CALCULUS REVIEW from Differential Equations by Murray R. Speigel

REVIEW EXERCISES

1. Find dy/dx for each of the following functional relations.*

(a) $y = x^4 - 3x^2 + x + 5.$

(b) $y = x^2 + \dfrac{3}{\sqrt{x}} - \sqrt[3]{4x} + \dfrac{2}{x^5}.$

(c) $y = \dfrac{\sin 3x}{\sqrt{x}} + \cos 3x.$

(d) $y = x \sin 4x - \cos^2 2x.$

(e) $y = e^x \ln (x^2 + 1) - e^{-2x}.$

(f) $y = e^{-3x}(A \sin 4x + B \cos 4x).$

(g) $y = \sqrt{x^2 + 3x} - 2 \csc 4x + 3 \sec 2x.$

(h) $y = \ln (\sec 3x + \tan 3x).$

(i) $y = \dfrac{x}{2} \sqrt{1 - x^2} + \tfrac{1}{2} \arcsin x.$

(j) $y = e^{\arctan x^2} + 3 \sin (2 \arccos x).$

(k) $x^2 + y^2 = cx.$

(l) $x^3y - y^4 = 4x^2.$

(m) $x \sin y + y \cos 3x = 5.$

(n) $x^2 \arcsin 2y - 3xy + 1 = 0.$

(o) $e^{x^2y} + \arctan xy = 2x.$

(p) $x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = a.$

(q) $y \ln (x + y) + 2 \cos (x + y) = 3x^2.$

(r) $y \sin^2 3x - x[\ln y]^3 = 10.$

(s) $x \sec y - y \tan^2 x = \sqrt{2x + 1}.$

(t) $\sqrt{x^2 + y^2} + \arcsin y/x = 4.$

2. Find d^2y/dx^2 for each of the following.

(a) $y = 3 \sin 2x - 4 \cos 2x.$

(b) $y = e^{-x}(\sin x + \cos x).$

(c) $x \ln y = x + 1.$

(d) $x^{2/3} + y^{2/3} = 1.$

3. Given the following sets of parametric equations, find dy/dx and d^2y/dx^2.

(a) $x = 2t^2 - t,\ y = t^3 + t.$

(b) $x = 3 \sin t,\ y = 3 \cos t.$

(c) $x = a(\theta - \sin \theta),\ y = a(1 - \cos \theta).$

(d) $x = \dfrac{e^u - e^{-u}}{2},\ y = \dfrac{e^u + e^{-u}}{2}$

4. If $\dfrac{dy}{dx} = 1 + y^2$ show that $\dfrac{d^3y}{dx^3} = 2(1 + y^2)(1 + 3y^2).$

5. Find each of the indicated partial derivatives.

(a) $V = 2 \sin (x + 3y)$: $\dfrac{\partial V}{\partial x}, \dfrac{\partial V}{\partial y}, \dfrac{\partial^2 V}{\partial x^2}, \dfrac{\partial^2 V}{\partial y^2}, \dfrac{\partial^2 V}{\partial x\, \partial y}, \dfrac{\partial^2 V}{\partial y\, \partial x}, \dfrac{\partial^3 V}{\partial y^2\, \partial x}.$

(b) $U = e^{-\lambda y} \sin \lambda x,\ \lambda =$ constant: $\dfrac{\partial^2 U}{\partial x^2}, \dfrac{\partial^2 U}{\partial y^2}.$

* In these exercises, as well as in later ones, we shall assume unless otherwise specified, that letters at the beginning of the alphabet represent constants. We shall use the notation "ln" for "$\log_e$" where $e = 2.71828 \cdots$ is the base of natural logarithms. Also we shall assume unless otherwise stated that the domain of each function considered will be suitably chosen so that the functions are real, single-valued, continuous, and have continuous derivatives.

ANSWERS TO EXERCISES

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1. (a) $4x^3 - 6x + 1$. (b) $2x - \frac{3}{2}x^{-3/2} - \frac{4}{3}(4x)^{-2/3} - 10x^{-6}$.

