2.01 *Factoring, Factoring, Factoring!*

PART I:Common Factor, Trinomial, Diff Squares, Diff/Sum of Cubes

Dr. Robert J. Rapalje

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

Of all the topics that you have studied in previous algebra courses, there is no topic more important, there is no topic that you need to review more than the topic of **factoring.** Let's begin with a working definition of factoring. What does it mean **to factor** something? What would you say if you were asked to **"factor the number 15"**? You would probably answer, **"3 times 5"** or **"5 times 3"**! The key word is **"times."** When asked to factor a given number, you answer with a product of two numbers.

DEFINITION

To FACTOR means to EXPRESS AS A PRODUCT!

Factoring is an important skill that goes all the way back to your first algebra course, and it will continue to be a most important skill, especially in calculus. There are many types of factoring that become more complex as well as more abstract in the higher math. Some of the exercises in this section will begin with a review of simple factoring exercises, such as $x^2 - 9$ or $x^2 - 6x$, and then progress into higher levels of factoring. Notice the increase in complexity and abstraction as you "grow" through this "one step at a time."

FACTORING THE COMMON FACTOR

The first step in any factoring problem should be to try to "factor the common factor." Factoring the common factor is simply using the distributive property in reverse.

EXAMPLES DISTRIBUTIVE PROPERTY:

$6(x + 7) = 6x + 42$

$7(2x + 3) = 14x + 21$

$9(3x - 4y) = 27x - 36y$

$12(2x + 1) = 24x + 12$

$5(3x - 2y + 4) = 15x - 10y + 20$

$5x\ (x + 4) = 5x^2 + 20x$

EXAMPLES OF FACTORING:

$6x + 42 = 6(x + 7)$

$14x + 21 = 7(2x + 3)$

$27x - 36y = 9(3x - 4y)$

$24x + 12 = 12(2x + 1)$

$15x - 10y + 20 = 5(3x - 2y + 4)$

$5x^2 + 20x = 5x\ (x + 4)$

When factoring the common factor, look for a number or variable that divides into both (or all) terms. If there is more than one common factor, be sure to get the largest common factor you can find. First write down the common factor. Then, open parentheses, and put down all the other factors that are left over. Again, follow the "one step" format of the following exercises.

EXERCISES: Factor completely.

1. $5x^2 + 15x$
   $= 5x\ (\qquad)$

2. $18x + 24y$
   $= 6(\qquad)$

3. $35xy + 7y$
   $= 7y\ (\qquad)$

4. $5x^3 - 45x^2$
   $= 5x^2(\qquad)$

5. $16a + 24b - 8$
   $= 8(\qquad)$

6. $12x - 24y + 48$
   $= 12(\qquad)$

7. $x^3 + 4x^2$

8. $4y^3 + 8y$

9. $16a^3 - 24a^2$

10. $12Z^3 - 18$

11. $24x^2 + 12$

12. $16b^2 + 48b^3$

13. $24x^3 + 24x^2$

14. $16x^2 - 32x^3$

15. $8a + 12b - 20c$

16. $40x - 32y + 64$

17. $42y^3 - 14y^2 + 49y$

18. $24x^3 + 24x^2 + 24x$

19. $19x^3 + 19x^2y + 38x^2$

20. $17x^3 - 34x^2$

21. $y^5 - 14y^3$

$= y^3(\qquad)$

22. $x^{10} + 5x^3$

$= x^3(\qquad)$

From these examples, observe the rule listed below!

RULE

When factoring powers, take out the lowest power of the factor. Then subtract exponents.

23. $y^{10} + 7y^4$

24. $x^7 + 8x^5$

25. $16x^2y^3 - 12x^3y^2$

$= 4x^2y^2(\qquad)$

26. $5x^5y^2 + 10x^4y^3$

27. $8x^5y^3 + 12x^3y^4$

28. $36x^3y^4 + 24x^2y^6$

In each of the next exercises, observe how you move from the simple to the more complicated; from the concrete to the abstract.

