Topics in geometric measures

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SEMINOLE COMMUNITY COLLEGE

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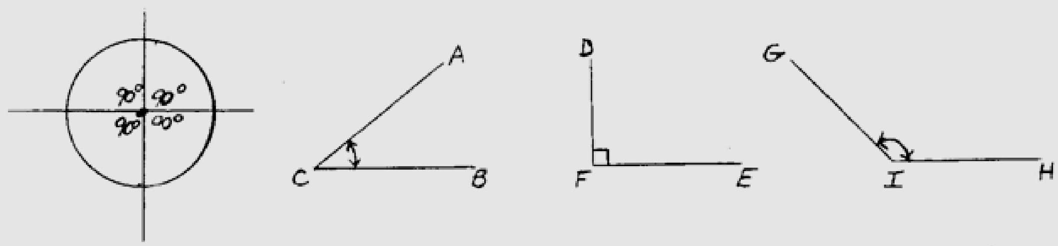
GEOMETRIC MEASURES

INTRODUCTION

Before beginning this section, some miscellaneous geometric facts and notations need to be set out. First, the circle has been divided into 360 equal units, each of which is called a degree, also denoted "1°". It then follows that an angle formed by perpendicular (denoted "$\perp$") lines would be 90°, which we call a "right" angle. An angle less than 90° is said to be an "acute" angle, while an angle of more than 90° is said to be an obtuse angle. The figure below shows $\angle$ACB (read "angle ACB") to be an acute angle, $\angle$DFE to be a right angle, and $\angle$GIH to be an obtuse angle. These angles could also be read $\angle$BCA, $\angle$EFD, and $\angle$HIG.



SHAPES AND FIGURES

The following is a list of some of the geometric figures that occur in nature, in everyday life, and hence in this section.

Number of Sides	Name	
--	Circle	
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
8	Octagon	
10	Decagon	
12	Dodecagon	
General	Polygon	A regular polygon is one in which all sides are equal.

TRIANGLES

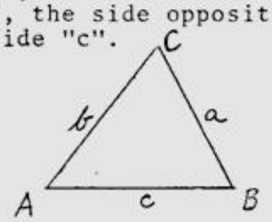
In any triangle, no matter what size or shape, the sum of the angles is always 180°. Based upon the sizes of the angles, there are three types of triangles:

1. Acute triangle -- all three angles are acute (less than 90°)
2. Right triangle -- one right angle. This requires that the other two be acute. (Why?)
3. Obtuse triangle - one obtuse (more than 90°) angle. (What does this make the other two angles?)

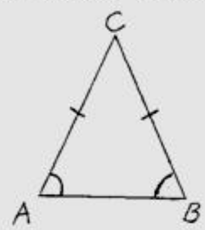
A further classification of triangles can be made based upon the number of equal sides and equal angles.

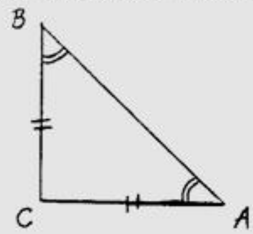
1. Equilateral -- all sides are equal.
2. Isosceles - - -two sides are equal.
3. Scalene - - - -no sides are equal.

The angles of a triangle are usually denoted by capital letters, as in the figure with A, B, C. The side which is opposite angle A is denoted lower case "a", the side opposite angle B is side "b", and opposite angle C is side "c".



If two sides of a triangle are equal, then the angles opposite those two sides are equal, and vice-versa. Equal sides may be denoted by one, two or three lines and angles as shown. In both figures, it is shown that a = b, and angle A equals angle B.



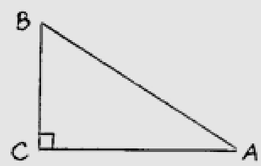


EXAMPLE 1: Using the fact that the sum of the angles of a triangle is 180°, if angle A = 38° and angle B = 94°, find angle C. Give two names to describe the triangle.

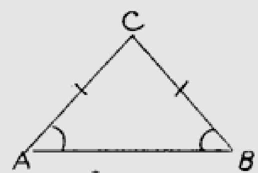
ANSWER: $38^\circ + 94^\circ = 132^\circ$ for $\angle A$ and $\angle B$, which leaves $180^\circ - 132^\circ = 48^\circ$ for $\angle C$. So $\angle C = 48^\circ$. Obtuse and scalene.

EXAMPLE 2: Find the angle B and angle C of the triangles pictured below if angle A = 38°.

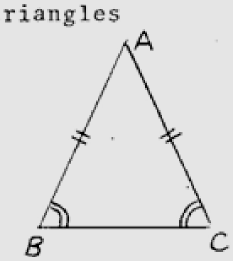
(a)



(b)



(c)



ANSWER: (a)

$$\begin{array}{r} \angle C = 90^\circ \\ \angle A = 38^\circ \\ \hline 128^\circ \end{array} \qquad \begin{array}{r} 180^\circ \\ -128^\circ \\ \hline 52^\circ \end{array}$$

So, $\angle C = 90^\circ$ and $\angle B = 52^\circ$

(b) $\angle A = \angle B$, so $\angle B = 38^\circ$

$$\begin{array}{r} 38^\circ \\ 38^\circ \\ \hline 76^\circ \end{array} \qquad \begin{array}{r} 180^\circ \\ -\ 76^\circ \\ \hline 104^\circ \end{array}$$

So, $\angle B = 38^\circ$ and $\angle C = 104^\circ$

(c) $\angle B = \angle C$.
Since $\angle A = 38^\circ$, that leaves $180^\circ - 38^\circ = 142^\circ$ to be divided evenly between $\angle B$ and $\angle C$. So both $\angle B$ and $\angle C = 71^\circ$.

EXAMPLE 3: Which of the triangles of Example 2 are:

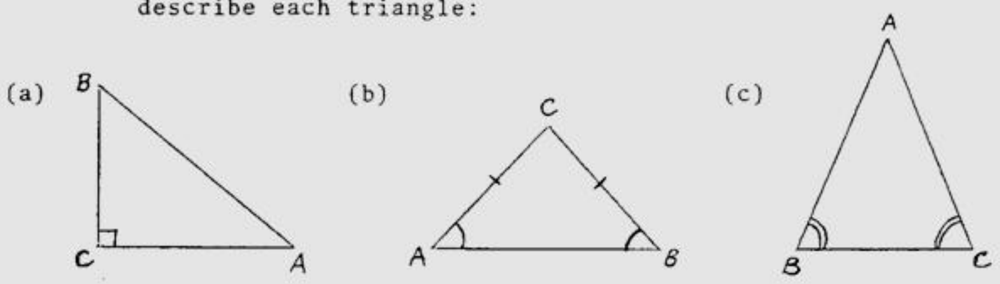
(1) Acute	(4) Equilateral
(2) Right	(5) Isosceles
(3) Obtuse	(6) Scalene

ANSWER: (to Example #3)

(1) c
(2) a
(3) b
(4) None
(5) b and c
(6) a

PROBLEMS:

1. If two angles of a triangle are 30° and 60°, find the third angle. Give two names to describe the triangle.
2. If two angles of a triangle are 23° and 85°, find the third angle. Give two names to describe the triangle.
3. If two angles of a triangle are 102° and 39°, find the third angle. Give two names to describe the triangle.
4. If two angles of a triangle are 45° and 90°, find the third angle. Give two names to describe the triangle.
5. In each of the following pictured triangles, angle A is 50°. Find angles C and B and give two names to describe each triangle:



6. Do problem #5 if angle A = 45°.
7. Do problem #5 if angle A = 10°.
8. In an equilateral triangle, the 180° of the triangle must be equally divided among all three angles. How many degrees are there in each angle of an equilateral triangle?
9. Which of the following are possible:
acute equilateral, acute isosceles, acute scalene, right equilateral, right isosceles, right scalene, obtuse equilateral, obtuse isosceles, obtuse scalene?

   Draw an example of those which are possible.

BASIC EQUATION SOLVING

As a tool for the next sections, the topic of equation solving must be considered. An equation is like a pan balance in which the two sides are equal or in balance. Weights may be added or removed from a pan balance without disturbing the balance. Likewise, with an equation, the same quantity may be added or subtracted from both sides, and both sides may be multiplied or divided by the same number (except zero). An equation usually has a letter (such as "x") which is called a variable. A variable simply takes the place of an unknown number until that unknown number can be determined by solving the equation.

Problem for Consideration: $x + 6 = 24$.

