Ratios and Proportions

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Some of the best answers to the age-old question, "What good is math?" may be found in this section on ratio and proportion. The concept of ratio and proportion is a convenient way of organizing given information, setting up a simple equation, and using this to determine an unknown quantity. The applications to everyday life, especially in the world of business, are innumerable.

First, what is a **ratio**, and what is a **proportion**? **A ratio is simply the quotient of two numbers.** This is where we get the word "**ratio**nal numbers." A **rational number** is any number that can be expressed as the **ratio** or quotient of two **integers (denominators cannot equal zero).** Every time you write a fraction, you have written a ratio. **A proportion is simply the equating of two ratios. Whenever one ratio** (or fraction) **equals another ratio** (or fraction)**, this is a proportion.**
The next examples involve reducing fractions to lowest terms.

Example 1: **Reduce the fraction $\frac{9}{12}$ to lowest terms.**

Solution: Divide the numerator and denominator each by 3: $\frac{3}{4}$.

Check: $\frac{9}{12}=\frac{3}{4}$ can be checked by seeing that $9 \cdot 4 = 3 \cdot 12$, or 36 = 36.

Example 2: **Reduce the fraction $\frac{8}{16}$ to lowest terms.**

Solution: Divide the numerator and denominator each by 8: $\frac{1}{2}$.

Check: $\frac{8}{16}=\frac{1}{2}$ can be checked by seeing that $8 \cdot 2 = 1 \cdot 16$, or 16 = 16.

Example 3: **Reduce the fraction $\frac{320}{480}$ to lowest terms. [Hint: divide by 160.]**

Solution: Dividing the numerator and denominator each by 160: $\frac{2}{3}$.

Check: $\frac{320}{480}=\frac{2}{3}$ can be checked by seeing that $320 \cdot 3 = 480 \cdot 2$, or 960 = 960.

In the examples above, each fraction, like $\frac{9}{12}$, $\frac{3}{4}$, or $\frac{2}{3}$, represented the ratio of two numbers.

Each time we stated that a fraction equals a fraction, like $\frac{9}{12} = \frac{3}{4}$, $\frac{8}{16} = \frac{1}{2}$, or $\frac{320}{480} = \frac{2}{3}$, these equalities are called **proportions**. When these proportions are stated in the general case, $\mathbf{\frac{a}{b} = \frac{c}{d}}$ (where b and d cannot equal zero), the result is called the **definition of equality of fractions:**

Definition of Equality of Fractions

Two fractions, $\frac{a}{b}$ and $\frac{c}{d}$, are equal if and only if **ad = bc, where b ≠ 0, d ≠ 0.**

In arithmetic problems, this formula gives a great way to check your answers when reducing fractions. In algebra problems, it gives a great way to convert complicated looking fractional equations into an equation that can often be solved very simply. In ratio problems from real life, it allows you to let x = an unknown quantity, write an equation, and solve the equation, with a little help from your calculator. Here's how it works.

Example 4: Solve the equation for x: $\frac{x}{20} = \frac{3}{4}$

Solution: $\frac{x}{20} = \frac{3}{4}$ means that $4 \cdot x = 20 \cdot 3$

$4x = 60$

Divide both sides of the equation by 4: $\frac{4x}{4} = \frac{60}{4}$

x = 15

Example 5: Solve the equation for x: $\frac{5}{6} = \frac{30}{x}$

Solution: $\frac{5}{6} = \frac{30}{x}$ means that $5 \cdot x = 30 \cdot 6$

$5x = 180$

Divide both sides of the equation by 5: $\frac{5x}{5} = \frac{180}{5}$

x = 36

Example 6: **Solve the equation for y:** $\frac{7}{6} = \frac{y}{9}$

Solution: $\frac{7}{6} = \frac{y}{9}$ means that $7 \cdot 9 = 6 \cdot y$

$63 = 6y$ or $6y = 63$

Divide both sides of the equation by 6: $\frac{6y}{6} = \frac{63}{6}$

y = 20.5

Example 7: **Solve the equation for y:** $\frac{13}{y} = \frac{29}{12}$

Solution: $\frac{13}{y} = \frac{29}{12}$ means that $13 \cdot 12 = 29 \cdot y$

$156 = 29y$ or $29y = 156$

Divide both sides of the equation by 29: $\frac{29y}{29} = \frac{156}{29}$

y = 5.38 (rounded to nearest hundredth).

