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MAC 1140 EXAM 4 A **NAME** ______________________

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. Circle answers.
TURN IN ALL WORKSHEETS. CALCULATORS ARE REQUIRED ON THIS TEST.

Simplify each of the following:

1. $\frac{50!}{40!} =$ 2. $\frac{(X+2)!}{X!} =$

In 3 - 4, give the next three terms of the sequence of numbers.
Write an expression for the nth term, where n begins n=1:

3. 0, 3, 8, 15, _____, _____, _____ $a_n =$ ____________

4. 2, -4, 6, -8, _____, _____, _____ $a_n =$ ____________

In 5 - 6, find the sum:

5. $\sum_{i=1}^{6} (2i - 3) =$ 6. $\sum_{i=0}^{4} 2^i =$

Given the formula $S = \frac{n}{2}(a_1 + a_n)$:

7. Find an expresssion for a_n, and find the sum of the first 100 terms:
2, 8, 14, 20, . . .

8. A brick patio is built with rows of bricks such that there are 24 bricks in the first row, 25 bricks in the second row, 26 bricks in the third row, etc. If there are 20 rows of bricks, how many bricks are there in the last (20th) row, and how many total bricks are in the patio?

In 9 - 12, solve for the unknown:

9. $\log_9 27 = X$ 10. $\log_b 3 = 3$ 11. $\log_2 8\sqrt{2} = X$

12. $\log_{10} 0 =$ ______ 13. $\log_b 16 = -2$ 14. $\log_8 X = -\frac{2}{3}$

In 15 - 19, simplify completely:

15. $\ln e^{X^2} =$ ______ 16. $e^{\ln 10} =$ ______ 17. $\ln(\sqrt[3]{e}) =$ ______

18. $\log_{10} 10^{-3} =$ ______ 19. $\log_5 7 =$ ______

20. $\ln(e^3 + e^3) =$ 21. $\log_5 \frac{25}{\sqrt[3]{5}} =$

In 22 - 23, solve for X:

22. $9^{(X+2)} = 16^{(3X-4)}$ 23. $\log_5 (X+4) - \log_5 X = -2$

24. $e^{(3X+2)} = 16$ 25. Given $A = P\left(1 + \frac{r}{n}\right)^{nt}$, solve for r in terms of the other variables.

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26. Given that $\log_b 2 = 0.3562$, $\log_b 3 = 0.5646$, and $\log_b 5 = 0.8271$, use the laws of logarithms to find:

a) $\log_b 12$

b) $\log_b 32$

c) determine the base "b" (Explain how you did it!)

27. The population of a rabbit farm is given by $Y = 450\ e^{0.06t}$, where t is in <u>*years*</u>.

a) Estimate the population in 20 years.

b) How long will it take the population to triple?

28. The population of a city in 1990 was 30,000. In 1993, the population was 52,000.

a) Assuming that $Y = Y_0e^{kt}$, find the value of k.

b) Use this value of k to predict the population of the city in 1998.

29. The learning curve for the number of units *N* produced per day after a new employee has worked *t* days is given by the formula $N = 30(1 - e^{kt})$, where *k* is a constant. After 20 days on the job a worker produces 19 units per day.

a) Find the value of *k*.

b) Sketch the graph.

c) Find the maximum N.

d) How many days pass before the worker produces 25 units per day?

MAC 1140 Exam 4A Solutions

1. $\frac{50!}{40!} = \boxed{3.728 \times 10^{16}}$

2. $\frac{(x+2)!}{x!} = \frac{(x+2)(x+1)\cancel{(x!)}}{\cancel{x!}} = \boxed{(x+2)(x+1)}$

3. $\overset{1}{0},\ \overset{2}{3},\ \overset{3}{8},\ \overset{4}{15},\ \boxed{\overset{5}{24},\ \overset{6}{35},\ \overset{7}{48}}$

$\boxed{a_n = n^2 - 1}$

4. $\overset{1}{2}\ \overset{2}{-4},\ \overset{3}{6},\ \overset{4}{-8}\ \boxed{\overset{5}{10},\ \overset{6}{-12},\ \overset{7}{14}}$

