Topics in Probability and Statistics

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SEMINOLE COMMUNITY COLLEGE

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P R O B A B I L I T Y

Introduction and Definition

One very important topic in Mathematics, and especially in Statistics, is that of Probability. With so many majors now requiring one or more courses in Statistics, and with so many students who have never worked with Probability, the latter seems an appropriate topic for Essential Math.

Some of the more common applications of Probability are: weather forecasting (probability of rain), gambling (cards, dice, roulette wheels, raffles, etc.), and insurance rates (based upon probability of having an accident, a fire or theft, or of dying, for automobile, homeowner, or life insurance, respectively.)

Perhaps the simplest example for illustration is the probability of rain. A 50% chance of rain means there is a probability of 50% or 1/2 that it will rain. Similarly, a 30% chance of rain means that the probability of rain is 30%, or since "%" means "per 100", it means 30 chances per 100 or 30 chances out of 100 or 30/100, which reduces to 3/10. We denote this: P(rain) = 3/10, read "Probability of rain equals 3/10." It could also be noted that if the chances of rain are 30%, i.e. P(rain) = 3/10, then the chances of it not raining are 70%, i.e. P(no rain) = 7/10.

Notice that the probability of an event would always be a fraction between 0 and 1, where probability of 0 means that the event is not possible and a probability of 1 would indicate a 100% chance that the event will occur. Notice also that the sum of the probability of rain and the probability of no rain is 1. In general, it is true that P(event will happen) + P(event will not happen) = 1, and moreover, the sum of the probabilities of every possible outcome of an event must equal 1.

A working definition of Probability, then, is:

$$P(\text{an event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

EXAMPLE 1

Consider the experiment of tossing one die. Find the probability of getting a five. A favorable outcome would be an outcome resulting in five. Since there is only one way of getting a five out of the total of six outcomes, P(five) = 1/6. For the same experiment of tossing one die, find P(even). There are 3 favorable outcomes (two, four, or six), out of the total of six outcomes, so P(even)=3/6, which reduces to 1/2.

C A R D S

(See page P-26 for breakdown of the cards.)

EXAMPLE 2

From a deck of 52 cards, one card is drawn at random. Find (a) P(ace), (b) P(black card), (c) P(heart), (d) P(red ace), (e) P(red spade), (f) P(red ace or heart), (g) P(red card or ace)

ANSWER

(a) P(ace) = 4/52 = 1/13

(b) P(black card) = 26/52 = 1/2

(c) P(heart) = 13/52 = 1/4

(d) P(red ace) = 2/52 = 1/26 (there are two red aces)

(e) P(red spade) = 0/52 = 0 (there are no red spades)

(f) P(red ace or heart) = 14/52 = 7/26 (there are two red aces and thirteen hearts, but one of them was counted twice, so the number of "red aces or hearts" is 14.

(g) P(red card or ace) = 28/52 = 7/13 (reduces by 4) (there are 26 red cards and four aces, two of which were counted twice, for a total of 28 cards that are "red or ace".

O D D S

Other means of describing the likelihood that an event will occur or will not occur are "odds in favor" and "odds against" defined:

$$\text{Odds in favor} = \frac{\text{no. favorable outcomes}}{\text{no. unfavorable outcomes}}$$

$$\text{Odds against} = \frac{\text{no. unfavorable outcomes}}{\text{no. favorable outcomes}}$$

They may be expressed as a fraction or as a ratio of two numbers. For example, odds in favor of tossing one die and getting a five would be 1/5 or 1 to 5 or 1:5, since there is 1 favorable outcome and 5 unfavorable outcomes. Odds in favor of getting an even number would be 3/3 or 3 to 3 or 3:3. However, the ratio should be reduced to 1/1 or 1 to 1 or 1:1. The last is an example of "even" odds.

Finally, comparing "probability" and "odds in favor", "probability" is the number of favorable compared to the total number of outcomes, while "odds in favor" is the number of favorable compared to the number of unfavorable.

EXAMPLE 3

If one card is drawn from a deck of 52, compare P(ace) to odds in favor of an ace.

ANSWER

P(ace) = 4/52 = 1/13 which means "1 favorable out of 13 (total)"

Odds in favor = 4/48 = 1/12 which means "1 favorable compared to 12 unfavorable"

With probability, it is correct to say 1 out of 13, but with odds in favor, it should be stated 1 to 12.

EXAMPLE 4

If P(event)= 3/17, find P(event will not occur).

ANSWER

P(event) = 3/17 means that there are 3 ways the event can occur out of a total of 17 outcomes, which leaves 14 ways that it will not occur.
So P(event will not occur) = 14/17 (14 out of 17).

EXAMPLE 5

If P(event) = 3/10, find P(event will not occur).

ANSWER

P(event will not occur) = 7/10

EXAMPLE 6

If P(event) = .325, find P(event will not occur).

ANSWER

P(event will not occur) = 1.000 - .325 = .675

EXAMPLE 7

If P(event) = 3/17, find the odds in favor.

ANSWER

P(event) = 3/17 means there are 3 favorable out of a total of 17 outcomes, which leaves 14 that are unfavorable. So odds in favor would be 3 (favorable) to 14 (unfavorable).

EXAMPLE 8

If the odds in favor of an event are 3 to 2, find P(event).

ANSWER

If odds in favor are 3 to 2, that makes a total of 5 outcomes, with 3 in favor, so P(event) = $\frac{3 \text{ (favorable)}}{5 \text{ (total)}}$

EXAMPLE 9

If the odds against are 3 to 2, find P(event).

ANSWER

This time odds <u>against</u> are 3 to 2 which means 3 unfavorable, to 2 favorable, with total of 5 outcomes, so:

P(event) = $\frac{2 \text{ (favorable)}}{5 \text{ (total)}}$

PROBLEMS:

In 1 through 18, find Probability and Odds in Favor given that one card is drawn from a deck of 52.