29a) $yx + 7x$

$= x(\quad)$

b) $ya + 7a$

$= a(\quad)$

c) Y\$ + 7\$

= \$(   )

d) y(Junk) + 7(Junk)

= (Junk) (   )

e) $y(3x+4) + 7(3x+4)$

$= (3x+4)(\quad)$

30a) $4xy + 3y$

b) $4xa + 3a$

c) $4x$\$ + 3\$

d) $4x$(Junk) + 3(Junk)

e) $4x(8y-7) + 3(8y-7)$

31. $a(3x+4) - 5(3x+4)$

32. $5q(8r+7) + 3(8r+7)$

33. $5u(3x+4) + 9v(3x+4)$

34. $10X(8y-7) - 3(8y-7)$

RULE

In order to factor a common factor, you must have an identical factor common to all terms. Be sure to count terms first.

EXAMPLE: Can you factor $5u(3x+4) + 9v(3x-4)$ in this manner?

NO! There is no factor common to both terms.

35. $x(x-y) - y(x-y)$

36. $x(x-y) - y(x-y) + 4(x-y)$

37. $x(x-y) + y(x-y) - 4(x-y)$

38. $x(2x+3y) - y(2x+3y) + 4(2x+3y)$

39. $(x+y)^2 - z(x+y)$

$= (\quad)[(\quad) - \_\_\_]$

$= (\quad)(\quad)$

40. $(x-y)^2 - z(x-y)$

41. $(x-y)^2 - y(x-y)$

42. $(x+y)^2 - y(x+y)$

43. $(2x+3y)^2 - 5(2x+3y)$

44. $(2x-3y)^2 - 5(2x-3y)$

45. $(2x+3y)^2 + 5(2x+3y)$

46. $(x+y)^2 + (x-y)(x+y)$

47. $(3x-2y)^2 - 2(3x-2y)(x-5y)$

48. $(5x+3y)^2 - 4(5x+3y)(x+3y)$

TRINOMIALS

Do you remember the product of two binomials, **F OI L**, and the fact that the result is usually in a **trinomial**? As examples, consider:

Product Binomials	F	OI	L
$(x + 2)(x + 5) =$	x^2	$+ 7x$	$+ 10$
$(x + 2)(x - 5) =$	x^2	$- 3x$	$- 10$
$(x - 2)(x + 5) =$	x^2	$+ 3x$	$- 10$
$(x - 2)(x - 5) =$	x^2	$- 7x$	$+ 10$

In each of these examples, you were given a **product** of two **binomials**, and with **F OI L**, in each case you obtained a **trinomial** with x^2. Now the problem will be to work these problems in reverse. What if you were given a **trinomial**, such as $x^2+7x+10$ and asked **to factor** it--that is, **to express it as a product.** This is the product of the two **binomials.** When you factor the trinomial: $x^2 + 7x + 10$

you expect the product of binomials: **( )( ).**

Also, when factoring a trinomial, instead of thinking **F OI L**, you need to change the order and think **F L OI.** In other words, you need to find the correct **F** (first times first) combination, then skip to the **L** (last times last). Finally, check to make sure the **OI** (outer times outer, inner times inner) **middle term** is correct.

F OI L	
$x^2 + 7x + 10$	**Given trinomial to factor;**
$(\quad)(\quad)$	Product of two **binomials;**
$(x \quad)(x \quad)$	**F term is** x^2, which is x times x;
$(x \quad)(x \quad)$	**L term is +10.** Find two numbers whose product is **+10.** Probably 2 times 5, or it could be 1 times 10. Try 2 times 5. Since the **last sign** is **positive**, it will be positive times positive, or negative times negative. Also, the middle terms **(O and I)** must be **added** together.
$(x + 2)(x + 5)$	**OI term is** $7x$. This means the **outer times outer** and the **inner times inner** terms must **add up** to $7x$.