$\boxed{a_n = (-1)^{n-1}(2n) \text{ or } (-1)^{n+1}(2n)}$

5. $\sum_{i=1}^{6}(2i-3)$

$= \overset{1}{(-1)} + \overset{2}{1} + \overset{3}{3} + \overset{4}{5} + \overset{5}{7} + \overset{6}{9}$

$= \boxed{24}$

6. $\sum_{i=0}^{4} 2^i = \overset{0}{1} + \overset{1}{2} + \overset{2}{4} + \overset{3}{8} + \overset{4}{16}$

$= \boxed{31}$

7. $\overset{1}{2},\ \overset{2}{8},\ \overset{3}{14},\ \overset{4}{20}, \ldots$

Arithmetic progression $d = 6$

$a_1 = 2,\ a_2 = 2+6,\ a_3 = 2+12,\ a_4 = 2+18$

$\boxed{a_n = 2 + (n-1)\cdot 6}$ or $\boxed{a_n = 6n-4}$

$a_{100} = 600 - 4 = 596$

$S = \frac{n}{2}(a_1 + a_n)$

$= \frac{100}{2}(2 + 596) = \boxed{29900}$

8. $\overset{1}{24} + \overset{2}{25} + \overset{3}{26} + \overset{\ldots}{\ldots} + \overset{20}{43}$

$S = \frac{20}{2}(24+43) = \boxed{670 \text{ Bricks}}$

9. $\log_9 27 = x$

$9^x = 27$

$3^{2x} = 3^3$

$2x = 3$

$\boxed{x = 3/2}$

10. $\log_b 3 = 3$

$b^3 = 3$

$\boxed{b = \sqrt[3]{3}}$

1.4422

11. $\log_2 8\sqrt{2} = x$

$2^x = 8\sqrt{2}$

$2^x = 2^3 \cdot 2^{1/2}$

$\boxed{x = 7/2}$

12. $\log_{10} 0 = \boxed{\text{undefined}}$

13. $\log_b 16 = -2 \quad (b > 0)$

$b^{-2} = 16 \qquad b^2 = \frac{1}{16}$

$\frac{1}{b^2} = \frac{16}{1} \qquad \boxed{b = \frac{1}{4}}$

14. $\log_8 x = -2/3$

$8^{-2/3} = x$

$\left(\sqrt[3]{8}\right)^{-2} = x$

$x = 2^{-2} = \boxed{\frac{1}{4}}$

15. $\ln e^{x^2} = \boxed{x^2}$

16. $e^{\ln 10} = \boxed{10}$

17. $\ln \sqrt[3]{e} = \ln e^{1/3} = \boxed{1/3}$

18. $\log_{10} 10^{-3} = \boxed{-3}$

19. $\log_5 7 = \frac{\ln 7}{\ln 5} = \boxed{1.210}$

20. $\ln(e^3 + e^3) = \ln(2e^3)$

$= \ln 2 + \ln e^3$

(calculator)

$= 3 + \ln 2$

$= \boxed{3.693}$

21. $\log_5 \frac{25}{\sqrt[3]{5}} = \log_5 25 - \log_5 \sqrt[3]{5}$

$= \log_5 5^2 - \log_5 5^{1/3}$

$= 2 - 1/3 = \boxed{5/3 \text{ or } 1.667}$

Also: $\dfrac{\ln\left(\frac{25}{\sqrt[3]{5}}\right)}{\ln 5}$

22. $9^{x+2} = 16^{(3x-4)}$

$\ln 9^{(x+2)} = \ln 16^{(3x-4)}$

$(x+2)\ln 9 = (3x-4)\ln 16$

$x \ln 9 - 2\ln 9 = 3x \ln 16 - 4\ln 16$

$4\ln 16 + 2\ln 9 = 3x\ln 16 - x\ln 9$

$4\ln 16 + 2\ln 9 = x(3\ln 16 - \ln 9)$

$x = \frac{(4\ln 16 + 2\ln 9)}{(3\ln 16 - \ln 9)} \approx \boxed{2.530}$

Easier to let

$y = 9^{(x+2)} - 16^{(3x-4)}$

$\boxed{\text{ROOT}}$

$\boxed{x = 2.530}$

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23. $\log_5 (x+4) - \log_5 x = -2$.