1. P(Queen)
2. P(Spade)
3. P(Red card)
4. P(Red queen)
5. P(Jack of spades)
6. P(3 of hearts or 2 of clubs)
7. P(Black card or heart)
8. P(Ace or heart)
9. P(Ace or black card)
10. P(Black ace or heart)
11. P(Black ace or spade)
12. P(Face card)
13. P(Black face card)
14. P(Ace or face card)
15. P(Face card or heart)
16. P(Face card or red card)
17. P(Red face card or ace)
18. P(Red face card or heart)

PROBLEMS: (cont'd)

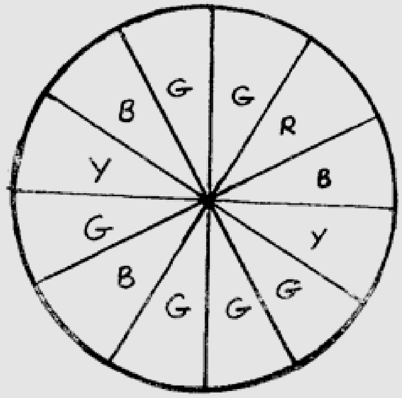
In 19 through 28, find the Probability.

19. P(not getting queen)
20. P(not getting spade)
21. P(not getting red card)
22. P(not getting red queen)
23. P(not getting a jack of spades)
24. P(not getting a black card or heart)
25. If a baseball player has a .300 batting average, it means his probability of getting a hit is .300. Find his probability of not getting a hit.
26. If a baseball player's batting average is .265, find P(not getting a hit).
27. If a basketball player has a shooting average of .415, find P(he misses the shot).
28. If a basketball player has a free throw average of .875, find P(he misses the shot).

29. If P(event) = 1/13, find odds in favor of the event.
30. If P(event) = 7/13, find odds in favor of the event.
31. If P(event) = 3/10, find odds in favor and odds against.
32. If P(event) = 15/17, find odds in favor and odds against.
33. If P(event) = 3/8, find odds against.
34. If P(event) = 19/53, find odds against.
35. If odds in favor of an event are 1 to 12, find P(event).
36. If odds in favor are 8 to 3, find P(event).
37. If odds against are 1 to 12, find P(event).
38. If odds against are 8 to 3, find P(event).
39. If odds against are 9 to 3, find P(event).
40. If odds against are 3 to 9, find P(event).

EXAMPLE 10

Consider the following fair game of spinning the pointer to obtain a color. Find the odds against and the probability of each color.



ANSWER

		Odds	Probability
2	Yellow	10 to 2 or 5 to 1	2/12 = 1/6 = .1666...
3	Blue	9 to 3 or 3 to 1	3/12 = 1/4 = .2500...
1	Red	11 to 1	1/12 = .0833...
6	Green	6 to 6 or 1 to 1	6/12 = 1/2 = .5000
			.9999...

Notice that the sum of the probabilities is .9999 which would have been 1 except for the round-off error. In a fair game, the sum of probabilities should be 1.

EXAMPLE 11

On the other hand, consider the following horse race:

3rd — Pace, Claiming $1250-1500, 1 Mile, Purse $850: 1. Ra Ra (Davis) (8-1); 2. Happy Dream D (Rauch) (10-1); 3. Tarport Bryn (Stanke) (6-1); 4. Caroliner (E.Hysell) (3-1); 5. Ben Omaga (Rau) (5-2); 6. Sunny Lermie (Britton) (7-2); 7. Last Stretch (Ihenfeld) (4-1); 8. Missy Penn (Brainard) (12-1).

EXAMPLE 11 (cont'd)

Note to "non-horse-racers": Odds of 8-1 means that for a $2 bet, if the horse wins it pays the ticket owner $16 plus his $2 bet for a total of $18. 5-1 odds would pay $10 + $2 = $12. 4-1 odds would pay $8 + $2 = $10. 5-2 odds would pay $5 + $2 = $7. In this race, the odds have Ben Omaga the favorite to win (5-2) while the long shot (dark horse) is Missy Penn (12-1). The odds are determined not by the track but by the way people are betting and are subject to constant change as betting changes. The odds at race time are used to determine pay-off, not the odds at the time the bet is placed.*

ANSWER

Entry	Odds	Probability	
1.	8-1	1/9	= .1111
2.	10-1	1/11	= .0909
3.	6-1	1/7	= .1429
4.	3-1	1/4	= .2500
5.	5-2	2/7	= .2857
6.	7-2	2/9	= .2222
7.	4-1	1/5	= .2000
8.	12-1	1/13	= .0769
			1.3797

What do you notice about the sum of the probabilities? Since the sum is more than 1, that means that the actual probabilities for each horse to win is actually less than what is given here, and so are the odds in favor of winning. In other words, it is not a "fair"[1] game. Of course, we already knew that since a percentage off the top goes to the track, while the rest is divided among the winners. Remember, of the money that goes in, only part of it comes back, so the odds are against gambling for profit.

PROBLEMS: (Optional)

On the following page, find the "probabilities" of each horse to win and determine the favorite (most likely to win) and the dark horse (least likely to win). See the previous example.

* **Thanks to Mack Blythe, Chairman Business Division, Seminole Community College**

1 *The precise meaning of a "fair" game will be explained in more detail in the section on Mathematical Expectation.*

1. 4th — Pace, Claiming $2500, 1 mile, Purse $850: 1. Gun Town (R.Faga) (12-1); 2. Bud Byrd (Bolton) (6-1); 3. Bombay Jo Anne (Buffamonte) (7-2); 4. Happy Race Time (Gosman) (4-1); 5. Kootney Brown (E.Hysell) (8-1); 6. Winning Angel (Regur) (10-1) 7. Edgars One N Only (M.Crank) (3-1); 8. Miss Maxy Byrd (J.Neely) (5-2).

2. 2nd — Trot, Claiming $1000-2000, 1 Mile, Purse $900: 1. Bart Van (E.Hysell) (20-1) 2. Vicars Rocket (Stetson) (15-1) 3. Raceway Rocket (Lynch) (6-1) 4. Miss Mac B (Smith) (5-2) 5. Gotcha Gal (Nikitas) (7-2) 6. Frosty Lew (J.Neely) (4-1) 7. Oh Aces Thor (Dokey) (10-1) 8. Flag Raiser (Brainard) (3-1).