Solution: The object is to isolate the x. Remember that we can do anything to one side as long as we do the same to the other. Since the x is "plus 6" (addition), we will do the opposite, "minus 6" (subtraction) to both sides. There are two ways to write this:

$$x + 6 = 24$$
$$x + 6 - 6 = 24 - 6$$
$$x = 18$$

$$\begin{array}{rcl} x + 6 & = & 24 \\ -\ 6 & & -6 \\ \hline x & = & 18 \end{array}$$

So the unknown number x is 18.

<u>EXAMPLE 4:</u> Solve the following equations for x:

(a) $x - 6 = 24$

(b) $6x = 24$

(c) $6x + 6 = 24$

(d) $6x - 6 = 24$

(e) $x^2 = 25$

(f) $x^2 + 16 = 25$

(a) $x - 6 = 24$
(Opposite of subtracting 6 is adding 6.)

$$x - 6 + 6 = 24 + 6$$
$$x = 30$$

(a) $x - 6 = 24$
(Opposite of subtracting 6 is adding 6.)

$$\begin{array}{rcl} & + 6 & +6 \\ \hline x & = & 30 \end{array}$$

(b) $6x = 24$ (The opposite of multiplication by 6 is division by 6, so we divide both sides by 6.)

$$\frac{6x}{6} = \frac{24}{6}$$
$$x = 4$$

(c) $6x + 6 = 24$ (First subtract 6)

$6x + 6 - 6 = 24 - 6$

$6x = 18$ (Next divide by 6)

$\frac{6x}{6} = \frac{18}{6}$

$x = 3$

(c) $6x + 6 = 24$ (First subtract 6)

$\underline{\quad - 6 \quad -6}$

$6x = 18$ (Next divide by 6)

$\frac{6x}{6} = \frac{18}{6}$

$x = 3$

(d) $6x - 6 = 24$ (First add 6)

$6x - 6 + 6 = 24 + 6$

$\frac{6x}{6} = \frac{30}{6}$ (Divide by 6)

$x = 5$

(d) $6x - 6 = 24$ (First add 6)

$\underline{\quad + 6 \quad +6}$

$\frac{6x}{6} = \frac{30}{6}$ (Divide by 6)

$x = 5$

(e) $x^2 = 25$

x^2 means a number "x" times itself.
What number times itself is 25?
Answer = 5, so $x = 5$

(f) $x^2 + 16 = 25$ (First subtract 16)

$x^2 + 16 - 16 = 25 - 16$

$x^2 = 9$

$x = 3$

(f) $x^2 + 16 = 25$ (subtract 16)

$\underline{\quad - 16 \quad -16}$

$x^2 = 9$

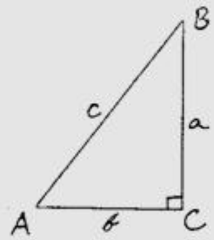
$x = 3$

What number times itself is 9?
Answer = 3

PROBLEMS: Solve for x. Show all steps.

1. $x + 12 = 30$
2. $x - 12 = 30$
3. $12x = 48$
4. $9x = 72$
5. $2x + 12 = 30$
6. $2x - 12 = 30$
7. $x^2 + 160 = 169$
8. $x^2 + 144 = 169$
9. $x^2 + 9 = 58$
10. $x^2 + 75 = 91$
11. $x^2 + 64 = 100$
12. $x^2 + 36 = 100$

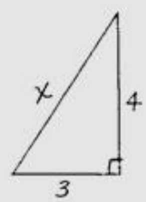
THEOREM of PYTHAGORAS



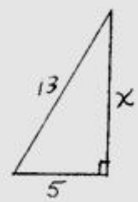
According to the Theorem of Pythagoras, in any <u>right</u> triangle, where "c" is the hypotenuse and "a" and "b" are legs, $a^2 + b^2 = c^2$.

EXAMPLE 5: Find the unknown side "x".

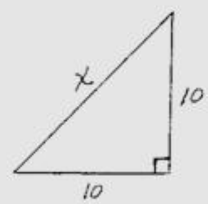
(a)



(b)



(c)



ANSWER:

(a) Which is the hypotenuse? "x"

$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$ or $x^2 = 25$

(What times itself equals 25?)

$x = 5$

(b) Which is the hypotenuse? "13"

$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

$-25 \qquad -25$

$x^2 = 144$

(What times itself equals 144?)

$x = 12$

(c) (For calculators with $\boxed{\sqrt{\ }}$)

Which is the hypotenuse? "x"

$10^2 + 10^2 = x^2$

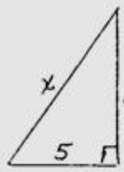
$100 + 100 = x^2$

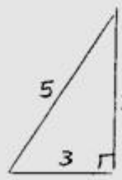
$x^2 = 200$

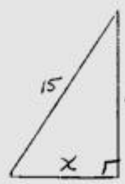
$x = 14.14$

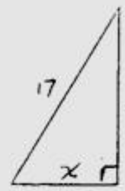
NOTE: In a right triangle, the hypotenuse is always the longest side of the triangle, and it is always the side which is "opposite" the right angle. The legs of the triangle must always be perpendicular to one another.

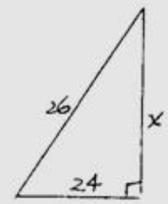
PROBLEMS: Find the unknown side.

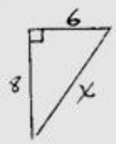
1. 

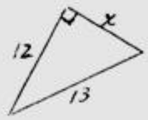
2. 

3. 

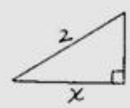
4. 

5. 

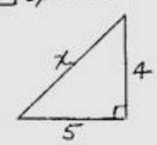
6. 

7. 

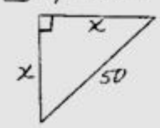
8. $\boxed{\sqrt{\ }}$ optional



9. $\boxed{\sqrt{\ }}$ optional



10. $\boxed{\sqrt{\ }}$ optional



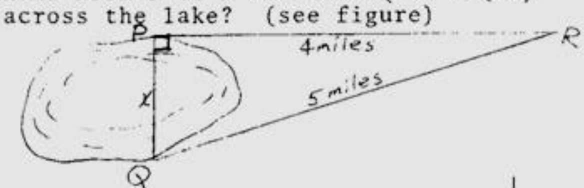
11. A rectangle has length 12 cm. and width 5 cm. Find the diagonal.

12. A rectangle has width 3 m and length 4 m. Find the diagonal.

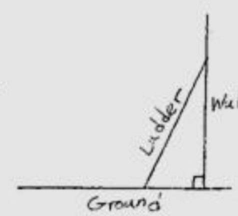
13. A rectangle has width 6 ft. and diagonal 10 ft. Find the length.

14. A rectangle has length 15 ft. and diagonal 17 ft. Find the width.

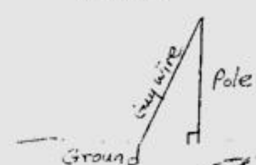
15. It is desired to find the distance across a lake. This is done by measuring the distances from P to R and from Q to R (dry land!) How far is it across the lake? (see figure)



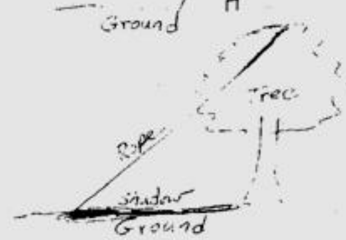
16. A 15-foot ladder leans against a wall with its foot 9 feet from the base of the wall. How high on the wall does the ladder reach?



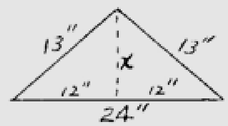
17. A guy wire to the top of a 15-foot pole reaches the ground 8 feet from the base of the pole. How long is the wire?



18. A 50-foot rope is stretched from the top of a tree to the tip of its shadow. If the shadow is 30 feet long, how tall is the tree?



19. Find the height of an isosceles triangle whose base is 24 inches and whose equal sides are 13 inches. (Hint: see figure.)



20. An isosceles triangle has a base of 10 cm. and height 12 cm. How long are the equal sides? (Hint: Draw a figure as in #19.)

21. $\boxed{\sqrt{\ }}$ Optional

A guy wire to the top of a 30-foot pole reaches the ground 8 feet from the base of the pole. How long is the guy wire?

22. $\boxed{\sqrt{\ }}$ Optional

A guy wire to the top of a pole is 30 feet and reaches the ground 8 feet from the base of the pole. How tall is the pole?

QUADRILATERALS

Square -- All sides are equal, opposite sides are parallel, and all angles are 90°.

