

CALCULUS

in

DIFFERENTIAL EQUATIONS

Dr. Robert J. Rapalje

I. DIFFERENTIATION: [Answers next page.]

1. If $1 + x^2y + 4y = 0$, find y'' .
2. If $y = Ae^{5x} + Be^{-2x} - \frac{1}{2}e^x$, find y'' .
3. If $y = 3x^2 + ce^{-2x}$, show that $y' = 6x^2 + 6x - 2y$.

II. INTEGRATION: [Answers next page.]

1. $\int \frac{3x}{x^2+2} dx$
2. $\int \frac{d\theta}{\theta^2+4} + \int \frac{d\theta}{\theta} + \int \frac{\theta d\theta}{\theta^2+4}$
3. $\int \frac{x dx}{\sqrt{1+x^2}}$
4. $\int \frac{dx}{\cos^2 x}$
5. $\int \frac{e^r dr}{1+e^{2r}}$
6. $\int \frac{\sin \theta}{3 + \cos 2\theta} d\theta$ [Hint: Use $\cos 2\theta = 2\cos^2 \theta - 1$]
7. $\int \cos^2 v dv$
8. Show that a) $\int e^{-\frac{2}{y}} dy = \frac{1}{y^2}$
b) $\int e^{2\cot y} dy = \frac{1}{\sin^2 y}$
9. $\int -y^3 e^{-y^2} dy$
10. $\int \frac{(2-v) dv}{v^2+3v+2}$

III. PARTIAL DERIVATIVES AND INTEGRATION: [Answers next page.]

1. If $m(x,y) = (x^2 - y^2)$, find $\frac{\partial m}{\partial x}$, $\frac{\partial m}{\partial y}$
2. If $m(x,y) = -\sin x - y$, find $\frac{\partial m}{\partial x}$, $\frac{\partial m}{\partial y}$
3. If $m(x,y) = 3x^2 + 2xy^2$, find $\frac{\partial m}{\partial x}$, $\frac{\partial m}{\partial y}$
4. If $\frac{\partial u}{\partial x} = x^2 - y^2$, find $u(x,y)$
5. If $\frac{\partial u}{\partial y} = -\sin x - y$, find $u(x,y)$
6. If $\frac{\partial u}{\partial x} = xy^2 + 2x^3$, find $u(x,y)$
7. If $\frac{\partial u}{\partial y} = \cos^2 x$, find $u(x,y)$. Show that an alternate form of the answer could be $\frac{y \cos 2x}{2} - \frac{y}{2} + f(x)$.
8. If $\frac{\partial u}{\partial x} = 2x - \frac{y}{x^2} - \frac{y^2}{x}$, find $u(x,y)$
9. If $\frac{\partial u}{\partial x} = (y^2 + xy + 1)e^{xy}$, find $u(x,y)$ [Hint: Must use integration by parts or a formula for $\int ue^u du$]

REQUIREMENTS FOR DIFFERENTIAL EQUATIONS

1. Trigonometric, Logarithmic, and Exponential Functions
2. Differentiation of all functions, including the above.
3. All techniques of integration, including the above.
4. Partial derivatives. See Sections 17.1, 17.2, 17.9, 17.13,
Also good practice in 17.8, 17.14, 17.15, 17.16.
[Sections refer to our Calculus text: Calculus with Analytic Geometry, by Proter & Murray, 3rd Edition]
5. Determinants. (Only one reference to these in D.E.)
See Appendix 5 of Calculus text.
6. Time to do homework every day (12 to 20 hours
per week).
7 week term.

ANSWERS TO CALCULUS in DIFFERENTIAL EQUATIONS

- | | |
|--|--|
| <p>I</p> <ol style="list-style-type: none"> 1. $y'' = \frac{2(4-3x^2)}{(x^2+4)^3}$ 2. $y'' = 25Ae^{5x} + 4Be^{-2x} - \frac{1}{2}e^x$ <p>II</p> <ol style="list-style-type: none"> 1. $\frac{3}{2} \ln(x^2+2) + C$ 2. $\frac{1}{2} \arctan \frac{\theta}{2} + \ln \theta + \frac{1}{2} \ln(\theta^2+4) + C$ 3. $\sqrt{1+x^2} + C$ 4. $\tan x + C$ 5. $\arctan(e^r) + C$ 6. $-\frac{1}{2} \arctan(\cos \theta) + C$ 7. $\frac{1}{2}v + \frac{1}{4} \sin 2v + C$ 8. $\frac{1}{2}(y^2+1)e^{-y^2} + C$ 9. $\ln \frac{(v+1)^3}{(v+2)^4}$ | <p>III</p> <ol style="list-style-type: none"> 1. $\frac{\partial M}{\partial x} = 2x; \frac{\partial M}{\partial y} = -2y$ 2. $\frac{\partial M}{\partial x} = 1 - y \cos x; \frac{\partial M}{\partial y} = -\sin x$ 3. $\frac{\partial M}{\partial x} = 6x + 2y^2; \frac{\partial M}{\partial y} = 4xy$ 4. $u(x,y) = \frac{x^3}{3} - xy^2 + f(y)$ 5. $u(x,y) = -y \sin x - \frac{y^2}{2} + f(x)$ 6. $u(x,y) = \frac{x^2 y^2}{2} + \frac{x^4}{2} + f(y)$ 7. $u(x,y) = y \cos^2 x + f(x)$ 8. $u(x,y) = x^2 + \frac{y}{x} - y^2 \ln x + f(y)$ 9. $u(x,y) = (y+x)e^{xy} + f(y)$ |
|--|--|