3. 7th — Pace, Condition, 1 Mile, Purse $900: 1. Mr. St. Patrick (Regur) (10-1) 2. Karnes Paula (Rau) (3-1) 3. Verging Shadow (R.Neely) (6-1) 4. Maxi Freight (Komers) (8-1) 5. Barts Mick Time (Spriggs) (7-2) 6. Good Knight Jewel (Nielsen) (4-1) 7. Busters Ace (M.Crank) (5-2) 8. Yank (Bridges) (12-1).

4. **Belmont Stakes**

PP.Horse, Jockey	Prob.odds
1.King Celebrity, Ned	50-1
2.General Assembly, Cordero	15-1
3.Quiet Crossing, No boy	50-1
4.Picturesqque, Hernandez	50-1
5.Spectacular Bid, Franklin	1-10
6.Screen King, Asmussen	20-1
7.Gallant Best, Martens	50-1
8.Golden Act, Hawley	15-1
9.s-Coastal, Hernandez	6-1
10.Mystic Era, Velasquez	50-1

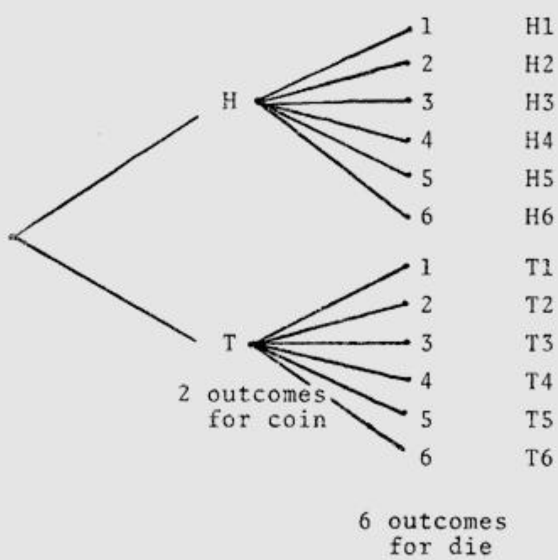
s-supplementary nominee.

T R E E   D I A G R A M S

When an experiment consists of two or more parts done in succession, (or at the same time, it really does not matter), it is often of interest to determine how many total outcomes there are and to list them. It is useful to draw what is known as a tree diagram.

EXAMPLE 12

The experiment is that of tossing a coin and a die. Show all possible outcomes with a tree diagram.

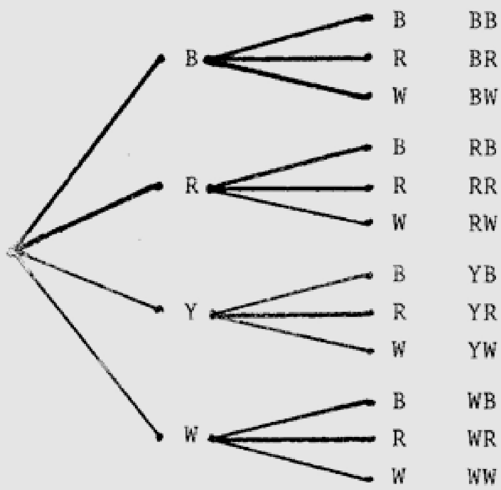


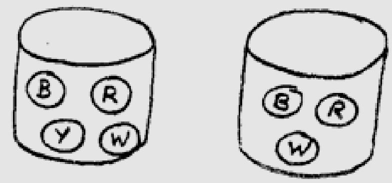
Total of 12 outcomes

(Note: 2 x 6 = 12)

EXAMPLE 13

A ball is drawn from each of the following jars. Show all possible outcomes with a tree diagram and find the probabilities requested below.[2]





Total of 12 outcomes
(Note: 4 x 3 = 12)

4 outcomes first jar   3 outcomes second jar

FIND:

P(both blue) = 1/12
P(at least one blue) = 6/12 = 1/2
P(no blue) = 6/12 = 1/2
P(at least one yellow) = 3/12 = 1/4
P(no white) = 6/12 = 1/2
P(no yellow) = 9/12 = 3/4
P(both yellow) = 0/12 = 0

PROBLEMS:

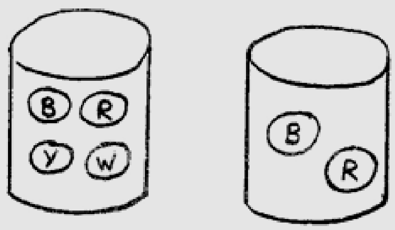
Draw a tree diagram for the following experiments and list all possible outcomes. How many are there?

1. Tossing a die and then a coin.
2. Tossing two coins.
3. Tossing three coins.
4. Having three children (choices would be "boy" or "girl").
5. Tossing two regular dice.
6. Tossing two tetrahedral dice (that is, a die with four sides instead of six.)
7. Tossing a regular die and a tetrahedral die.
8. Tossing a tetrahedral die and a coin.

[2] *To determine probabilities, it is necessary that each outcome is equally probable.*

Draw a tree diagram for the experiment of drawing a marble from each jar, list the outcomes and find: (a) P(both blue), (b) P(both red), (c) P(at least one blue), (d) P(no blue), (e) P(exactly one red).

9.



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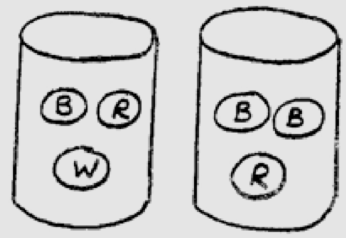
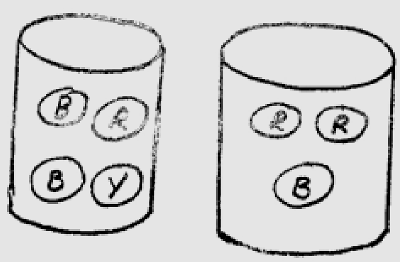
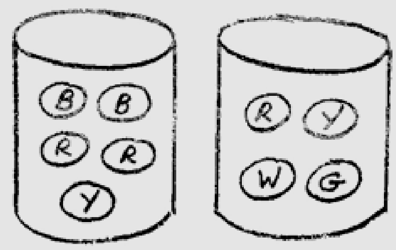


11.