Rectangle -- All angles are 90°, opposite sides are parallel and equal.

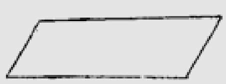
Rhombus -- All sides are equal, opposite sides are parallel.

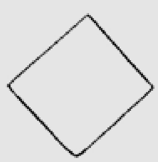
Parallelogram -- Opposite sides are parallel and equal.

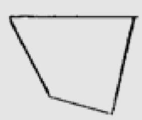
Trapezoid -- One pair of opposite sides are parallel.

The sum of the angles of a quadrilateral is 360°.

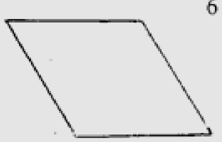
PROBLEMS: Give the best name to describe each of the following:

1. 

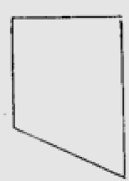
2. 

3. 

4. 

5. 

6. 

7. 

8. 

PERIMETERS and AREAS

Perimeter is simply the total distance around a figure, and will always be given in plain units such as feet, inches, meters, centimeters, etc. Just add the sides. Area will always be given in square units such as square feet (sq. ft. or ft^2), square inches (sq. in. or in^2), square meters (sq. m. or m^2), square centimeters (sq. cm. or cm^2), etc. The volume of a three-dimensional solid will always be given in cubic units such as cubic feet (cu. ft. or ft^3), cubic inches (cu. in. or in^3), cubic meters (cu. m. or m^3), cubic centimeters (cc or cm^3). Before any perimeter, area or volume can be found, be sure to have the same units for all sides being used. (See example 13). Before attempting any problems, some formulas must be established.

RECTANGLE

$P = 2w + 2\ell$

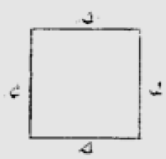
$A = \ell w$

The perimeter consists of two widths and two lengths.

SQUARE

$P = 4s$

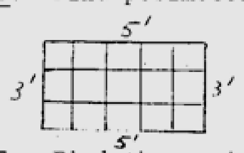
$A = s^2$



The area is simply the length times width.

EXAMPLE 6: Find perimeter and area of a rectangle 3' by 5'.

ANSWER:

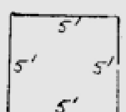


$P = 2w + 2\ell$
$= 6' + 10'$
$= 16'$

$A = \ell \times w$
$= 5' \times 3'$
$= 15$ sq. ft.

EXAMPLE 7: Find the perimeter and area of a 5-foot square.

ANSWER:



$P = 4 \times 5' = 20'$

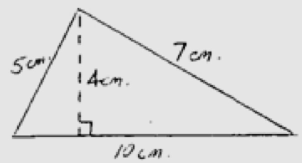
$A = \ell \times w$
$= 5' \times 5'$
$= 25$ sq. ft.

TRIANGLE

The perimeter of a triangle is simply the sum of the three sides.

It can be seen from the figure that the area of a triangle which has a base "b" and a height "h" (b and h must be perpendicular) is actually half the area of the rectangle shown. So: $A = \frac{bh}{2}$ or $\frac{1}{2}bh$.

EXAMPLE 8: Find the perimeter and area of the triangle shown.



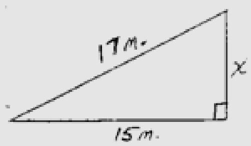
ANSWER: $P = 10 + 5 + 7 = 22 \text{ cm.}$

$$A = \frac{bh}{2}$$

$$= \frac{10 \times 4}{2} = \frac{40}{2} = 20 \text{ sq. cm. or } 20 \text{ cm}^2$$

EXAMPLE 9: Find the perimeter and area of a right triangle whose hypotenuse is 17 m. and whose base is 15 m.

ANSWER:



The height is missing, so the Theorem of Pythagoras is used:

$$15^2 + x^2 = 17^2$$
$$225 + x^2 = 289$$
$$x^2 = 64$$
$$x = 8 \text{ m.}$$

NOW: $P = 15 + 8 + 17 = 40 \text{ m.}$

$$A = \frac{bh}{2}$$

$$= \frac{15 \times 8}{2} = \frac{120}{2} = 60 \text{ sq. m or } 60\text{m}^2$$

CIRCLE

The Greeks, especially a mathematician by the name of Euclid, deserve credit for most of what we know about circles and triangles, and Geometry in general for that matter. Instead of watching T.V. they worked math problems for recreation and education. One of their great discoveries regarding the circle was that the ratio of the distance around (called "circumference") to the distance across (called "diameter") was always about the same (called a "constant"). No matter how small or large the circle, "C" divided by "d" always came out a little more than 3. In fact, it always came out about 3.14. This constant became the subject of much study and was named after one of the letters of their alphabet, the Greek letter π "pi".

So π was defined as the ratio of the circumference to the diameter of a circle.

$$\pi = \frac{\text{circumference}}{\text{diameter}} \quad \text{or} \quad \pi = \frac{C}{d}.$$

It is then a simple algebraic trick to multiply both sides of that equation by "d" to obtain:

$$\pi d = C \text{ or } C = \pi d$$

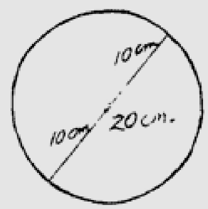
Further, since in a circle d = 2r, C = $\pi(2r)$ or $2\pi r$. This gives two formulas for finding the circumference (or perimeter) of a circle. After all, it is not possible in a circle to "add up the sides", as in other geometric figures.

Further study shows the area of a circle to be $A = \pi r^2$.

Circle Formulas: $C = \pi d$ or $C = 2\pi r$
$A = \pi r^2$

EXAMPLE 10: Find the circumference and area of a circle whose diameter is 20 cm. (Use $\pi \approx 3.14$)

ANSWER: $C = \pi d$
$= 3.14 \times 20 = 62.8$ cm.
$A = \pi r^2$ (if diameter is 20, then radius is 10)
$= 3.14 \times 10^2$ or $3.14 \times 10 \times 10$
= 314 sq. cm or 314 cm^2



EXAMPLE 11: Find the circumference and area of a circle whose radius is 20 cm.
NOTE: This makes the diameter 40 cm.

ANSWER: $C = \pi d$
$= 3.14 \times 40$
= 125.6 cm.

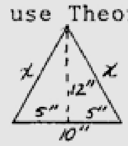
$A = \pi r^2$
$= 3.14 \times 20 \times 20$
= 1256 sq. cm. or 1256 cm^2

PROBLEMS: Find the perimeter and area of each of the following:

1. A rectangle whose length is 18 cm. and whose width is 10 cm.
2. A rectangle whose length is 38 m and whose width is 30 m.
3. A square whose sides are each 12 m.
4. A square whose sides are each 30 ft.
5. A right triangle whose legs are 12 cm. and 5 cm.
6. A right triangle whose hypotenuse is 15 in. and with one of the legs 12 in.

7. An equilateral triangle whose sides are 20' and whose height is 17.3'.

8. An isosceles triangle whose base is 10" and whose height is 12". (HINT: First use Theorem of Pythagoras to find the equal sides.)



9. An isosceles triangle whose base is 8' and whose height is 3'.

10. An isosceles triangle whose base is 12' and whose height is 8'.

11. An isosceles triangle whose equal sides are each 10 m. and whose base is 12 m. (HINT: First use Theorem of Pythagoras to find the height.)

12. An isosceles triangle whose equal sides are each 13 ft. and whose base is 10 ft.

13. A circle whose radius is 10 cm.

14. A circle whose diameter is 10 cm.

15. A circle whose diameter is 6 m.

16. A circle whose radius is 6 m.

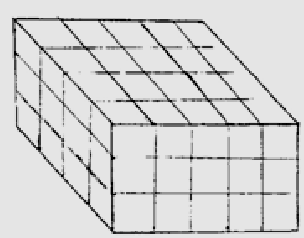
17. A circle whose radius is 25 m. (calculator).

18. A circle whose diameter is 25 m. (calculator).

VOLUMES

Volume may be defined as a measure of the inside capacity of a solid and must be measured in cubic units.