Example 8: **Solve the equation for x:** $\frac{5 \text{ oz}}{100 \text{ sq ft}} = \frac{x \text{ oz}}{1500 \text{ sq ft}}$

Solution: $\frac{5 \text{ oz}}{100 \text{ sq ft}} = \frac{x \text{ oz}}{1500 \text{ sq ft}}$ means that $5 \cdot 1500 = 100 \cdot x$

$7500 = 100x$ or $100x = 7500$

Divide both sides of the equation by 100: $\frac{100x}{100} = \frac{7500}{100}$

x = 75 oz.

Example 9: **Solve the equation for x:** $\frac{500 \text{ mg}}{15 \text{ cc}} = \frac{175 \text{ mg}}{x \text{ cc}}$

Solution: $\frac{500 \text{ mg}}{15 \text{ cc}} = \frac{175 \text{ mg}}{x \text{ cc}}$ means that $500 \cdot x = 15 \cdot 175$

$500x = 2625$

Divide both sides of the equation by 100: $\frac{500x}{500} = \frac{2625}{500}$

x = 5.25 cc.

Now, after solving a few equations, it's time to apply this technique to problems in the real world. Consider the following examples.

Example 10: If 5 packages of a product sell for $6.00, how much should 8 packages cost?

Solution: One obvious solution is to find the cost of one package by dividing $6 by 5 (which is $1.20), then multiply by the number of packages that you want (8 pkg), which is $9.60. With a calculator, this is all very easy.

However, as an introduction to a larger and very useful method, you should set up a proportion. Remember in this, that the expression "**how much"** always means **the unknown,** which will be the variable, usually **x.** While there are many ways to set up a proportion, let's begin with the ratio: $\dfrac{\textbf{packages}}{\textbf{price}}$. The proportion will be $\dfrac{\textbf{packages}}{\textbf{price}} = \dfrac{\textbf{packages}}{\textbf{price}}$. However you set it up, it is very important to be consistent. $\dfrac{\textbf{packages}}{\textbf{price}}$**:** $\dfrac{\textbf{5 packages}}{\textbf{\$ 6}} = \dfrac{\textbf{8 packages}}{\textbf{\$ x}}$

From this you can see:

$$5 \cdot x = \$\,6 \cdot 8$$
$$5x = \$\,48$$
$$x = \mathbf{\$\,48/5 \text{ or } \$9.60.}$$

Alternate Solution: $\dfrac{\textbf{price}}{\textbf{packages}}$**:** $\dfrac{\textbf{\$ 6}}{\textbf{5 packages}} = \dfrac{\textbf{\$ x}}{\textbf{8 packages}}$

Notice--equation is same as before:

$$5 \cdot x = \$\,6 \cdot 8$$
$$5\,x = \$\,48$$
$$x = \mathbf{\$\,48/5 \text{ or } \$\,9.60}$$

Example 11: If 5 packages of a product sell for $6.00, how many packages can you buy for $20?

Solution: Set up a ratio $\dfrac{\textbf{cost}}{\textbf{packages}}$**:** $\dfrac{\textbf{\$ 6}}{\textbf{5 packages}} = \dfrac{\textbf{\$ 20}}{\textbf{x packages}}$

$$\mathbf{6\,x = 100}$$
$$\mathbf{x = 100/6 \text{ or } 16.7 \text{ pkg}}$$

(Technically, you could only buy **16 packages**, since they come in packages, and $20 is not enough to buy the 17th package.)