(c) $3x^{-1/2}\cos 3x - \frac{1}{2}x^{-3/2}\sin 3x + 3\sin 3x$. (d) $4x\cos 4x + 3\sin 4x$.

(e) $\dfrac{2xe^x}{x^2+1} + e^x\ln(x^2+1) + 2e^{-2x}$.

(f) $e^{-3x}[(4A - 3B)\cos 4x - (3A + 4B)\sin 4x]$.

(g) $-4(x^2 + 3x - 2)^{1/2}\csc 4x\cot 4x + (x + \frac{3}{2})(x^2 + 3x - 2)^{-1/2}\csc 4x$
$+ 6\sec 2x\tan 2x$.

(h) $3\sec 3x$. (i) $\sqrt{1-x^2}$. (j) $\dfrac{2xe^{\arctan x^2}}{1+x^4} - \dfrac{6\cos(2\arccos x)}{(1-x^2)^{1/2}}$.

(k) $\dfrac{c-2x}{2y}$. (l) $\dfrac{8x-3x^2y}{x^3-4y^3}$. (m) $\dfrac{3y\sin 3x - \sin y}{x\cos y + \cos 3x}$.

(n) $\sqrt{1-4y^2}\left(\dfrac{3y - 2x\arcsin 2y}{2x^2 - 3x\sqrt{1-4y^2}}\right)$. (o) $\dfrac{(2 - 2xye^{x^2y})(1 + x^2y^2) - y}{x^2(1 + x^2y^2)e^{x^2y} + x}$.

(p) $-\sqrt{\dfrac{1-y^2}{1-x^2}}$. (q) $\dfrac{6x(x+y) + 2(x+y)\sin(x+y) - y}{y + (x+y)\ln(x+y) - 2(x+y)\sin(x+y)}$.

(r) $\dfrac{y(\ln y)^3 - 6y^2\sin 3x\cos 3x}{y\sin^2 3x - 3x(\ln y)^2}$.

(s) $\dfrac{1 - \sqrt{2x+1}\sec y + 2y\sqrt{2x+1}\tan x\sec^2 x}{\sqrt{2x+1}(x\sec y\tan y - \tan^2 x)}$.

(t) $\dfrac{y\sqrt{x^2+y^2} - x^2\sqrt{x^2-y^2}}{x\sqrt{x^2+y^2} + xy\sqrt{x^2-y^2}}$.

2. (a) $16\cos 2x - 12\sin 2x$. (b) $2e^{-x}(\sin x - \cos x)$. (c) $y[(\ln y)^2 - 1]/x^2$.

(d) $\frac{1}{3}x^{-4/3}y^{-1/3}$.

3. (a) $\dfrac{3t^2+1}{4t-1}, \dfrac{12t^2-6t-4}{(4t-1)^3}$. (b) $-\tan t, -\frac{1}{3}\sec^3 t$.

(c) $\dfrac{\sin\theta}{1-\cos\theta}, \dfrac{-1}{a(1-\cos\theta)^2}$. (d) $\dfrac{e^u - e^{-u}}{e^u + e^{-u}}, \dfrac{8}{(e^u + e^{-u})^3}$.

5. (a) $2\cos(x+3y), 6\cos(x+3y), -2\sin(x+3y), -18\sin(x+3y)$,
$-6\sin(x+3y), -6\sin(x+3y), -18\cos(x+3y)$.

(b) $-\lambda^2e^{-\lambda y}\sin\lambda x, \lambda^2e^{-\lambda y}\sin\lambda x$. (c) $\dfrac{2x}{8z+y}, \dfrac{-z}{8z+y}, \dfrac{8z^2+2yz}{(8z+y)^3}$.

(d) $\dfrac{10z+1}{4x^3y}, -\dfrac{x}{4y}$. (e) $\dfrac{2(y^2+z^2-x^2)}{(x^2+y^2+z^2)^2}, \dfrac{2(x^2+z^2-y^2)}{(x^2+y^2+z^2)^2}, \dfrac{2(x^2+y^2-z^2)}{(x^2+y^2+z^2)^2}$.