EXAMPLE 12: Find the volume of a box which is 5 inches long, 4 inches wide, and 3 inches high. (See figure)



ANSWER: A "cubic inch" is a cube that is one inch by one inch by one inch, such as the small cubes in the figure on p. G-13. There are 4 x 5 = 20 such cubes in the top layer, another 20 cubes in the middle layer, and 20 cubes in the bottom layer, for a total of 60 "cubic inches". Of course, the easy way to find the volume of a box is:

$$\boxed{V = l \cdot w \cdot h}$$
$$= 5 \times 4 \times 3 = 60 \text{ cubic inches}$$

In general, the volume of any solid that is the same from bottom to top (called a prism) is the area of the base (A) times height (h).

prism : $\boxed{V = A \cdot h}$

A special case of this is a cylinder. If the base is a circle of radius r and the height is h, then the volume of the cylinder is given by:

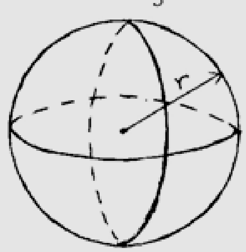
cylinder : $\boxed{V = \pi r^2 h}$ (why?)

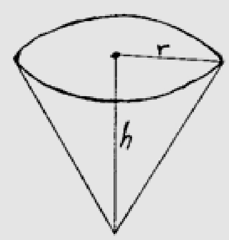
EXAMPLE 13: Find the volume of a cylindrical can whose height is 10 inches and whose base has a diameter of 10 inches.

ANSWER: $V = \quad r^2h$, where $r = 5$ and $h = 10$.
$= 3.14 \times 5^2 \times 10 = 785$ cu. in.

Two other important solids which are not prisms are the sphere and the cone. Their volume formulas will be given without detailed explanation.

Sphere: $V = \frac{4}{3}\pi r^3$ Cone: $V = \frac{1}{3}\pi r^2 h$





PROBLEMS: Find the volumes of each of the following:

1. A rectangular box which is 6 m. by 4 m. by 3 m.

2. A rectangular box which is 9 ft. by 5 ft. by 4 ft.

3. A cube whose edge is (whose sides are) 5 inches.

Problems: (cont'd)

4. A cube whose edge is 30 cm.

5. A circular cylinder whose height is 5 meters and whose base radius is 10 meters.

6. A circular cylinder whose height is 10 meters and whose base radius is 5 meters.

7. A circular cylinder whose height is 10 meters and whose base is of diameter 4 meters.

8. A circular cylinder whose height is 10 meters and whose base is of diameter 5 meters. (Calculator)

9. Find the volume of a sphere of radius 10 cm.

10. Find the volume of a sphere of diameter 5 cm. (Calculator)

11. Find the volume of a cone whose base is of radius 10 inches and whose height is 10 inches.

12. Find the volume of a cone whose base is of diameter 10 inches and whose height is 12 inches. (Calculator)

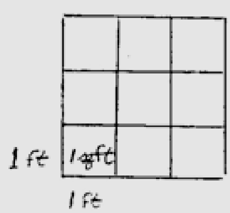
PROBLEMS FOR CONSIDERATION:

(a) How many square feet are in one square yard?

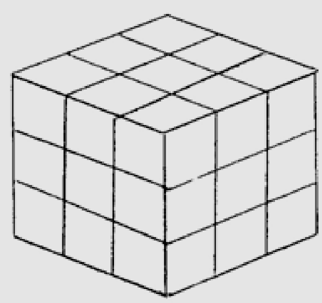
(b) How many cubic feet are in one cubic yard?

ANSWERS: The critical question is: how many feet are in one yard? (Answer: It depends upon how many people are the yard!)

(a)



(b)



Answer: 3' by 3' = 9 sq. ft.

Answer: 3' by 3' by 3' = 27 cu. ft.

PROBLEMS:

1. How many: (a) square inches are in one square foot? (b) cubic inches are in one cubic foot?

2. How many: (a) square inches are in one square yard? (b) cubic inches are in one cubic yard?

3. How many: (a) square centimeters are in one square meter? (b) cubic centimeters are in one cubic meter?

4. How many: (a) square millimeters are in one square centimeter? (b) cubic millimeters are in one cubic centimeter?

5. If there are 6 giggles in one gaggle, (a) how many square giggles are in one square gaggle? (b) how many cubic giggles are in one cubic gaggle?

6. If there are 4 rupples in one scrupple, (a) how many square rupples are in one square scrupple? (b) how many cubic rupples are in one cubic scrupple?

7. If there are 1,000,000 bubbles in a box, a) how many square bubbles are in one square box? b) how many cubic bubbles are in one cubic box?

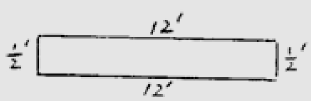
EXAMPLE 14: Find the perimeter and area of a rectangle of length 12 feet and width 6 inches.

ANSWER: The problem here does not have like units.

(1) So either change the 6 in. to 1/2 (or .5) ft., or

(2) Change the 12 ft. to 144 in.

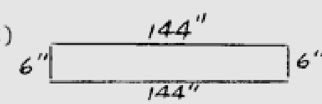
(1)



P=12' + 12' + 1' = 25'

A= w = 12 x 1/2 = 6 sq.ft

(2)



P=288" + 12" = 300"

A= w = 144" x 6" = 864 sq. in.

Comparing these two solutions, both are correct. (Why?) But which was easier?

EXAMPLE 15: Find the volume of a rectangular parallelopiped (box) if the dimensions are 4 meters by 50 centimeters by 50 centimeters.

ANSWER: It will be easiest to change everything to meters.
50 centimeters = .5 or 1/2 meter.
V = 4 m. x .5 m. x .5 m. = 1 cubic meter

PROBLEMS:

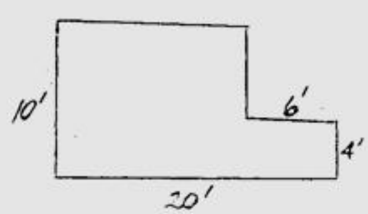
1. A rectangle is 6" by 4'. Find perimeter and area.
2. A rectangle is 50 cm. by 40 mm. Find perimeter and area.
3. A triangle has base 9 yards; 3 ft. height. Find area.
4. A triangle has base 120 m. and 1000 cm. height. Find area.

In # 5 - 11, find the volume of a box which is:

5. 6" x 6' x 7'
6. 4' x 10' x 3 yds.
7. 4' x 10 yds. x 6"
8. 3 yds. x 5' x 4"
9. 40 cm. x 2 m. x 1 m.
10. 3 m x 12 m x 10 cm.
11. 4 m. x 20 m. x 4 mm.
12. Find the volume of a cylinder of height 10 m. and of base radius 50 cm.
13. Find the volume of a cylinder of height 10 m. and of base diameter 20 cm.
14. Find the volume of a cylinder of height 20 cm. and of base diameter 1 m.
15. Find the volume of a cylindrical can which is 1 yd. high and 4 feet in diameter.
16. Find the volume of a cylindrical can which is 1 yard in diameter and 4 feet high.

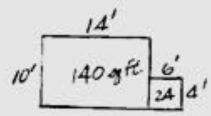
ADDITION and SUBTRACTION of AREAS

EXAMPLE 16: Find the area of the figure:



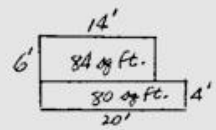
ANSWER: There are three approaches to this problem:

(1)



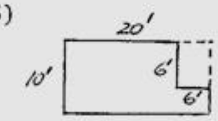
Total = 140
+24
164 sq. ft.

(2)



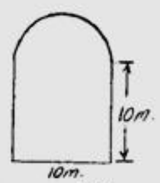
Total = 84
+80
164 sq. ft.

(3)



Whole = 20 x 10
= 200
- 36
164 sq. ft.

EXAMPLE 17: Find the area and perimeter of the figure:



ANSWER: The figure is square with a semi-circle of radius 5m. Find the area of the square and of the semi-circle and add.

Area Circle $A = \pi r^2$

$= 3.14 \times 5^2$

$= 78.5$ square meters

Semi-circle = 1/2 of 78.5 = 39.25 square meters.
+ Square = 10 x 10 = +100.00 square meters

Total 139.25 square meters

Perimeter

Circle $C = \pi d$

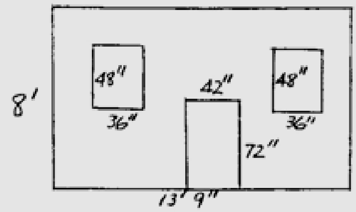
$= 3.14 \times 10 = 31.4$

Semicircle = 1/2 of 31.4 = 15.7 m.

Three sides of square = 30.0

Total 45.7 m.

EXAMPLE 18: Find the area of the following wall of a house excluding the door and windows:



ANSWER: Total Area = 8' x 13.75' = 110 sq. ft.

