12 by 12 = **144** square inches in 1 square foot.
**b)** 100 by 100 = **10,000** square centimeters in 1 square meter.
**c)** 6 by 6 = **36** square giggles in 1 square gaggle.

**EXAMPLE 11.** How many cubic feet are in 1 cubic yard?

Solution: A cubic yard is 3 feet by 3 feet by 3 feet.
Notice in the figure to the right, there are
3 x 3 x 3 = **27 cubic feet** in the cubic yard.

**EXAMPLE 12a)** How many cubic inches are in 1 cubic foot?
**b)** How many cubic millimeters are in 1 cubic centimeter?
**c)** How many cubic centimeters are in 1 cubic meter?
**d)** If there are 7 rubics in a zircon, how many cubic rubics are in 1 cubic zircon?

Solution: **a)** $12^3$ = 1728 cubic inches in 1 cubic foot.
**b)** $10^3$ = 1000 cubic millimeters in 1 cubic centimeter.
**c)** $100^3$ = 1,000,000 cubic centimeters in 1 cubic meter.
**d)** $7^3$ = 343 cubic rubics in 1 cubic zircon.

**EXAMPLE 13.** Find the perimeter and area of a rectangle of length 12 feet and width 6 inches.

Solution:        In this exercise, the units of measure are not the same.  So, you have to either
**a)** change the 6 inches to ½ (or 0.5) foot, or
**b)** change the 12 feet to 144 inches.
**c)** Compare the answers to see if they are consistent.

**a)** 12 ft

½ ft [rectangle] ½ ft

12 ft

**b)** 144 in

6 in [rectangle] 6 in
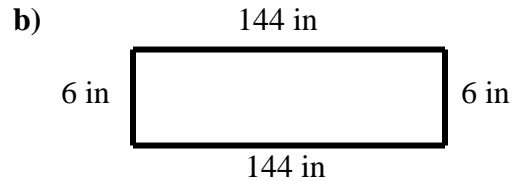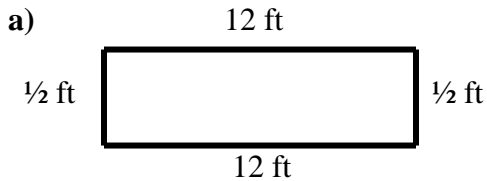
144 in

**P = 12 + 12 + ½ + ½ = 25 ft.**
**A = 12 ft. x ½ ft. = 6 sq. ft.**

**P = 144 + 144 + 6 + 6 = 300 in.**
**A = 144 in.  x  6 in. = 864 sq. in.**

**c)**        **Compare the answers**.
There are 12 feet in 1 yard, so **P = 25 feet x 12 = 300 inches.**
There are 144 square inches in 1 square yard, so **A = 6 sq ft x 144 = 864 sq in.**

**EXAMPLE 14.**        Find the area of a rectangle whose width is 60 cm and whose length is 5 m.

Solution:        You may convert to 5 meters to 500 cm.        A = 60 x 500 = **30,000 sq. cm.**
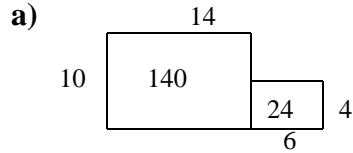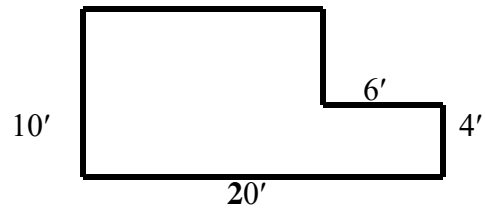        -OR- You may convert 60 cm to 0.60 m.        A = 0.60 x 5  = **3 sq. m.**

**EXERCISES.**

31.     How many square feet are in 1 square yard?

32.     How many square millimeters are in 1 square meter?

33.     How many cubic millimeters are in 1 cubic meter?

34.     How many cubic inches are in 1 cubic foot?

35.     A rectangle is 6 inches by 4 feet.  Find the perimeter and area.

36.     A rectangle is 3 yards by 12 feet.  Find the perimeter and area.

37.     A rectangle is 30 cm by 4 m.  Find the perimeter and area.

38.     A rectangle is 50 cm. by 40 mm.  Find the perimeter and area.

39.     A triangle has base 9 yards and height 3 feet.  Find the area.

40.     A triangle has base 120 m. and height 1000 cm.  Find the area.
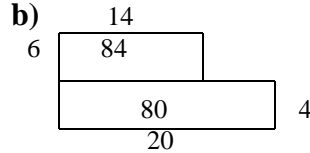
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**EXAMPLE 15.**     Find the area of the figure:

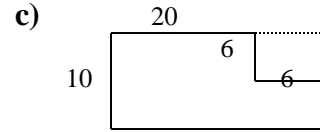Solution:  There are three solutions:

10′

6′

4′

20′

**a)**

14

10   |   140

24   |   4

6

**b)**

14

6   |   84

80   |   4

20

**c)**

20

6

10

6

Total:   140
        + 24
     **164 sq. ft.**

Total:   84
        + 80
     **164 sq. ft.**

Whole:  20 x 10 =  200
        - 6 x 6   = - 36
              **164 sq. ft.**


**EXAMPLE 16.**     Find the perimeter and the area of the figure.

Solution:     The figure is a square with a semi-circle of
              radius 5 m.  Find the area of the square and
              of the semi-circle.

10m

10 m
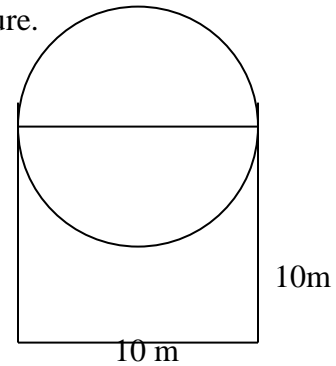
              Area:  Circle  $A = \pi r^2$
                            $= \pi \times 5^{\,2}$
                            $= 78.5$ square meters
              Semicircle $= \frac{1}{2}$ of $78.5 = $  39.25 sq. m.

              Square  $A = 10 \times 10$   $=$   100.00 sq. m.
                    **Total Area      = 139.25 sq. m.**

              Perimeter:  Circle  $C = \pi d$
                              $= \pi \times 10 = 31.4$ m.

              Semicircle $= \frac{1}{2}$ of $31.4 = $          15.7 m.
              Three sides of square  $= 3 \times 10 =$ 30.0 m.
                    **Total Perimeter        = 45.7 m.**

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# APPLICATIONS

Media

1. A series of photographs is to be taken of an aircraft, automobile, or other object. With the object at the center of a circle of radius 100 feet, it is desired that a picture be taken every ten degrees. How many pictures must be taken, and what distance along the circle should the photographer move between consecutive pictures? (Total circumference = $200\pi$ feet. 36 pictures every 17.45 feet)

Construction

1. You've been asked to participate in a Habitat for Humanity project in a lower socio-economic community. The community is asking to make improvements for children and families in a local park. You are working with a team of people and they have given you the assignment of designing and constructing a gazebo. You've decided to build the gazebo in the shape of a hexagon. Find the length of each side if the gazebo is to be 20 feet across (from side to side).

2. Does it make a difference if the gazebo described in #1 is to be 20 feet across (from corner to corner) instead of from side to side? Is this design larger or smaller? Find the length of each side of the gazebo according to this construction.

3. Concrete blocks are 8"x8"x16". How many blocks will it take to build a wall that is 20 feet long and 10 feet high?

4. Concrete blocks cost $1.50 each, and labor for laying the blocks is $1 each for the first 100 blocks and $.75 for each block thereafter. How much will it cost for blocks and labor to build a wall that is 20 feet long and 10 feet high? (See previous exercise.)

5. Wooden fence comes in prefabricated 8-foot lengths, costing $21.95 per unit. If a rectangular back yard is 40 feet on the sides and 100 across the back, how many units will it take to fence both sides and the back of the yard? If a post is required at each corner and between each unit, how many posts will be required? If the posts cost $7.95 each, what is the total cost of the prefabricated units and posts for the fence?

6. A 4-foot sidewalk, 6 inches thick, is to be built from the dining room to a dormitory, a distance of a quarter mile. How many cubic yards of concrete will be needed for the job?

Interior Design.

1. A rectangular room is 12 feet by 15 feet. How many square yards of carpet will be needed to cover the floor?

2. A rectangular room is 12 feet by 15 feet, with 10 foot ceiling. If paint can be applied at the rate of 300 square feet per gallon, how many gallons of paint will be needed to paint the walls of the room (for simplicity sake, neglect windows and doors)?

3. If the rectangular room in the previous exercise has 4 windows that are each 6 feet by 8 and 2 doorways that are each 4 feet by 8 feet, does this reduce the number of gallons of paint that you will need to paint the room?
_____

Agriculture.