[NOTICE: The order does not matter! If you wrote $(x+5)(x+2)$, this is exactly equivalent to $(x + 2)(x + 5)$. **WHY??**]

The following is a helpful summary of **trinomial (F L OI) factoring.**

RULES:

1. **When the sign of the LAST term is positive, the signs are the SAME. Find middle term by ADDING the O and I terms.**

2. **When the sign of the LAST term is negative, the signs are OPPOSITE. Find middle term by SUBTRACTING the O and I terms.**

It is also worth noting that not all trinomials can be factored. For examples, the trinomials $x^2 + x + 2$, $x^2 + 2x + 6$, and $x^2 - 4x - 6$ cannot be factored. These are called **prime trinomials.**

EXERCISES. Factor each of the following trinomials.

1. $x^2 + 3x + 2$	2. $x^2 + 9x + 14$	3. $x^2 + 7x + 10$
4. $x^2 -12x + 35$	5. $x^2 - 13x + 22$	6. $x^2 - 9x + 18$
7. $x^2 + 3x - 40$	8. $x^2 + 18x - 40$	9. $x^2 + 39x - 40$
10. $x^2 - 15x - 16$	11. $x^2 - 6x - 16$	12. $x^2 - x - 72$
13. $x^2 - 7x - 18$	14. $x^2 + 3x - 18$	15. $x^2 - 19x + 60$
16. $x^2 - 6x - 27$	17. $x^2 + 2x - 24$	18. $x^2 - 5x - 24$

19. $x^2 - 32x + 60$ 20. $x^2 + 11x - 60$ 21. $x^2 + 61x + 60$

22. $x^2 - 23x + 60$ 23. $x^2 - 17x - 60$ 24. $x^2 - 59x - 60$

25. $x^2 - x - 56$ 26. $x^2 - x - 72$ 27. $x^2 + 17x + 72$

28. $x^2 + 17x + 42$ 29. $x^2 - 19x - 42$ 30. $x^2 - 23x + 42$

Frequently, it is necessary to **FACTOR THE COMMON FACTOR FIRST (FCFF).** When there is a common factor in the problem, always remember to **FCFF**! **(NOTE: These exercises require TWO steps!)**

EXERCISES: Factor the trinomials completely. Be sure to FCFF!!

31. $3x^2 + 6x - 9$ 32. $5x^2 + 15x - 20$ 33. $8x^2 + 8x - 48$

$3(X^2 + 2X - 3)$ 5(   ) ___(   )

(   )(   ) 5(   )(   ) ___(   )(   )

34. $6x^2 + 6x - 12$ 35. $12x^2 + 36x - 120$ 36. $10x^2 + 30x - 100$

___(   )

___(   )(   )

37. $x^3 - 4x^2 - 5x$ 38. $x^3 + 5x^2 + 6x$ 39. $2x^3 - 14x^2 + 20x$

40. $5x^3 + 5x^2 - 10x$ 41. $7x^3 + 49x^2 + 42x$ 42. $8x^3 - 40x^2 + 32x$

43. $x^4 + x^3 - 20x^2$ **44.** $x^4 + 2x^3 - 35x^2$ **45.** $15x^4 - 45x^3 - 60x^2$

46. $20x^4 - 20x^3 - 120x^2$ **47.** $30x^4 + 90x^3 - 300x^2$ **48.** $18x^4 + 54x^3 + 36x^2$

In the factoring of trinomials of the previous exercises, it may have been assumed that either the coefficient of x^2 is 1 or that the coefficient can be factored as a common factor of the entire trinomial. However, consider the example $5x^2 + 6x + 1$. Remember that this should be factored by **F L OI**, where the **F** term must be $5X^2$, **L** must be **1**, and the **OI** term must add up to $6x$, as follows:

$$5x^2 + 6x + 1$$

$$(\qquad)(\qquad)$$

Try the following examples (solutions are given below):

$3x^2 + 4x + 1$ $8x^2 + 9x + 1$ $8x^2 + 6x + 1$

$(\qquad)(\qquad)$ $(\qquad)(\qquad)$ $(\qquad)(\qquad)$

Solutions:

$(3x + 1)(x + 1)$ $(8x + 1)(x + 1)$ $(4x + 1)(2x + 1)$

Of course, with larger numbers, with many more combinations of numbers this can become a very lengthy process of trial and error. There are some systematic methods of factoring these trinomials, one of which will be presented at the end of the "Factoring by Grouping" section. In problems that are not too difficult, the trial and error method will be fairly simple and more than adequate for now.