12.



FUNDAMENTAL COUNTING PRINCIPLE and PERMUTATIONS

How do you find the total number of outcomes without drawing the tree diagram? The answer to this question is called the "fundamental counting principle", that is, the fundamental way of counting the total number of outcomes on successive events. We multiply the number of outcomes of each event, assuming that the later events are independent of (that is, not affected by) the first events.

EXAMPLE 14

How many outcomes are there if three coins are tossed in a fountain?

ANSWER

There are two outcomes (heads or tails) for each coin, so $2 \times 2 \times 2 = 8$.

EXAMPLE 15

How many outcomes are there if ten coins are tossed?

ANSWER

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$

This is a bit cumbersome, so in math we have a shorthand notation: 2^{10} read "two to the tenth power" which means two times itself ten times.

EXAMPLE 16

If a person has tossed a coin nine times and gotten heads each time, find the probability of getting a head on the tenth toss. (Assume the coin is fair.)

ANSWER

Probability of getting a head on any toss is 1/2. The tenth toss is independent of the previous nine tosses.

EXAMPLE 17

How many outcomes are there if two dice and three coins are tossed?

ANSWER

$$\underbrace{6 \cdot 6}_{\text{dice}} \cdot \underbrace{2 \cdot 2 \cdot 2}_{\text{coins}} = 288$$

EXAMPLE 18

How many ways can four people be seated in a row?

ANSWER

The first person has four choices of seats, but the second person only has three choices(since one has already been taken), the third person has two choices, and the last has only one seat left.
So, $4 \times 3 \times 2 \times 1 = 24$

EXAMPLE 19

How many ways can ten people be seated in a row?

ANSWER

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3,628,800$

Again, this became rather cumbersome, so again in math we have a shorthand notation. We write "10!" read "ten factorial", which means "10 x 9 x 8 x 7 etc. all the way down to 1."

NOTATIONAL DRILL

Find the value of each of the following:

1. 3^2	2. $3!$	3. 5^2	4. 2^3	5. $5!$
6. 2^4	7. 10^3	8. 7^2	9. $8!$	10. 3^5

11. Do you suppose 20! would be 2 x 10! ?

ANSWERS

1. 9	2. 6	3. 25	4. 8	5. 120
6. 16	7. 1000	8. 49	9. 40,320	10. 243

11. No. 20! = 20 x 19 x 18 etc. which equals 2.4329×10^{18}

EXAMPLE 20

If there are ten flags from which to choose and it is desired to fly three of the flags from a flag pole, how many ways can the flags be arranged?

ANSWER

There are 10 choices for the first, there are 9 choices for the second and 8 choices for the third. $10 \times 9 \times 8 = 720$

EXAMPLE 21

If eight horses are in a horse race, in how many different ways can they place first, second, and third?

ANSWER

$8 \times 7 \times 6 = 336$

Examples 18 through 21 form a special type of problem which is called a permutation. Example 21 was a permutation of 8 things taken 3 at a time. Example 20 was a permutation of 10 things taken 3 at a time. Example 19 was a permutation of 10 things taken 10 at a time, while Example 18 was a permutation of 4 things taken 4 at a time. Notations often used for a permutation of n things taken r at a time include:

$$P^n_r, \quad P\binom{n}{r}, \quad {}_nP_r, \text{ and } P(n, r).$$

EXAMPLE 22

Find P^6_4 and P^6_6

ANSWER

$P^6_4 = 6 \times 5 \times 4 \times 3 = 360$ (counting down 4 places beginning with 6)

$P^6_6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ (counting down 6 places beginning with 6)

PROBLEMS:

In 1-6, how many outcomes are there if:

1. Three coins and a die are tossed?
2. Two coins and two dice are tossed?
3. Three dice and one coin are tossed?
4. Three coins and a tetrahedral (four-sided) die are tossed?
5. Two regular dice, a tetrahedral die and a coin are tossed?
6. Three tetrahedral dice, a regular die and two coins are tossed?
7. If there are eight flags from which to choose and it is desired to fly four flags, how many ways can the flags be arranged?
8. If six people decide to sit in a row, how many seating arrangements are possible?
9. If ten horses are entered in a horse race, in how many ways can they come in first, second, and third?
10. In how many ways can four books be arranged on a shelf?
11. In how many ways can two flags be flown if there are 8 flags from which to choose?

12. If six horses are entered in a horse race, in how many ways can they come in first, second and third?

13. If there are six people playing musical chairs with three chairs, how many outcomes are there?

14. If three flags are to be flown from a flag pole and there are six from which to choose, how many different arrangements of flags could there be?

15. If there are ten flags from which to choose, in how many ways can four of them be arranged on a flagpole?

16. In how many ways can eight people sit in a row?

17. In how many ways can ten people sit in a row?

18. In how many ways can a field of six horses come in first and second?

19. Find P_3^6

20. Find P_4^8

21. Find P_5^5

22. Find P_6^6

23. Find P_2^{12}

24. Find P_3^{12}

MISCELLANEOUS

EXAMPLE 23

How many four-letter combinations can be formed using our alphabet (a) if repetition of letters is allowed; (b) if repetition is not allowed?

ANSWER

(a) There are 26 choices for each letter
$26 \times 26 \times 26 \times 26 = 456,976$

(b) If repetition is not allowed, then there are 26 choices for the first letter, but the one letter is used up, leaving 25 choices for the second, 24 choices for the third, and 23 choices for the fourth.