Example 12: **In a grocery store, a certain brand of peanut butter comes in two sizes. The large jar is 32 ounces and sells for \$2.99, while the smaller size is 18 ounces which sells for \$1.59. Which size is the better buy?**

Solution: Set up ratio of $\mathbf{\dfrac{cost}{ounce}}$ for each jar. The jar with the lowest cost per ounce is the best buy.

$$\mathbf{Lg: \frac{cost}{ounce} = \frac{\$2.99}{32} = \$0.0934}$$

$$\mathbf{Sm: \frac{cost}{ounce} = \frac{\$1.59}{18} = \$0.0833}$$

Therefore, the smaller size is actually the better buy.

Example 13: **A patient in the hospital is to receive 1200 mg. of a medication over the next 24 hours in an intravenous solution of 6 liters of saline.**

a) How many milligrams of medication should be administered per hour?
b) How many milliliters of saline should be used per hour?

Solution: a) Set up a ratio of mg of medication to hours.

$$\mathbf{\frac{mg.}{hrs.}: \qquad \frac{1200\,mg.}{24\,hrs.} = \frac{x\ \ mg.}{1\,hour}}$$

x = 1200/24 or **50 mg/hr.**

b) The first step is to convert 6 liters (move decimal three places to the right) to 6000 milliters of saline. Next, set up a ratio of ml. of medication to hours.

$$\mathbf{\frac{ml.}{hrs.}: \qquad \frac{6000\,mg.}{24\,hrs.} = \frac{x\ \ ml.}{1\,hour}}$$

x = 6000/24 or **250 ml/hr.**

Example 14: Thirty-five milligrams of a medication are to be dispensed in 300 milliliters of an intravenous saline solution.

a) How many milligrams of medication should be dispensed with 8 liters of saline solution?

b) How many milliliters (or liters) of saline should be used to dispense 500 milligrams of the medication?

Solution: a) Set up a ratio of milligrams of medication to milliliters of saline, noting that 8 liters is actually 8000 milliliters of saline solution.

$$\frac{\textbf{mg. medication}}{\textbf{ml. saline}} : \qquad \frac{\textbf{35 mg.}}{\textbf{300 ml.}} = \frac{\textbf{x mg.}}{\textbf{8000 ml.}}$$

$$300\text{ x} = 35\ (8000)$$
$$300\text{ x} = 280000$$
$$\text{x} = 280000/300 \text{ or } \mathbf{933.333...\ mg}$$

b) Using the same ratio,

$$\frac{\textbf{mg. medication}}{\textbf{ml. saline}} : \qquad \frac{\textbf{35 mg.}}{\textbf{300 ml.}} = \frac{\textbf{500 mg.}}{\textbf{x ml.}}$$

$$35\text{ x} = 500\ (300)$$
$$\text{x} = 150000/35 \text{ or } \mathbf{4285.72\ ml.}$$
$$\mathbf{4.3\ liters\ (approximately).}$$

Example 15: a) There are 16 ounces in a pint, 2 pints in a quart, and 4 quarts in a gallon. How many ounces are in 1 gallon?

b) How many milliliters are in 1 liter?

c) How many milliliters are in 1 kiloliter?

d) Which system, English or metric, is easier to work with and why?

Solution: a) Begin with 1 gallon of liquid. This gallon represents 4 quarts, each of which contains 2 pints, for a total of 8 pints. Now, each of the 8 pints contains 16 ounces, for a total of 8 x 16 or **128 ounces**.

b) **1000 milliliters**.

c) **1,000,000 milliliters**.

d) **Metric system**. All conversion units in the metric system are powers of 10.

Example 16: A chemical spray is to be diluted at a rate of 3 ounces of chemical to a gallon of water and then applied to 200 square feet of coverage. If there are 43,560 square feet per acre,

a) how many ounces of the chemical will be needed per acre of coverage?
b) how much water should be used to dilute it?
c) use the answer to the previous example (96 ounces per gallon) to express the number of ounces of chemical needed to spray an acre in gallons.