Consider the examples $5x^2 + 8x + 3$ and $5x^2 + 16x + 3$. Again, these are factored by **F L OI**, where the **F** term must be $5x^2$, **L** must be **3**, and the **OI** term must add up to $8x$ and $16x$ respectively, as follows:

$$5x^2 + 8x + 3 \qquad\qquad 5x^2 + 16x + 3$$

$$(\quad)(\quad) \qquad\qquad (\quad)(\quad)$$

Try the following examples (again solutions are given below):

$$3x^2 + 10x + 7 \qquad 3x^2 + 22x + 7 \qquad 3x^2 + 4x - 7$$

$$(\quad)(\quad) \qquad (\quad)(\quad) \qquad (\quad)(\quad)$$

Solutions:

$(3x + 7)(x + 1)$ $(3x + 1)(x + 7)$ $(3x + 7)(x - 1)$

REMEMBER: 1. **When the L term is *positive*, add the O and I terms.**

2. **When the L term is *negative*, subtract the O and I terms.**

EXERCISES:

1. $3x^2 + 4x + 1$
2. $4x^2 + 5x + 1$
3. $7x^2 - 8x + 1$

4. $3x^2 + 2x - 1$ 5. $4x^2 + 3x - 1$ 6. $7x^2 + 6x - 1$

7. $10x^2 - 7x + 1$ 8. $10x^2 - 9x - 1$ 9. $10x^2 + 3x - 1$

10. $3x^2 + 8x + 5$ 11. $3x^2 + 16x + 5$ 12. $3x^2 + 2x - 5$

13. $3x^2 - 14x - 5$ 14. $3x^2 - 34x + 11$ 15. $3x^2 + 8x - 11$

16. $5x^2 + 3x - 8$ 17. $5x^2 + 39x - 8$ 18. $5x^2 + 6x - 8$

19. $6x^2 - 19x + 8$ 20. $6x^2 - 13x - 8$ 21. $6x^2 + 13x - 8$

22. $6x^2 + 19x + 10$ 23. $6x^2 + 11x - 10$ 24. $6x^2 + 17x + 10$

DIFFERENCE OF SQUARES; PERFECT SQUARE TRINOMIALS

EXAMPLES: **GENERALIZATION:**

$$x^2 - 25 = (x - 5)(x + 5)$$

$$x^2 + 10x + 25 = (x + 5)(x + 5) = (x + 5)^2$$

$$x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$$

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$$

1. $x^2 - 64$

$(x - \_\_)(x + \_\_)$

2. $x^2 - 100$

3. $x^2 - 81$

4. $x^2 - 169$

5. $x^2 - a^2$

6. $x^2 - b^2$

7. $4x^2 - 9$

$(2x - \_\_\_)(2x + \_\_\_)$

8. $64x^2 - 121$

9. $16x^2 - 49$

10. $81x^2 - 25y^2$

11. $49x^2 - 36y^2$

12. $25x^2 - 144a^2$

Don't forget that the first step in any factoring problem is to **factor the common factor first.** This means that each of the following exercises will require **two steps.** This is known as the "factoring two-step."