$26 \times 25 \times 24 \times 23 = 358,800$

EXAMPLE 24

In having a new home built, a couple has several decisions (choices) to make. They can have it built on one of 4 lots; they can have three or four bedrooms; exterior design of brick, block, wood, stone or aluminum siding; interior of plaster, drywall, panel or wallpaper; fireplace or no fireplace; wall-to-wall carpet or congoleum; patio or no patio; two-car, one-car garage, or carport. How many possible combinations of house can be built using these choices?

ANSWER

$$\underset{\text{lot}}{4} \times \underset{\text{bdrm.}}{2} \times \underset{\text{ext.}}{5} \times \underset{\text{inter.}}{4} \times \underset{\text{firepl.}}{2} \times \underset{\text{floor}}{2} \times \underset{\text{patio}}{2} \times \underset{\text{garage}}{3} = 3840$$

PROBLEMS:

1. Using the letters: a, b, c, d, e, f, how many two-letter "words" (i.e. combinations -- it doesn't have to spell anything!) can be formed (a) if repetition is not allowed, (b) if repetition is allowed?
2. Using the letters: a, b, c, d, e, f, how many three-letter designations can be formed (a) if repetition is not allowed, (b) if repetition is allowed?
3. How many different license plates can be made with three letters followed by two digits if the letters must be g, h, i, or j and the digits must be 1, 3, 5 or 7 (a) if repetition is not allowed, (b) if repetition is allowed?
4. How many different license plates can be made using a letter followed by two digits if the letter could be any letter of the alphabet, but the numbers must be 5 or 4 and (a) repetition is not allowed, (b) repetition is allowed?
5. How many different license plates can be made using three letters followed by three digits if the first letter must be "B" and the last number must be "2" and the letters and digits between are A, E, I, U, and 4, 6, 8, (a) and repetition is not allowed, (b) repetition is allowed?
6. How many radio station call letters can be made using the letters K or W for the first digit, followed by two letters selected from the word "RADIO" (a) if repetition is not permitted, (b) if repetition is permitted?
7. Do problem 6 if the K or W is followed by three letter.
8. How many three-letter words can be made using the letters of the word, "RADIO" if the middle letter must be a vowel and the first and last a consonant, (a) if repetition is not permitted, (b) if repetition is permitted.

9. Do problem #8 for four-letter words from the word "RADIO" where the middle two must be vowels and the first and last be consonants.

10. When ham radio call-letters were first issued, they used a choice of K or W followed by a digit 0 through 9, and then three letters. How many combinations of call-letters are possible if the last 3 letters are chosen from the word, "RADIO"?

PROBLEMS FOR CALCULATORS:

11. There are 24 Greek letters. How many two-letter designations of Greek letters (fraternities) are possible (a) if repetition is not allowed, (b) if repetition is allowed?

12. Do problem #11 for three-letter Greek words.

13. How many three-letter words can be formed using our 26-letter alphabet (a) if repetition is not allowed, (b) if repetition is allowed?

14. How many different license plates can be formed using six digits (from all 10 digits), (a) if repetition is not allowed, (b) if repetition is allowed?

15. How many different license plates can be formed using three letters followed by three digits, (a) if repetition is not allowed, (b) if repetition is allowed?

    Why did Florida change?

16. Radio stations in the 1920's and before used three letters, of which the first was W or K and the last two, any of the 26. If repetition was allowed, how many such stations were there?

17. After the 1920's, running out of three-letter combinations, a change was made to add a fourth letter. Allowing repetition, how many choices are there now?

18. How many total ham radio combinations could be made using the old system of K or W followed by a digit and then three letters?

The system of ham operator call-letters in Problem #10 did not provide enough combinations for the growing hobby, so a new system was developed and is used today, as the example illustrates:*

W A 4 ORS

ORS: These are any 3 letters of the alphabet.

4: This is one of 10 digits that denotes an area of the country.

A: This is a letter A, B, C, etc. used to give more combinations as they are needed. (Today, they are up to C)

W: All begin with W.

19. How many combinations are there using the letter "A" for the second letter?

20. How many combinations are there using the letters "A" through "C" for the second letter?

21. How many combinations are there using all letters "A" through "Z"?

22. How many combinations are there using the letters A, B, or C for the second letter, and "ORS" in that order for the last three letters?

EXPECTED VALUE or MATHEMATICAL EXPECTATION

In an experiment, or game of chance, it is often of interest to know, if the experiment or game were to continue indefinitely, what would be the average outcome, or what price for the game would be "fair".[3]

EXAMPLE 25

On "Let's Make a Deal", a woman has won $1,000 and must decide whether to "choose a door" or keep the $1,000. Behind each of the three doors, respectively, is a $3,000 vacation, a $1,200 furniture prize and a booby prize of $60 worth of dog food. What should she do?

3 *"Fair" game simply means neither the house nor the player has an advantage. Everyone is equally probable to win or lose.*

* Thanks to John D. Rapalje, Jr.

ANSWER (to #25)

(1) Compute for each possible outcome, the probability thereof.

(2) Multiply the prize times the probability of winning that prize.

(3) Sum the results. This is the expected value.

E = 1/3 x $3,000 + 1/3 x 1200 + 1/3 x $60 = $1,000 + $400 + $20 = $1420 = Expected Value of Winnings.

She should go for it since she expects to win more than $1,000. (She may come out ahead, she may end up with dog food, but the laws of probability in the long run show her ahead.)

EXAMPLE 26

Suppose someone gave you $1,000,000 and then offered to let you toss a coin with the condition that if it comes up heads, you win $4,000,000; if it comes up tails, you lose the $1,000,000. Would you play or would you keep the $1,000,000? Mathematically, what should you do?

ANSWER

Probabilities are 1/2 and 1/2, so

E = 1/2 x $4 million + 1/2 x 0 = $2 million.

Mathematically, you should play, but I, personally, would take the $1,000,000!

EXAMPLE 27

A bag contains 5 red balls, 3 white balls, 2 blue balls. One ball is drawn at random and the prize for red is $1, for white is $5 and for blue is $10. If the price of the game is $4, is it a fair game?