Door = 6' x 3.5' = 21 sq. ft.

Window = 3 x 4 = 12 ft.

Window = 3 x 4 = 12 ft.

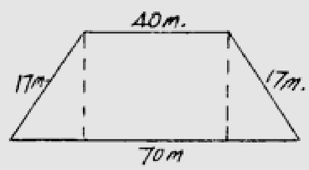
45 sq. ft.

110 sq. ft.

-45 sq. ft.

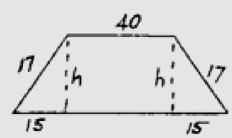
65 sq. ft.

EXAMPLE 19: Find the area of the isosceles trapezoid:



ANSWER: First, solve the missing sides of the triangles.

70 - 40 = 30 to be equally divided with 15 m. on each side. Second, the Theorem of Pythagoras is used to find the height.



$15^2 + h^2 = 17^2$

$225 + h^2 = 289$

$h^2 = 64$

$h = 8$

Finally, Rectangle (in middle) Area = 40 x 8' = 320 sq. m.

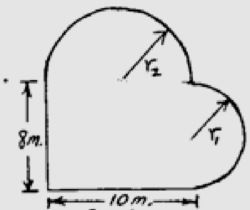
Triangle (left) Area = $\frac{bh}{2} = \frac{15 \times 8}{2} = 60$ sq. m.

Triangle (right) (Same) = 60 sq. m.

TOTAL 440 sq. m.

(Note: The formula $A = \frac{B+b}{2} \cdot h$ could also have been used.)

EXAMPLE 20: Find the area and perimeter.



ANSWER: $r_1 = 4$, $r_2 = 5$ (Why?)

A_1 of whole circle $= \pi r^2 = 3.14 \times 4^2 = 50.24$ sq.m.

A_2 of whole circle $= \pi r^2 = 3.14 \times 5^2 = 78.5$ sq.m.

A_1 of half circle $= 25.12$ sq. m.

A_2 of half circle $= 39.25$ sq. m.

A of rectangle $= +80.00$ sq. m.

144.37 sq. m.

C_1 of whole circle $= \pi d = 3.14 \times 8 = 25.12$ m.

C_2 of whole circle $= \pi d = 3.14 \times 10 = 31.40$ m.

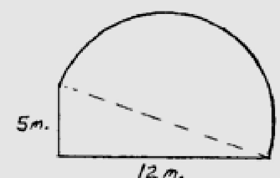
C_1 of half circle $= 12.56$ m

C_2 of half circle $= 15.70$ m

Two straight sides $= +18.00$ m

46.26 m

EXAMPLE 21: Find the area and perimeter.



ANSWER: Find the hypotenuse of right triangle which is 13. This is the diameter of the circle, so: r = 13/2 or 6.5

A of whole circle $= \pi r^2 = 3.14 \times 6.5^2 = 132.665$ sq.m.

A of half circle $= 66.33$ sq.m.

A triangle $= +30.00$ Why?

96.33 sq. m.

C of whole circle $= \pi d = 3.14 \times 13 = 40.82$ m

C of half circle $= 20.41$ m

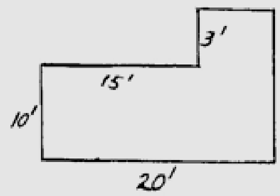
+17.00 Why?

37.41 m.

PROBLEMS:

In 1-25, find the perimeter and area.

1.



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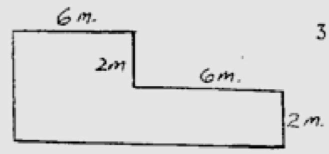

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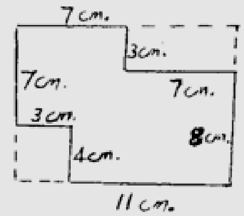

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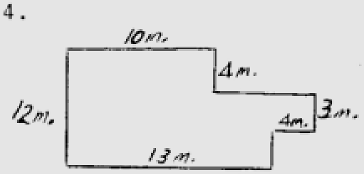

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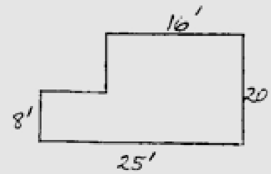

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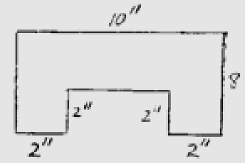

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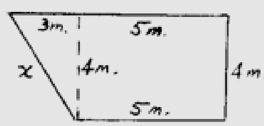
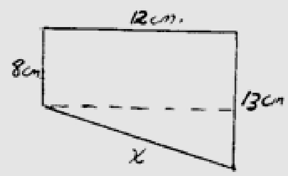


(Hint: Use T/P* to find x in this & in later problems.)

8.



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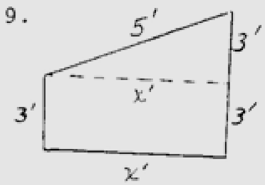

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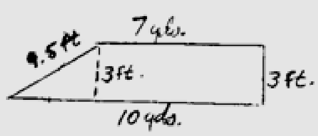

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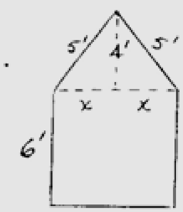
12.



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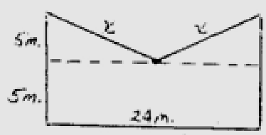

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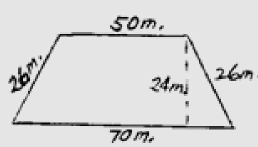

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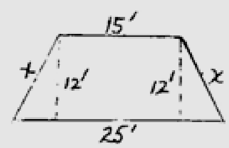

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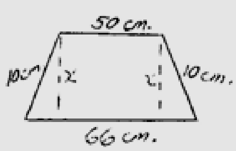

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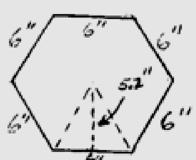
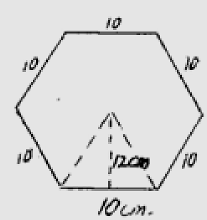
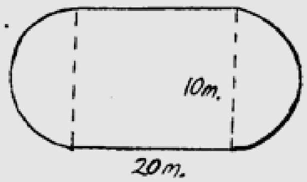


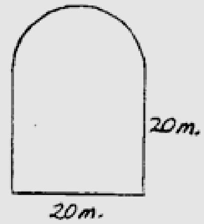
(Hint: Divide area into triangles & find area of each triangle)

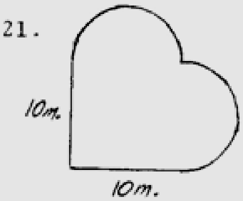
18.

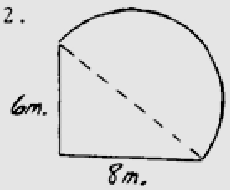


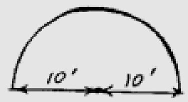
* Theorem of Pythagoras

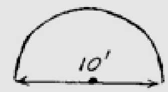
19. 

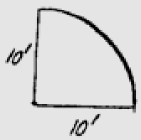
20. 

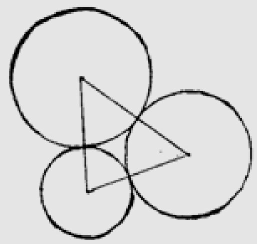
21. 

22. 

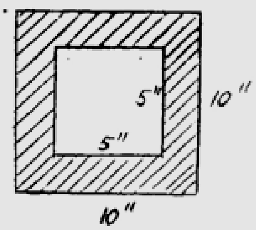
23. 

24. 

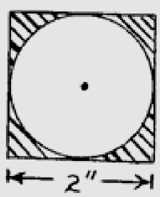
25. 

26. 

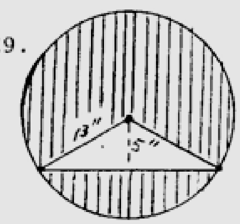
Find triangle perimeter if circles have radius 2",3",4".

27. 

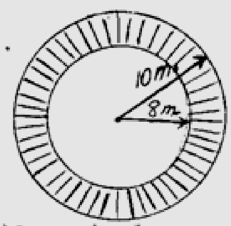
(a)Lg. rectangle are^?
(b)Sm. rectangle ar.
(c)Shaded Area?

28. 

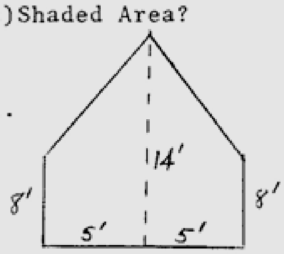
(a)Square Area?
(b)Circle Area?
(c)Shaded Area?