$\log_5 \frac{x+4}{x} = -2$

$5^{-2} = \frac{x+4}{x}$

$\frac{1}{25} = \frac{x+4}{x}$

$x = 25x + 100$

$-24x = 100$

$\cancel{x = -\frac{25}{6}}$

$\boxed{\text{No Solution}}$

Reject log of negative

24. $e^{(3x+2)} = 16$

$\ln e^{3x+2} = \ln 16$

$3x+2 = \ln 16$

$3x = \ln 16 - 2$

$x = \boxed{\frac{\ln 16 - 2}{3}} \approx \boxed{.258}$

25. $A = P\left(1+\frac{r}{n}\right)^{nt}$

$\frac{A}{P} = \left(1+\frac{r}{n}\right)^{nt}$ or $\ln\left(\frac{A}{P}\right) = \ln\left(1+\frac{r}{n}\right)^{nt}$

$\sqrt[nt]{\frac{A}{P}} = 1 + \frac{r}{n}$

$\frac{r}{n} = \sqrt[nt]{\frac{A}{P}} - 1$

$\boxed{r = n\left(\sqrt[nt]{\frac{A}{P}} - 1\right)}$

$\ln\left(\frac{A}{P}\right) = nt \ln\left(1+\frac{r}{n}\right)$

$\ln\left(1+\frac{r}{n}\right) = \frac{1}{nt} \ln\left(\frac{A}{P}\right)$

$1 + \frac{r}{n} = e^{\left[\frac{1}{nt}\ln\left(\frac{A}{P}\right)\right]}$

$\frac{r}{n} = e^{[\quad]} - 1$

$\boxed{r = n\left[e^{\frac{1}{nt}\ln\left(\frac{A}{P}\right)} - 1\right]}$

26. $\log_b 2 = 0.3562$

$\log_b 3 = 0.5646$

$\log_b 5 = 0.8271$

a) $\log_b 12 = \log_b 2^2 \cdot 3$

$= 2\log_b 2 + \log_b 3$

$= 2(0.3562) + (0.5646)$

$= \boxed{1.277}$

b) $\log_b 32 = \log_b 2^5$

$= 5 \log_b 2$

$= 5(0.3562)$

$= \boxed{1.781}$

c) $\log_b 2 = 0.3562$

$b^{.3562} = 2$ also $\sqrt[.3562]{2}$

Use $\boxed{\text{ROOT}}$ $\boxed{b \approx 7}$

27a) $y = 450e^{.06(20)}$

$= \boxed{1494}$

b) $3y_0 = y_0 e^{.06t}$

$3 = e^{.06t}$

$\ln 3 = \ln e^{.06t}$

$t = \frac{\ln 3}{.06} = \boxed{18.31 \text{ yrs}}$

28. $y = y_0 e^{kt}$

$52\,000 = 30,000 e^{3k}$

$\frac{52}{30} = e^{3k}$

$\ln \frac{52}{30} = \ln e^{3k} = 3k$

$k = \frac{\ln \frac{52}{30}}{3} \approx .183348778973$

$y = 30,000\, e^{(\text{ANS}).8} \approx \boxed{130,059}$

29d) $25 = 30(1 - e^{kt})$

$25 = 30 - 30e^{kt}$

$30e^{kt} = 5$

$e^{kt} = \frac{1}{6}$

$kt = \ln \frac{1}{6}$

$t = \frac{\ln \frac{1}{6}}{k} = 35.7$

$\boxed{\approx 36 \text{ days}}$

29. $N = 30(1 - e^{kt})$

$19 = 30(1 - e^{20k})$

$= 30 - 30e^{20k}$

$30e^{20k} = 11$

$e^{20k} = \frac{11}{30}$

$\ln e^{20k} = \ln \frac{11}{30}$

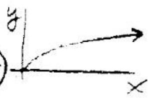
$20k = \ln\left(\frac{11}{30}\right)$

$k = \frac{\left(\ln \frac{11}{30}\right)}{20} \approx \boxed{-.05}$

$= -.050165105443$

b) Set $x = 0$ to 100

$y = 0$ to 50



c) MAX = 29.8

$x = 0$ to 100

$\boxed{\text{MAX} = 50}$