ANSWER

P(red) = 5/10 = 1/2

P(white) = 3/10

P(blue) = 2/10 = 1/5

E = 1/2 x $1 + 3/10 x $5 + 1/5 x $10 = $.50 + $1.50 + $2 = $4.00
"Fair" game to play for $4.

EXAMPLE 28

A raffle sells tickets for $1. The prizes are: 1st - $100, 2nd - $50, 3rd - $10. If 200 tickets are sold, find expected value. Is it a good raffle to buy tickets for?

ANSWER

P(winning $100) = 1/200

P(winning $50) = 1/200

P(winning $10) = 1/200

E = 1/200 x $100 + 1/200 x $50 + 1/200 x $10 =

$.50 + $.25 + $.05 = $.80 BAD GAME!!! (They usually are!)

PROBLEMS:

1. A coin is tossed. If heads comes up, you win $1; if tails comes up, you win $.50. What is a fair price to pay for the game?

2. A die is tossed and the player receives the no. of dollars equal to that number that comes up --($1 for a one, $2 for a two, $6 for a six, etc.) Find the mathematical expectation.

3. On a roulette wheel, there are 18 reds, 18 blacks, 4 greens. If the ball lands on red, the player wins $10, on green, wins $100, and on black, wins nothing. Find the mathematical expectation or fair price.

4. A bag contains seven white balls and three red balls. A prize of $10 is given for drawing a red ball, no prize for white. Find E.

5. A man has 5 tickets in a lottery with one prize and 29 blanks. If the prize is $120, find his expectation.

6. The odds in favor of winning a $10,000 prize are 3 to 2. Find E. (Hint: find probability of winning first.)

7. If the probability of your living through this year is 99/100, (from mortality tables, probability of dying 1/100), find the fair price for a $10,000 life insurance policy. Do you think this is the price you would actually pay?

8. Three coins are tossed, the prize being a dollar for each head. Find E.

9. One million lottery tickets are sold for $1. The grand prize is $300,000; two second prizes of $50,000; four prizes of $10,000; ten prizes of $1,000; one-hundred prizes of $100; one-thousand prizes of $20. What are the expected winnings?

10. Which has the highest expected salary: Sales, Teaching or Managing?

    Sales has 10% chance of $60,000, 20% chance of $20,000, 70% chance of $7,000.

    Teaching has 80% chance of $13,000, 20% chance of $18,000.

    Managing has 30% chance of $17,000, 70% chance of $11,000.

    Find E of each.

11. The following odds chart published by a well-known super-market chain for a promotional game is actually a <u>probability</u> (not odds) table. From this chart it may be determined that there are a total of 13,500,000 tickets to be given away.
    a) Find the expected value of one ticket.
    b) Find the average number of tickets required to win a prize.
    c) Find the probability that a given ticket is a winner.

(Calculator required!)

ODDS CHART

PRIZE VALUE	NO. OF PRIZES	ODDS FOR ONE STORE VISIT
$2,002.00	35	385,714 to 1
1,001.00	100	135,000 to 1
200.00	200	67,500 to 1
100.00	750	18,000 to 1
20.00	1,000	13,500 to 1
10.00	2,000	6,750 to 1
5.00	5,000	2,700 to 1
2.00	20,000	675 to 1
1.00	109,920	123 to 1
TOTAL PRIZES	139,005	97 to 1

P R O B A B I L I T Y   R E V I E W   P R O B L E M S

1. (a) Define "Probability".
   (b) Define "Odds in Favor".
   (c) Define "Odds Against".

2. How many outcomes are there if:
   (a) 3 coins are tossed?
   (b) 2 dice are tossed?
   (c) one coin and one die are tossed?
   (d) one regular and one tetrahedral die are tossed?
   (e) 4 books are arranged on a shelf?
   (f) 5 flags are arranged using all 5?
   (g) 5 flags are arranged using 3 at a time?
   (h) there are 3 true-false questions?
   (i) there are 3 multiple-choice questions with 4 choices?
   (j) there are 3 multiple-choice questions with 5 choices?

3. How many ways can:
   (a) two digits followed by a letter be arranged?
   (b) six people be seated in a row?
   (c) the letters of the word "favor" be arranged?
   (d) the letters of the word "love" be arranged?
   (e) license plates with 4 digits be made (repetition allowed)?
   **(f) license plates with 4 letters be made (repetition allowed)?
   (g) license plates with 4 digits, but no repetition of digits?
   **(h) license plates with 4 letters, but no repetition of letters?
   **(i) license plates with 3 letters, followed by 3 digits, (repetition allowed)?
   **(j) license plates with 3 letters, 3 digits, no repetitions?

** Calculator.

4. (a) A woman has three skirts, two vests and 6 blouses. How many combinations of clothes can she make?
   (b) A car comes with the following choices: 6 colors, 3 styles, 2 transmissions, 3 types of brakes, air or no air (2), radio or no radio (2). How many combinations are there?

(c) How many four-letter words can be made from the letters A, C, D, F, G, K if repetition of letters is allowed?

(d) Do part (c) if repetition is not allowed?

5. Draw a tree diagram for:

(a) Tossing two coins.

(b) Tossing three coins. List the outcomes

(c) Tossing one coin and one die.

6. Using a deck of 52 cards, find the probability and odds in favor of: (reduce all fractions)

(a) P(ace)

(b) P(spade)

(c) P(red card)

(d) P(red king)

(e) P(ace of spades)

(f) P(red card or king)

(g) P(face card)

(h) P(heart or spade)

(i) P(red heart)

(j) P(red spade)

7. If two dice are tossed: (See page P-26 for outcomes of two dice)

Optional

(a) P(sum of 2)

(b) P(sum of 7)

(c) P(sum of 12)

(d) P(sum of 3 or less)

(e) P(sum of 3 or more)

(f) P(at least one "2")

(g) P(double "2")

(h) P(double "6")

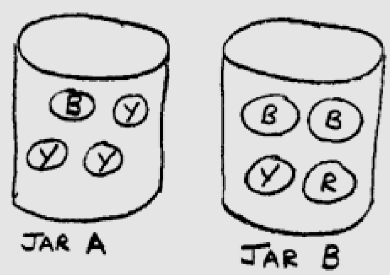
8. Using jar A only, draw one ball.