13. $9x^2 - 9$

9(   )

9(   )(   )

14. $3x^2 - 12$

3(   )

3(   )(   )

15. $5x^2 - 45$

___(   )

___(   )(   )

16. $4x^2 - 64$

17. $4x^2 - 100$

18. $8x^2 - 72$

19. $3x^3 - 75x$ **20.** $5x^3 - 80x$ **21.** $2x^3 - 50x$

22. $64x^4 - 4x^2y^2$ **23.** $12y^4 - 12x^2y^2$ **24.** $79y^4 - 79x^2y^2$

Factor each of the following perfect square trinomials:

25. $x^2 + 4x + 4$ **26.** $x^2 + 14x + 49$ **27.** $x^2 + 20x + 100$

$= (\quad)(\quad)$

$= (\quad)^2$

28. $x^2 - 12x + 36$ **29.** $x^2 - 18x + 81$ **30.** $x^2 - 24x + 144$

Remember to **factor the common factor first:**

31. $5x^2 - 20x + 20$ **32.** $2x^2 - 20x + 50$ **33.** $3x^2 + 6x + 3$

34. $3x^2 - 60x + 300$ **35.** $6x^2 - 36x + 54$ **36.** $4x^2 + 32x + 64$

37. $x^3 + 4x^2 + 4x$ **38.** $x^3 - 12x^2 + 36x$ **39.** $9x^3 - 18x^2 + 9x$

40. $6x^4 - 36x^3 + 54x^2$ **41.** $12y^4 - 48y^3 + 48y^2$ **42.** $2y^4 - 28y^3 + 98y^2$

In the next exercises, remember that **sum of squares**, such as x^2+9 **or** x^2+4, does **not** factor by this method.

43. $x^4 - 16$

$= (x^2 - \_\_\_\_\_)(x^2 + \_\_\_\_\_)$

$= (x - \_\_\_)(x + \_\_\_)(x^2 + \_\_\_)$

44. $x^4 - 1$

45. $x^4 - 81$ **46.** $x^4 - y^4$

47. $81x^4 - 16y^4$ **48.** $16x^4 - 81$

49. $x^4 + 10x^2 + 9$ **50.** $x^4 + 13x^2 + 36$

51. $x^4 - 13x^2 + 36$

52. $x^4 - 29x^2 + 100$

53. $y^4 + 5y^2 - 36$

54. $y^4 - 18y^2 + 81$

55. $25x^4 - 64x^2$

56. $9x^4 - 81x^2$

57. $9x^4 - 36x^2Y^2$

58. $9x^3 - 54x^2 + 81x$

59. $8x^3 + 80x^2 + 200x$

60. $8y^4 + 16y^3 + 8y^2$

SUM AND DIFFERENCE OF CUBES

In this section, formulas and procedures will enable you to factor expressions in the form $x^3 - y^3$ and also $x^3 + y^3$. Recall from previous sections that $x^2 - y^2 = (x - y)(x + y)$ and that $x^2 + y^2$ cannot be factored. Begin with the multiplication problems:

$$
\begin{aligned}
(X - Y)(X^2 + XY + Y^2) &= X^3 + X^2Y + XY^2 \\
&\quad \underline{- X^2Y - XY^2 - Y^3} \\
&= X^3 - Y^3
\end{aligned}
$$

$$
\begin{aligned}
(X + Y)(X^2 - XY + Y^2) &= X^3 - X^2Y + XY^2 \\
&\quad \underline{+ X^2Y - XY^2 + Y^3} \\
&= X^3 + Y^3 .
\end{aligned}
$$

This derives the formulas, known as the "**sum and difference of cubes formulas.**"

SUM AND DIFFERENCE OF CUBES FORMULAS

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Translated into words, this means the sum or difference of two cubes can be factored into the product of a binomial times a trinomial. Begin by taking the cube root of the perfect cubes.

In the difference formula, the binomial is "the first minus the second". Then use this binomial to "build" the trinomial that follows: take the "square of the first" plus the "product of the first and second" plus the "square of the second."

In the sum formula, the binomial is "the first plus the second". Then use this binomial to "build" the trinomial that follows: take the "square of the first" minus the "product of the first and second" plus the "square of the second."