Solution: a) Set up a ratio comparing ounces of chemical to square feet of coverage.

$$\frac{\textbf{ounces}}{\textbf{sq. ft.}}: \qquad \frac{\textbf{3 oz.}}{\textbf{200 sq. ft.}} = \frac{\textbf{x oz.}}{\textbf{43560 sq. ft.}}$$

200 x = 3(43560)
200 x = 130680
x = 130680/200 or **653.4 ounces.**

b) Set up a ratio comparing ounces of chemical to gallons of water.

$$\frac{\textbf{ounces}}{\textbf{gal water}}: \qquad \frac{\textbf{3 oz.}}{\textbf{1 gal}} = \frac{\textbf{653.4 oz.}}{\textbf{x gal}}$$

3 x = 653.4
x = 653.4/3 or **217.8 gallons.**

c) Set up a ratio of ounces of chemical to gallons of chemical, using the answer to part a) of this problem.

$$\frac{\textbf{ounces}}{\textbf{gallons}}: \qquad \frac{\textbf{96 oz.}}{\textbf{1 gal}} = \frac{\textbf{653.4 oz.}}{\textbf{x gal}}$$

96 x = 653.4
x = 653.4/96 or 6.80625 gallons of chemical.
6.8 gallons (approximately).

Exercises: Write a proportion for each of the following. Solve the resulting equations.

1. If a 9-oz. bag of potato chips costs $1.50, what would you expect to pay for a 16 oz. bag?

2. If 8 pounds of dog food costs $6.50, what would you expect to pay for 25 pounds?

3. If a 20-oz. bottle of ketchup costs $0.90, what would you expect to pay for a 44 oz. bottle?

4. If 6 packages of a product cost $2.50, what would you expect to pay for 40 packages?

5. If on the interstate it takes 3 hours to travel 200 miles, how far can you travel at this rate in 16 hours?

6. If on the interstate it takes 3 hours to travel 200 miles, how long will it take to travel 750 miles at this rate?

7. If it takes 3 hours to drive 65 miles in the mountains, how long will it take to drive 100 miles at the same rate?

8. If it takes 35 minutes to drive 15 miles in the mountains, how far can you drive at this rate in 2 hours?

9. If a typist can type 5 pages in 13 hours, how long will it take him to type a 14-page report?

10. If a typist can type 13 pages in 5 hours, how many pages can she type in 14 hours?

11. If an author can complete 5 sections in 2 days, how many sections can she complete in 17 days?

12. If an author can complete 2 sections in 5 days, how long will it take him to complete 17 sections?

13. A gardening chemical is to be applied at 5 teaspoons per 300 sq. ft. How many teaspoons should be applied for 10,000 sq. ft.?

14. A gardening chemical is to be applied at 7 teaspoons per 300 sq. ft. How many sq. ft. can be treated by 100 teaspoons of the chemical?

15. A salt water brine is to be made at the rate of 3 pounds of salt for each 8 gallons of water. How much salt should be used for 25 gallons of water?

16. A salt water brine is to be made at the rate of 3 pounds of salt for each 8 gallons of water. How much water should be used with 25 pounds of salt?

17. A certain brand of dish detergent, which normally sells for $2.29 for in the 28-ounce size, is on sale for $2.19. The larger size is 42.7 ounces, selling for $3.19, but it is not on sale. Which is the better price, the smaller size on sale, or the larger size at the regular price?

18. There are 16 ounces in a pint, 2 pints in a quart, and 4 quarts in a gallon. How many ounces are in 1 gallon? (By the way, where did we get such strange units? Is there a better way? See Metric System section.)

19. A chemical spray is to be diluted at a rate of 3 ounces of chemical to a gallon of water and then applied to 200 square feet of coverage. If there are 43560 square feet per acre, how many ounces of the chemical will be needed per acre of coverage, and how much water should be used to dilute it?

20, Use the answers to the previous exercises to express the number of ounces of chemical in gallons.

ANSWERS TO EXERCISES

1. \$2.67;	**2.** \$20.31;	**3.** \$1.98;	**4.** \$16.67;
5. 1066.67 mi;	**6.** 11.25 hr;	**7.** 4.62 hr;	**8.** 51.43 mi;
9. 36.4 hr;	**10.** 36.4 pg;	**11.** 42.5 sec;	**12.** 42.5 days;
13. 166.67 tsp;	**14.** 4285.71 sq ft;	**15.** 9.375 lb;	**16.** 66.67 gal.

17. Larger size at regular price.