(a) P(B) = _____ (b) P(Y) = _____

9. Using jar B only, draw one ball.

(a) P(B) = _____ (B) P(Y) = _____

(c) P(R) = _____



10. Draw one ball from each jar:

(a) Draw a tree diagram showing all possibilities.

(b) How many outcomes are there?

(c) P(both be Y)

(d) P(red from jar B)

(e) P(both B)

(f) P(at least one Y)

(g) P(at least one B)

11. Using the above problems, find the Probability of <u>NOT</u>:

(a) drawing an ace

(b) drawing a face card

**(c) getting a sum of 7

**(d) getting a double 6

(e) drawing two balls both blue

(f) drawing both yellow

(g) drawing a red from jar B only

(h) drawing a Y from jar A only

(i) drawing at least one blue

(j) drawing a red from jar A only

**-optional.

12. Find the odds against each of Problem 11 (a-j) experiments.

P R O B A B I L I T Y   P R A C T I C E   T E S T

Compute all answers with calculator or by hand (or head) unless otherwise indicated.

All fractions must be reduced completely. SHOW ALL WORK.

1. Give the textbook (class) definition of Probability. 1.______

2. Give the definition of odds against. 2.______

3. How many outcomes are there if two dice are tossed? 3.______

4. How many outcomes are there if three coins are tossed? 4.______

5. How many outcomes are there if a regular die and two tetrahedral dice are tossed? 5.______

6. In how many ways can five flags be flown from 3 poles? (that is, using three flags at a time) 6.______

7. In how many ways can five flags be flown using all five flags? 7.______

8. Given the letters a, b, c, d, e, f, g, how many three-letter "words" can be made if repetition of letters is permitted? 8.______

9. Given the letters in #8, how many words can be formed if repetition is not permitted? 9.______

10. Find 4! 10.______

11. A car dealer has 3 styles of cars in 5 colors, with the following options: transmissions, 2 choices; air, 2 choices; brakes, 3 choices; interior, 3 types; How many possible choices are there? 11.______

12. How many of the cars in #11 are green? (Assume only one shade of green!) 12.______

In 13-17, assume that 1 card is drawn from a deck of 52.

13. P(ace) = ______

14. P(black card)= ______

15. P(black king) = ______

16. P(face card) = ______

17. P(black card or queen) = ______

Probability Practice Test (Cont'd)

On 18-22, find the odds in favor of drawing from a deck of 52:

18. an ace 18.________

19. a black card 19.________

20. a black king 20.________

21. a face card 21.________

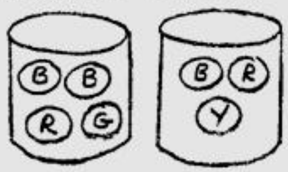
22. a black card or queen 22.________

23. A baseball player has a probability of .305 of getting a hit. Find P(not getting a hit). 23.________

24. A jar contains 6 red, 5 white, 3 blue balls. Find P(not drawing a blue ball). 24.________

25. In #24, find probability of drawing a "white or blue" ball. 25.________

26. A ball is drawn from each of the jars. Draw a tree diagram showing all possible outcomes. How many possible outcomes are there? 26.________



In relation to #26, find:

27. P(both blue) 27.________

28. P(at least one blue) 28.________

29. P(yellow) 29.________

30. If the probability of an event is 3/5, find the odds in favor of it. 30.________

31. If the odds in favor of an event are 5 to 3, find the probability of the event. 31.________

32. If the odds against an event are 5 to 3, find the probability of the event. 32.________

33. In a raffle, 200 tickets are sold. 1st prize is $50, 2nd prize is $20, 3rd prize is $10. Find the expected value for a person having one ticket. (What should he have to pay as a <u>fair</u> price?) 33.________

34. Find the expected salary for a profession in which 10% earn $70,000, 40% earn $20,000, 40% earn $12,000 and 10% earn $6,000. 34.________

T H E   C A R D S

52 Total

BLACK (26)			RED (26)	
SPADES (13)	CLUBS (13)		HEARTS (13)	DIAMONDS (13)
Ace	Ace		Ace	Ace
King	King	FACE CARDS	King	King
Queen	Queen	FACE CARDS	Queen	Queen
Jack	Jack	FACE CARDS	Jack	Jack
10	10		10	10
9	9		9	9
8	8		8	8
7	7		7	7
6	6		6	6
5	5		5	5
4	4		4	4
3	3		3	3
2	2		2	2

T W O   D I C E

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	((3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

ANSWERS

P-4: P-5

1.	1/13;	1:12	11. 7/26;	7:19	21. 1/2	31. 3:7;	7:3
2.	1/4;	1:3	12. 3/13;	3:10	22. 25/26	32. 15:2;	2:15
3.	1/2;	1:1	13. 3/26;	3:23	23. 51/52	33. 5:3	
4.	1/26;	1:25	14. 4/13;	4:9	24. 1/4	34. 34:19	
5.	1/52;	1:51	15. 11/26;	11:15	25. .700	35. 1/13	
6.	1/26;	1:25	16. 8/13;	8:5	26. .735	36. 8/11	
7.	3/4;	3:1	17. 5/26;	5:21	27. .585	37. 12/13	
8.	4/13;	4:9	18. 4/13;	4:9	28. .125	38. 3/11	
9.	7/13;	7:6	19. 12/13		29. 1:12	39. 1/4	
10.	15/52;	15:37	20. 3/4		30. 7:6	40. 3/4	

P-8:

1.

Entry	Odds	Probability	
1	12-1	1/13 = .0769	D
2	6-1	1/7 = .1429	
3	7-2	2/9 = .2222	
4	4-1	1/5 = .2000	
5	8-1	1/9 = .1111	
6	10-1	1/11 = .0909	
7	3-1	1/4 = .2500	
8	5-2	2/7 = .2857	F
		1.3797	

2.