29. 

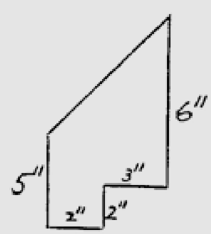
(a)Circle area?
(b)Triangle area?
(c)Shaded area?

30. 

(a)Lg. circle area?
(b)Sm. circle area?
(c)Shaded area?

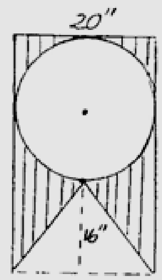
31. 

Find the area only

32. 

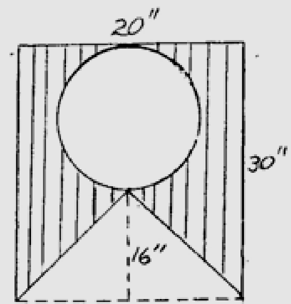
Find the area only

33.



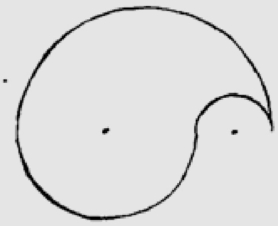
(a) Area of circle, triangle, & rectangle?
(b) Shaded area?

34.



(a) Area of circle, triangle, & rectangle?
(b) Shaded area?

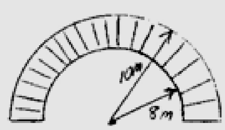
35.



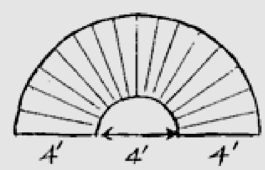
Area & perimeter of teardrop formed by three semicircles, of radius 1",2" and 3"?

In 36 and 37, find the perimeter and area of the figure.

36.

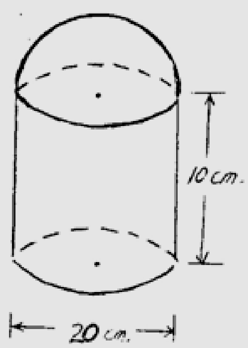


37.

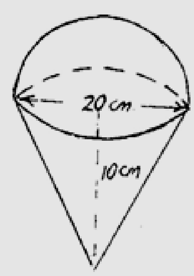


In 38 to 40, find the volumes.

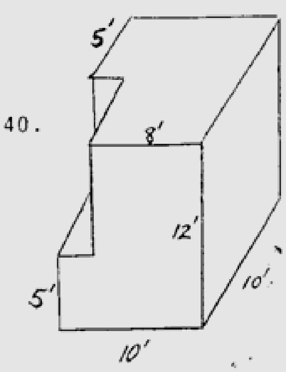
38.



39.



40.



EXAMPLE 22: A rectangular room is 20' by 27'. If carpet costs \$12 per square yard, how much will it cost to carpet a room?

ANSWER:

(1) 20' x 27' = 540 sq. ft.

(2) How many sq. ft. in one sq. yd.? (Ans. 9)

(3) 540 ÷ 9 = 60 sq. yds.

(4) 60 sq. yds. x \$12 = \$720 (Big room!!)

EXAMPLE 23: A rectangular lot is 120' by 150'.

(a) If fence costs \$5 per yard, how much would it cost to fence in the lot?

(b) If sod costs \$1 per square yard, how much would it cost to sod the lot?

(c) How many cubic yards of sand would be required to cover the lot with 6" of sand?

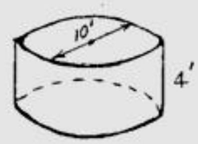
ANSWER: Since both 120 and 150 are divisible by 3, it is easy to say the lot is 40 yds. by 50 yds.

(a) Perimeter = 80 + 100 = 180 yds. (Why?)
So the cost is 180 yds. x \$5 = \$900.

(b) Area = 40 x 50 = 2,000 sq. yds.
So the cost is 2,000 x \$1 = \$2,000 (Big lot!!)

(c) 6" is $\frac{6}{36} = \frac{1}{6}$ yard,
So $V = 40 \times 50 \times \frac{1}{6}$
= 333.3 cubic yards

EXAMPLE 24: A circular swimming pool is 10 feet across and 4 feet deep. If there are 7½ gallons of water in one cubic foot, how many gallons of water would it take to fill the tank?



ANSWER: The swimming pool is a cylinder with base radius 5' and of height 4'.

$V = A\,h$, where A = Area of base = πr^2 (Why?)

$A = 3.14 \times 5^2$

$= 78.5$ sq. ft.

$V = 78.5 \times 4$

= 314 cubic feet, each of which is 7.5 gallons

$V = 314 \times 7.5 =$ 2355 gallons

EXAMPLE 25: A cake was made in the shape of a rectangle with dimensions 1' by 2' by 6". The cake was so good, it was decided to double all the dimensions and make it "twice" as big. Find the volume of the original cake and of the new cake. What actually happened to the volume of the new cake?

ANSWER:

Original cake: $V = 1' \times 2' \times \frac{1}{2}' = 1$ cubic foot

New cake: $V = 2' \times 4' \times 1' = 8$ cubic feet

The new cake is EIGHT times the original cake. (That's a big cake!) This means that all ingredients in the cake must not be doubled, but multiplied by eight.

PROBLEMS:

1. How much will it cost to carpet a rectangular room that is 9 yds. long and 5 yds. wide if carpet costs $2 per sq. ft.?

2. How much will it cost to tile a bathroom floor that is 12 sq. yds. if the tile cost is $.80 per square foot?

3. How much will it cost to carpet a rectangular room that is 18 ft. by 10 ft. if carpet is $12 per sq. yd.?

4. What happens to the area of a floor if:
   (a) The dimensions are doubled?
   (b) The dimensions are tripled? (Hint: make up a simple example)

5. A room would cost $100 to carpet.
   (a) If the dimensions were doubled, what would it cost?
   (b) If the dimensions were tripled, what would it cost?

6. (a) 120 sq. ft. = __________ sq. yds.
   (b) 45 sq. yds. = __________ sq. ft.

7. Does "9 square feet" mean the same as a "9 foot square"?

8. How many yards of fence would be required to fence a rectangular area 45' by 34'?
   How many square yards of carpet would be required to carpet a rectangular area 45' by 34'?

9. An asphalt walkway 4' wide, 12' long and 3" thick is to be built.
   (a) Find the volume.
   (b) If asphalt comes in 1.5 cubic foot bags, how many bags would be needed?
   (c) At $3 per bag, what will the job cost?

In Problems 10-13, round to the next larger cubic yard and find the cost of the cement at $35 per yard. If the amount is less than 3 yards, a $20 delivery fee should be added to the cost.

10. How many cubic yards of cement will be required to pour a 10' by 30' patio that averages 3" thick?

11. How many cubic yards of cement will be required to pour a 10' by 30' patio that averages 4" thick?
    How much will it cost?

12. A foundation for a utility shed is to be 10' by 10' and 3" thick. How many cubic yards is this?
    How much will it cost?

13. A driveway is to be 10' wide, 50' long and 4" thick.
    How many cubic yards is this?
    How much will it cost?

14. A lot is 100' by 180'.
    (a) How many yards of fence would it take to surround the property?
    (b) How many square yards are in its area?
    (c) If building codes require the ground level to be raised 3 inches, how many cubic yards of dirt must be hauled in?

15. A rectangular pool is 4' across, 5' long and 18" deep.
    How many gallons will it take to fill it if there are 7.5 gallons per cubic foot?

16. A circular pool is 20' across and 5' deep.
    Find the volume in cubic feet and in gallons.

17. A rectangular pool is 5 yards across, 50 feet long and has an average depth of 6 feet.
    How many gallons of water does it hold?

18. A rectangular pool is 3 m. long, 2 m. wide and 1/2 m. deep.
    If there are 1000 liters of water in a cubic meter, how many liters of water will it take to fill the pool?

19. A circular pool is 4 meters across and 1 meter deep.
    How many liters of water will the pool hold? (See #18.)

20. A circular pool is 3 meters across and 1 meter deep.
    How many liters of water will the pool hold?

21. A rectangular pool is 20 meters long, 5 meters across and an average depth of 2 meters.
    Find the volume kiloliters. (Hint: How many kiloliters are in one cubic meter?)