These formulas are really easy to remember. Notice that the $x^3 - y^3$ formula begins with $(x - y)$. The $x^3 + y^3$ formula begins with $(x + y)$. Next notice that the trinomial factor in both formulas is the same except for one sign. This trinomial factor in each formula involves the "square of the first," the "product of the two," and the "square of the second." The first sign in the trinomial is the opposite of the sign of the binomial, and the last sign is always positive. Finally, you will never be able to factor the resulting trinomial (by ordinary trinomial methods), so you need not even try (at this level).

Before getting into the exercises, be sure to be familiar with the perfect cubes: $1^3 = 1$; $2^3 = 8$; $3^3 = 27$; $4^3 = 64$; and $5^3 = 125$. **Be able to recite these from memory: 1, 8, 27, 64, 125. Practice them as you drive down the highway!**

In each of the following, factor completely:

1. $x^3 - 8$
   $= x^3 - 2^3$ [X=first; 2=second]
   $= (\quad - \quad)(x^2 + 2x + 2^2)$
   $= (\qquad)(\qquad)$

2. $x^3 - 125$
   $= (\quad)^3 - (\quad)^3$
   $= (\quad - \quad)(\quad + \quad + \quad)$
   $= (\qquad)(\qquad)$

3. $x^3 - 64$
   $= (\quad)^3 - (\quad)^3$
   $= (\qquad)(\qquad)$

4. $x^3 - 27$
   $= (\quad)^3 - (\quad)^3$
   $= (\qquad)(\qquad)$

5. $x^3 + 8$
   $= x^3 + 2^3$ [x=first; 2=second]
   $= (\quad + \quad)(x^2 - \quad + \quad)$

6. $x^3 + 64$
   $= (\quad)^3 + (\quad)^3$
   $= (\quad + \quad)(\quad - \quad + \quad)$

7. $x^3 + 125$

$= (\quad)^3 + (\quad)^3$

$= (\quad)(\quad)$

8. $x^3 + 27$

$= (\quad)^3 + (\quad)^3$

$= (\quad)(\quad)$

9. $8x^3 - 125$

$= (2x)^3 - 5^3$ [2 x =first; 5=second]

$= (\quad - \quad)[(2x)^2 + (2x)(5) + 5^2]$

$= (\quad)(\quad)$

10. $27x^3 - 8y^3$

$= (\quad)^3 - (\quad)^3$

$= (\quad - \quad)[\quad + \quad + \quad]$

$= (\quad)(\quad)$

11. $64x^3 + 125$

$= (\quad)^3 + (\quad)^3$

$= (\quad)[\quad]$

$= (\quad)(\quad)$

12. $27x^3 + 8y^3$

$= (\quad)^3 + (\quad)^3$

$= (\quad)[\quad]$

$= (\quad)(\quad)$

13. $8x^3 - 27y^3$

14. $125y^3 - 8x^3$

15. $8x^3 + 1$

16. $125y^3 - 1$

In the next exercises, don’t forget the common factor first.

17. $16x^4 - 54x$

$= 2x\ (8x^3 - 27)$

$= 2x\ [(2x)^3 - (3)^3]$

18. $3x^3 - 24y^3$

$= 2x\ (\_\_\_ - \_\_\_)(\_\_\_\_ + \_\_\_\_ + \_\_\_\_)$

19. $5x^4 + 40x$

20. $10x^5y + 80x^2y^4$

21. $3x^5y^5 - 81x^2y^2$

22. $16x^2y^2 + 250x^2y^5$

23. $x^6 - y^9$

$= (x^2)^3 - (y^3)^3$

= ________________________________

24. $x^6 + y^9$

$= (\quad)^3 + (\quad)^3$

= ________________________________

25. $8x^6 + 125y^6$

$= (\quad)^3 + (\quad)^3$

=

=

26. $8x^6 - 125y^6$

Remember, if there is a common factor, factor this first.