Entry	Odds	Probability	
1	20-1	1/21 = .0476	D
2	15-1	1/16 = .0625	
3	6-1	1/7 = .1429	
4	5-2	2/7 = .2857	F
5	7-2	2/9 = .2222	
6	4-1	1/5 = .2000	
7	10-1	1/11 = .0909	
8	3-1	1/4 = .2500	
		1.3018	

3.

Entry	Odds	Probability	
1	10-1	1/11 = .0909	
2	3-1	1/4 = .2500	
3	6-1	1/7 = .1429	
4	8-1	1/9 = .1111	
5	7-2	2/9 = .2222	
6	4-1	1/5 = .2000	
7	5-2	2/7 = .2857	F
8	12-1	1/13 = .0769	D
		1.3797	

4.

Entry	Odds	Probability		
1	50-1	1/51 = .0196	D	
2	15-1	1/16 = .0625		
3	50-1	1/51 = .0196	D	
4	50-1	1/51 = .0196	D	
5	1-10	10/11= .9090	FF	(Lost!)
6	20/1	1/21 = .0476		
7	50-1	1/51 = .0196	D	
8	15-1	1/16 = .0625		
9	6-1	1/7 = .1429		(Won!)
10	50-1	1/51 = .0196	D	
		1.3225		

P-9:

1. 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T
   Total: 12

2. HH, HT, TH, TT
   Total: 4

3. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT
   Total: 8

4. BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG
   Total :8

5.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3.4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Total 36 Outcomes

6.

1,1	1,2	1,3	1,4
2,1	2,2	2,3	2,4
3,1	3,2	3,3	3,4
4,1	4,2	4,3	4,4

Total 16 Outcomes

7.

1,1	1,2	1,3	1,4
2,1	2,2	2,3	2,4
3,1	3,2	3,3	3,4
4,1	4,2	4,3	4,4
5,1	5,2	5,3	5,4
6,1	6,2	6,3	6,4

Total 24 Outcomes

8. 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T
   Total: 8

P-10:

9. a) 1/8
   b) 1/8
   c) 5/8
   d) 3/8
   e) 1/2

10. a) 2/9
    b) 1/9
    c) 7/9
    d) 2/9
    e) 4/9

11. a) 1/6
    b) 1/6
    c) 2/3
    d) 1/3
    e) 7/12

12. a) 0
    b) 1/10
    c) 2/5
    d) 3/5
    e) 9/20

ANSWERS

P-13:	1. 48	7. 1680	13. 120	19. 120
P-14	2. 144	8. 720	14. 120	20. 1680
	3. 432	9. 720	15. 5040	21. 120
	4. 32	10. 24	16. 40320	22. 720
	5. 288	11. 56	17. 3628800	23. 132
	6. 1536	12. 120	18. 30	24. 1320

P-15:	1a) 30	5a) 72	9a) 12	13a) 15600	18. 351,520
P-16	b) 36	b) 144	b) 36	b) 17576	19. 175,760
P-17	2a) 120	6a) 40	10a) 1200	14a) 151,200	20. 527,280
	b) 216	b) 50	b) 2500	b) 1,000,000	21. 4,569,760
	3a) 288	7a) 120	11a) 552	15a) 11,232,000	22. 30
	b) 1024	b) 250	b) 576	b) 17,576,000	
	4a) 52	8a) 6	12a) 12144	16. 1352	
	b) 104	b) 12	b) 13824	17. 35,152	

P-19:	1. $.75	4. $3	7. $100	10. S $14,900	11a) 3.7¢
P-20	2. $3.50	5. $20	8. $ 1.50	T $14,000	b) 97
	3.$14.50	6. $6000	9. $ .48	M $12,800	c) 1/97 or .0103

P-21:	1a) See P-1	3a) 2600	5a) See P-9 #2	7a) 1/36	10a) BB	YB
P-22	b) See P-2	b) 720	b) See P-9 #3	b) 1/6	BB	YB
P-23	c) See P-2	c) 120	c) See P-9 #1	c) 1/36	BY	YY
		d) 24	(reversed)	d) 1/12	BR	YR
	2a) 8	e) 10,000		e) 35/36	YB	YB
	b) 36	f) 456,976	6a) 1/13; 1:12	f) 11/36	YB	YB
	c) 12	g) 5040	b) 1/4; 1:3	g) 1/36	YY	YY
	d) 24	h) 358,800	c) 1/2; 1:1	h) 1/36	YR	YR
	e) 24	i) 17,576,000	d) 1/26; 1:25		b) 16 Outcomes	
	f) 120	j) 11,232,000	e) 1/52; 1:51	8a) 1/4	c) 3/16	
	g) 60		f) 7/13; 7:6	b) 3/4	d) 1/4	
	h) 8	4a) 36	g) 3/13; 3:10		e) 1/8	
	i) 64	b) 432	h) 1/2; 1:1	9a) 1/2	f) 13/16	
	j) 125	c) 1296	i) 1/4; 1:3	b) 1/4	g) 5/8	
		d) 360	j) 0 --	c) 1/4		

P-23:	11a) 12/13	f) 13/16	12a) 12:1	f) 13:3
	b) 10/13	g) 3/4	b) 10:3	g) 3:1
	c) 5/6	h) 1/4	c) 5:1	h) 1:3
	d) 35/36	i) 3/8	d) 35:1	i) 3:5
	e) 7/8	j) 0	e) 7:1	

P-24:	1. See P-1	11. 540	21. 3:10	27. 1/6
P-25	2. See P-2	12. 108	22. 7:6	28. 2/3
	3. 36	13. 1/13	23. .695	29. 1/3
	4. 8	14. 1/2	24. 11/14	30. 3:2
	5. 96	15. 1/26	25. 4/7	31. 5/8
	6. 60	16. 3/13	26. BB BR BY	32. 3/8
	7. 120	17. 7/13	BB BR BY	33. $.40
	8. 343	18. 1:12	RB RR RY	34. $20,400
	9. 210	19. 1:1	GB GR GY	
	10. 24	20. 1:25	12 Outcomes	