1. A privacy hedge is to be planted, with one bush every 5 feet.  If the hedge must extend for a total distance of 300 yards, how many bushes will be needed?  If the bushes cost $6 each, what is the cost of the hedge?

2. There are 16 ounces in a pint, 2 pints in a quart, and 4 quarts in a gallon.  How many ounces are in 1 gallon?  (By the way, where did we get such strange units?  Is there a better way?  See "Metric System" section.)

3. A chemical spray is to be diluted at a rate of 3 ounces of chemical to a gallon of water and then applied to 200 square feet of coverage.  If there are 43560 square feet per acre, how many ounces of the chemical will be needed per acre of coverage, and how much water should be used to dilute it?  Use the answer to the previous exercise to express the number of ounces of chemical in gallons.

**NOTE:**   The following exercises are taken from The CLAST Study Guide for the Mathematics Subtest (1994) and the CLAST Item Specifications Sample Problems (1990), State of Florida, Department of State.

**105.   The figure below shows a water tank in the shape of a cylinder with a hemisphere on top.**

**106.   The figure below shows a running track having the shape of a rectangle with semicircles at each end.**

**107.** A patio is to be built of concrete. The base of the patio is to be a slab of concrete 15 feet long by 12 feet wide by 6 inches thick. If one cubic yard of concrete costs $39, how much will the concrete for the patio cost?

  A. $65      B. $130      C. $1560      D. $3510


**108.** A fence that costs $6.50 per yard is to be placed around a rectangular yard that is 90 feet by 120 feet. What is the total cost of the fence?

  A. $910      B. $1365      C. $2730      D. $7800

**109.** A rectangular flower bed measures 5 feet by 54 inches. The outside dimensions of a path around the bed are 7¼ feet by 81 inches. What is the area of the path?

  A. 29 sq feet  B. 317 sq feet  C. 317.25 sq feet  D. 3807 sq inches


**110.** The trunk of a tree has a 1.2-meter diameter. What is its circumference?

 A. 0.36  sq meters  B. 0.6  meters  C. 1.2  meters  D. 1.44  sq meters


**111.** What will be the cost of carpeting an office that measures 12 feet by 15 feet if the carpet costs $12.50 per square yard?

  A. $250      B. $650      C. $750      D. $2250


**112.** The outside dimensions of a picture frame are 2 feet by 30 inches. If its outside dimensions are 1¾ feet by 27 inches, what is the area of the frame?

 A. 12 sq feet   B. 12.75 sq feet   C. 153 sq inches   D. 162 sq inches


### ANSWERS TO EXERCISES

**1.** P=56 cm, A=180 sq cm;      **2.** P=136 m, A=1140 sq m;   **3.** P=48 m, A=144 sq m;
**4.** P=120 ft, A=900 sq ft;      **5.** P=30 cm, A= 30 sq cm;     **6.** P=36 in, A= 54 sq in;
**7.** P=60 ft, A=173 sq ft;       **8.** P=36 in, A= 60 sq in;      **9.** P=18 ft, A= 12 sq ft;
**10.** P=32 ft, A=48 sq ft;       **11.** P=32 m, A= 48 sq m;      **12.** P=36 ft, A= 60 sq ft;
**13.** P=62.8 cm, A=314 sq cm;       **14.** P=31.4 cm, A= 78.5 sq cm;
**15.** P=18.84 m, A= 28.26 sq m;      **16.** P=37.68 m, A=113.04 sq m;
**17.** P=157 m, A= 1962.5 sq m;       **18.** P=78.5 m, A= 490.63 sq m;

**19.** $\pm\sqrt{120}$ , $\pm 10.95$;  **20.** $\pm 15$;  **21.** $\pm 58$;  **22.** $\pm 7$;  **23.** $\pm 24$;  **24.** $\pm 85$;  **25.** 12;  **26.** 10;
**27.** 8;  **28.** 13;  **29.** 6;  **30.** 7;
**31.** 10;  **32.** 25;  **33.** 17;  **34.** $\sqrt{119}$ , 10.91;  **35.** $\sqrt{55}$ , 7.42;  **36.** $\sqrt{73}$ , 8.54;
**37.** 5 m;  **38.** 10 ft;  **39.** 20 cm;  **40.** 15 ft;  **41.** 7 cm;  **42.** 20 cm;  **43.** 85 cm;  **44.** 17 ft;
**45.** 39.36 ft;  **46.** 30.02 ft;  **47.** 55 ft;