22. The tiles in a bathroom floor are 1 inch squares. How many tiles are there if the area of the floor is 2.5 sq. yds.?

23. If a rectangle is 4 ft. by 12 ft., how many two-inch tiles would have to be put around the outside edge to completely frame the rectangle? Draw a figure.

24. A picture is 20" by 24". If the picture is framed by a 3-inch frame:
(a) find the perimeter of the outside edge of the frame
(b) find the area of the frame.

25. A field bordering a river is to be fenced on three sides with the length along the river unfenced. If the width of the rectangular field is 75 ft., and the total amount of fence is 400 ft., find the area enclosed. (Draw figure.)

26. Find the radius of a circle if:
(a) its circumference is 36π m.
(b) its area is 36π m.

27. Five equal squares are placed end to end to form a rectangle whose perimeter is 120 cm. Find the dimensions of the squares and the area of each square.

28. If the five equal squares placed end to end form a rectangle whose perimeter is 372 cm., find the dimensions of the squares and the area of each square.

<u>In Problems 29-32, suppose that the price per pallet of sod is listed as follows</u>, depending upon how many pallets are required. Each pallet covers 400 sq. ft.

	Bahia	Floratem
1-2	$48 ea.	$82.50 ea.
3-6	$44 ea.	$80.00 ea.
7-10	$38 ea.	$77.50 ea.
11-15	$34 ea.	$75.00 ea.
16 or more	$30 ea.	$72.50 ea.

For example, if 2 pallets of Bahia are required, they would cost $48 x 2 = $96. If 10 pallets of Bahia are required, they would cost $38 x 10 = $380. If 11 pallets are required, they would cost $34 x 11 = $374. (Strange, but true!)

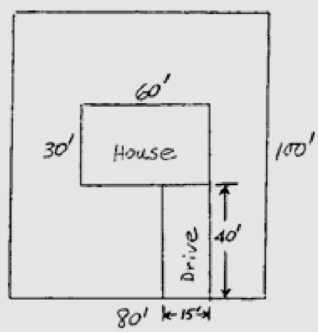
PROBLEMS: For each of the following lots, determine how many pallets of sod would be required to cover the yard, excluding house driveway, sidewalks, etc. Then find the cost to do the yard:

(a) in Bahia?

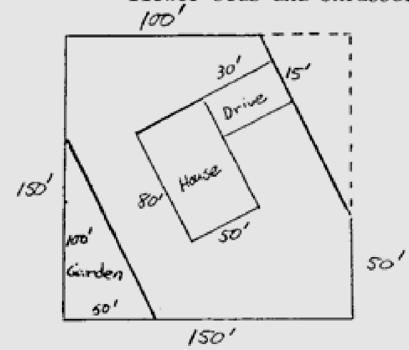
(b) in Floratem?

In 31 and 32, subtract 1500 sq ft for flower beds and shrubbery.

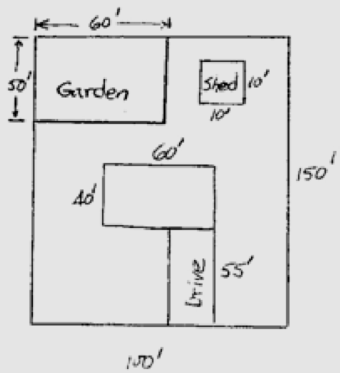
29.



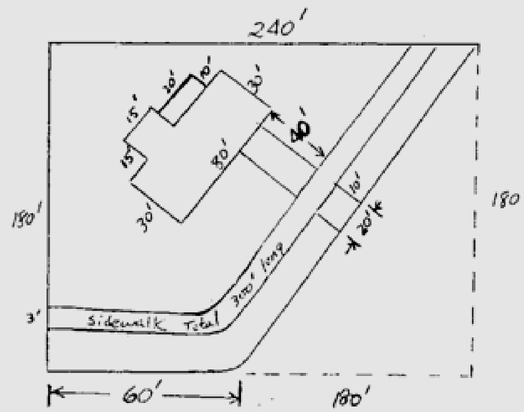
31.



30.



32.



GEOMETRIC MEASURES

Practice Test

Calculators permitted. Let $\pi = 3.14$

All problems 2 points per answer blank.

1. In any triangle, the sum of the angles is____. 1.____________

2. In a triangle in which the angles are all equal, each angle would be _____ degrees. 2.____________

3. A triangle in which all sides are equal is called a(n) _____ triangle. 3.____________

4. A triangle in which all angles are less than 90° is called a(n) ____ triangle. 4.____________

5. If two sides of a triangle are equal, it is said to be ____. 5.____________

6. The Greeks defined π to be (not 3.14) . 6.____________

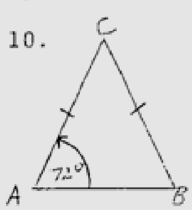
7. If two angles of a triangle are 25°, 120°, then the third angle is (a)_____. What kind of triangle is it: (b) based upon the size of the angles? (c) based upon the number of equal angles and sides?

   7.(a)____________
   (b)____________
   (c)____________

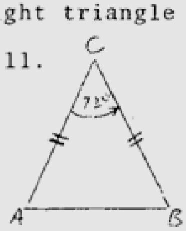
8. According to the Theorem of (a)_____, which applies only to (b)____ triangles, if side c is the (c)_____ and sides a and b are (d)_____, it may be concluded that (e)____.

   8.(a)____________
   (b)____________
   (c)____________
   (d)____________
   (e)____________

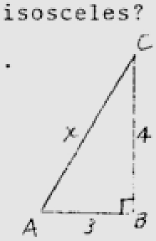
9. Can a right triangle be isosceles? 9.____________

10.


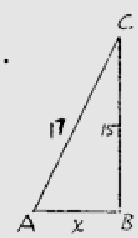
Find angle C

11.


Find angle A

12.


Find x.

13.


Find x.

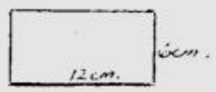
10.____________
11.____________
12.____________
13.____________

14. Any quadrilateral with opposite sides parallel is called a(n)_____.   14.______________

15. Any quadrilateral with only one set of opposite sides parallel is called a(n)_____.   15.______________

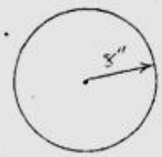
16. Any quadrilateral with opposite sides parallel and equal is called a(n)_____.   16.______________

17.



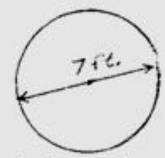
(a)perimeter
(b)area

18.



(a)circumference
(b)area

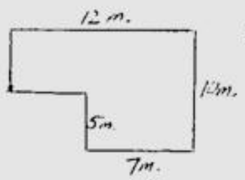
19.



(a)circumference
(b)area

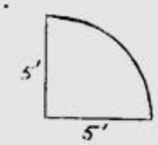
17.(a)____________
(b)____________
18.(a)____________
(b)____________
19.(a)____________
(b)____________

20.



(a)perimeter
(b)area

21.



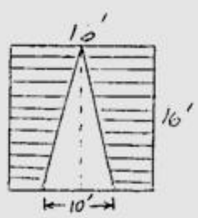
(a)perimeter
(b)area

22.

(a)perimeter
(b)area

20.(a)____________
(b)____________
21.(a)____________
(b)____________
22.(a)____________
(b)____________

23.



Find shaded area.   23.______________

24. Find the height of a telephone pole with a 13-foot guy wire anchored 5 feet from the base of the pole.   24.______________

25. Find the volume of a box which is 10 ft. x 6 ft. x 1 yd.   25.______________

26. Find the volume of a cylinder if the diameter of the base is 6 m. and the height is 4 m. $(V = \pi r^2 h)$ 26.______________

27. How many square inches are in one square foot? 27.________

28. How many square feet are in one square yard? 28.________

29. How many cubic feet are in one cubic yard? 29.________

30. How many cubic centimeters are in one cubic meter? 30.________

31. If the dimensions of a porcelain figurine are doubled, what happens to the volume? 31.________

32. If the dimensions of a room are tripled, what happens to the area of the room? 32.________

33. A parking lot which is a 100 ft. x 300 ft. rectangle is to be made using a 6-inch layer of limerock. How many cubic yards of lime-rock are needed? 33.________

34. A rectangular lot that is 20 yds. by 60 yds. is to be fenced in at a cost of $1.50 per foot. Find the cost. 34.________

35. A rectangular room that is 20 ft. by 60 ft. is to be carpeted at a cost of $12 per square yard. Find the cost. 35.________

36. A picture which is 8" x 10" is to be surrounded by a 2-inch border. Find the area of the border. 36.________

37. How many cubic inches are in one cubic foot? 37.________

EXTRA CHALLENGE: (Show all work) EC. P = ________

A = ________

Find the perimeter and area.
(Hint: must use "thm. of guess who"!)

(BE SURE TO PUT UNITS ON ANSWERS REQUIRING THEM.)

GEOMETRIC MEASURES

G-4:
1. 90°; right; scalene
2. 72°; acute; scalene
3. 39°; obtuse; isosceles
4. 45°; right; isosceles

5a) $B=40^\circ$; $C=90^\circ$ right; scalene
b) $B=50^\circ$; $C=80^\circ$ acute; isosceles
c) $B=65^\circ$; $C=65^\circ$ acute; isosceles

6a) $B=45^\circ$; $C=90^\circ$; right; isosceles
b) $B=45^\circ$; $C=90^\circ$; right; isosceles
c) $B=67\frac{1}{2}^\circ$; $C=67\frac{1}{2}^\circ$; acute; isosceles

7a) $B=80^\circ$; $C=90^\circ$; right; scalene
b) $B=10^\circ$; $C=160^\circ$; obtuse; isosceles
c) $B=85^\circ$; $C= 85^\circ$; acute; isosceles

8. 60°
9. All except right equilateral and obtuse equilateral.

G-6:

1. 18	4. 8	7. 3	10. 4
2. 42	5. 9	8. 5	11. 6
3. 4	6. 21	9. 7	12. 8

G-8: G-9

1. 13	6. 10	11. 13	16. 12	21. 31.05
2. 4	7. 5	12. 5	17. 17	22. 28.91
3. 12	8. 1.73	13. 8	18. 40	
4. 8	9. 6.40	14. 8	19. 5	
5. 10	10. 35.36	15. 3	20. 13	

G-9:

1. Parallelogram	4. Trapezoid	7. Trapezoid
2. Square	5. Rhombus	8. Rhombus
3. Quadrilateral	6. Rectangle	

G-12: G-13
1. P= 56 cm; A= 180 sq cm
2. P=136 m; A=1140 sq m
3. P= 48 m; A= 144 sq m
4. P=120 ft; A= 900 sq ft
5. P= 30 cm; A= 30 sq cm
6. P= 36 in; A= 54 sq in
7. P= 60 ft; A= 173 sq ft
8. P= 36 in; A= 60 sq in
9. P= 18 ft; A= 12 sq ft
10. P= 32 ft; A= 48 sq ft
11. P= 32 m; A= 48 sq m
12. P= 36 ft; A= 60 sq ft
13. P= 62.8 cm; A= 314 sq cm
14. P= 31.4 cm; A= 78.5 sq cm
15. P= 18.84 m; A= 28.26 sq m
16. P= 37.68 m; A= 113.04 sq m
17. P=157 m; A=1962.5 sq m
18. P= 78.5 m; A= 490.63 sq m

G-14: G-15

1. 72 cu m	5. 1570 cu m	9. 4186.67
2. 180 cu ft	6. 785 cu m	10. 65.42
3. 125 cu in	7. 125.6 cu m	11. 1046.67 cu in
4. 27,000 cu cm	8. 196.25 cu m	12. 314 cu in

G-16:

1a) 144	2a) 1,296	3a) 10,000	4a) 100	5a) 36	6a) 16
b) 1728	b) 46,656	b) 1,000,000	b) 1000	b) 216	b) 64

G-17:
1. P= 9 ft (108 in) A= 2 sq ft (288 sq in)
2. P= 108 cm (1080 mm) A= 200 sq cm (20000 sq mm)
3. A= 4.5 sq yd (40.5 sq ft)
4. A= 600 sq m
5. V= 21 cu ft
6. V= 360 cu ft
7. V= 60 cu ft
8. 15 cu ft
9. .8 cu m (800,000 cu cm)
10. 3.6 cu m
11. .32 cu m
12. 7.85 cu m
13. .31 cu m (314,000 cu cm)
14. .16 cu m (157,000 cu cm)
15. 37.68 cu ft
16. 28.26 cu ft

G-21: G-22
1. P= 66 ft; A= 215 sq ft
2. P= 32 ft; A= 36 sq m
3. P= 50 cm; A= 121 sq cm
4. P= 58 m; A= 156 sq m
5. P= 90 ft; A= 392 sq ft
6. P= 40 in; A= 68 sq in
7. P= 22 m; A= 26 sq m
8. P= 46 cm; A= 126 sq cm
9. P= 18 ft; A= 18 sq ft
10. P= $63\frac{1}{2}$ ft; A= $76\frac{1}{2}$ sq ft
11. P= 62 m; A= 228 sq m
12. P= 28 ft; A= 48 sq ft
13. P= 70 m; A= 180 sq m
14. P= 172 m; A=1440 sq m
15. P= 66 ft; A= 240 sq ft
16. P= 136 cm; A= 348 sq cm
17. P= 36 in; A= 93.6 sq in
18. P= 60 cm; A= 360 sq cm
19. P= 71.4 m; A= 278.5 sq m
20. P= 91.4 m; A= 557 sq m

G-21
G-22:
G-23

21. P= 51.4 m; A= 178.5 sq m
22. P= 29.7 m; A= 63.25 sq m
23. P= 51.4 ft; A= 157 sq ft
24. P= 25.7 ft; A= 39.25 sq ft
25. P= 35.7 ft; A= 78.5 sq ft
26. P= 18 in

27a) 100 sq in
b) 25 sq in
c) 75 sq in

28a) 4.00 sq in
b) 3.14 sq in
c) .86 sq in

29a) 530.66 sq in
b) 60.00 sq in
c) 470.66 sq in

30a) 314.00 sq m
b) 200.96 sq m
c) 113.04 sq m

31. 110 sq ft
32. 26.5 sq in
33a) Circle: 314 sq in
Tri: 160 sq in
Rect: 720 sq in
b) Shaded: 246 sq in
34a) Circle: 153.86 sq in
Tri: 160 sq in
Rect: 600 sq in
b) Shaded: 286.14 sq in

35a) 18.84 in
b) 18.84 sq in
36a) 60.52 m
b) 56.52 sq m
37a) 33.12 ft
b) 50.24 sq ft
38. 5233.33 cu cm
39. 3140 cu cm
40. 1130 cu ft

G-25:
G-26
G-27

1. $810
2. $ 86.40
3. $240
4a) 4 times
b) 9 times
5a) $400
b) $900
6a) 13.33
b) 405
7. No
8a) 52 2/3
b) 170
9a) 12 cu ft
b) 8 bags
c) $24
10. 3 cu yd; $105
11. 4 cu yd; $140
12. 1 cu yd; $ 55
13. 7 cu yd; $245
14a) 186.67 yd
b) 2000 sq yd
c) 166.67 cu yd
15. 225 gal
16. 1570 cu ft
11,775 gal
17. 33,750 gal
18. 3,000 gal
19. 12,560 liters
20. 7,065 liters
21. 200 kiloliters
22. 3240
23. 196
24a) 112 in
b) 300 sq in
25. 18,750 sq ft
26a) 18 m
b) 6 m
27. 10 cm by 10 cm
A=100 sq cm
28. 31 cm by 31 cm
A= 961 sq cm
29. 14; a) $ 476 b) $1050
30. 22; a) $ 660 b) $1595
31. 29; a) $ 870 b) $2102.50
32. 52; a) $1560 b) $3770

G-29:
G-30
G-31

1. 180°
2. 60°
3. Equilateral
4. Acute
5. Isosceles
6. $\frac{\text{Circumference}}{\text{diameter}}$
7a) 35°
b) obtuse
c) scalene
8a) Pythagoras
b) right
c) hypotenuse
d) legs
e) $a^2+b^2=c^2$
9. Yes
10. 36°
11. 54°
12. 5
13. 8
14. Parallelogram
15. Trapezoid
16. Parallelogram
17a) 36 cm
b) 72 sq cm
18a) 50.24 in
b) 200.96 sq in
19a) 21.98 ft
b) 38.465 sq ft
20a) 44 m
b) 95 sq m
21a) 17.85 ft
b) 19.625 sq ft
22a) 36 in
b) 60 sq in
23. 176 sq ft
24. 12 ft
25. 180 cu ft
26. 113.04 cu in
27. 144
28. 9
29. 27
30. 1,000,000
31. Eight times as great
32. Nine times as great
33. 555.56
34. $720.
35. $1600.
36. 88 sq in
37. 1728

E.C. P= 67.4 ft
A= 61 sq ft