27. $5x^7 + 40xy^9$

$= 5x(\qquad\qquad)$

$= 5x[(\quad)^3 + (\quad)^3]$

$=$

$=$

28. $5x^7 - 40xy^9$

29. $16x^4 - 2xy^6$

30. $25x^5 + 200x^8y^9$

31. $x^6 + 2x^3 + 1$

$= (x^3 \qquad)(x^3 \qquad)$

$=$

$=$

32. $x^6 - 9x^3 + 8$

33. $x^6 - 7x^3 - 8$

34. $x^6 - 64$

$= (x^3 - \qquad)(x^3 + \qquad)$

$=$

ANSWERS 2.01 Part I

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1. $5X(X+3)$; **2.** $6(3X+4Y)$; **3.** $7Y(5X+1)$; **4.** $5X^2(X-9)$;
5. $8(2a+3b-1)$; **6.** $12(X-2Y+4)$; **7.** $X^2(X+4)$; **8.** $4y(y^2+2)$;
9. $8a^2(2a-3)$; **10.** $6Z^2(2Z-3)$; **11.** $12(2X^2+1)$; **12.** $16b^2(1+3b)$;
13. $24X^2(X+1)$; **14.** $16X^2(1-2X)$; **15.** $4(2a+3b-5c)$;
16. $8(5X-4Y+8)$; **17.** $7Y(6Y^2-2Y+7)$; **18.** $24X(X^2+X+1)$;
19. $19X^2(X+Y+2)$; **20.** $17X^2(X-2)$; **21.** $Y^3(Y^2-14)$; **22.** $X^3(X^7+5)$;
23. $Y^4(Y^6+7)$; **24.** $X^5(X^2+8)$; **25.** $4X^2Y^2(4Y-3X)$; **26.** $5X^4Y^2(X+2Y)$;
27. $4X^3Y^3(2X^2+3Y)$; **28.** $12X^2Y^4(3X+2Y^2)$; **29a)** $X(Y+7)$; **b)** $a(Y+7)$;
29 c) $\$(Y+7)$; **d)** (Junk)$(Y+7)$; **e)** $(3X+4)(Y+7)$;
30a) $Y(4X+3)$; **b)** $a(4X+3)$; **c)** $\$(4X+3)$; **d)** (Junk)$(4X+3)$;
e) $(8Y-7)(4X+3)$; **31.** $(3X+4)(a-5)$; **32.** $(8r+7)(5q+3)$;
33. $(3X+4)(5u+9v)$; **34.** $(8Y-7)(10X-3)$; **35.** $(X-Y)^2$;
36. $(X-Y)(X-Y+4)$; **37.** $(X-Y)(X+Y-4)$; **38.** $(2X+3Y)(X-Y+4)$;
39. $(X+Y)(X+Y-Z)$; **40.** $(X-Y)(X-Y-Z)$; **41.** $(X-Y)(X-2Y)$;
42. $(X+Y)(X)$ or $X(X+Y)$; **43.** $(2X+3Y)(2X+3Y-5)$;
44. $(2X-3Y)(2X-3Y-5)$; **45.** $(2X+3Y)(2X+3Y+5)$; **46.** $2X(X+Y)$;
47. $(3X-2Y)(X+8Y)$; **48.** $(5X+3Y)(X-9Y)$.

p.117-119:

1. $(X + 2)(X + 1)$ **2.** $(X + 7)(X + 2)$ **3.** $(X + 5)(X + 2)$
4. $(X - 7)(X - 5)$ **5.** $(X -11)(X - 2)$ **6.** $(X - 6)(X - 3)$
7. $(X - 5)(X + 8)$ **8.** $(X +20)(X - 2)$ **9.** $(X +40)(X - 1)$
10. $(X -16)(X + 1)$ **11.** $(X - 8)(X + 2)$ **12.** $(X - 9)(X + 8)$
13. $(X - 9)(X + 2)$ **14.** $(X + 6)(X - 3)$ **15.** $(X -15)(X - 4)$
16. $(X - 9)(X + 3)$ **17.** $(X + 6)(X - 4)$ **18.** $(X - 8)(X + 3)$
19. $(X -30)(X - 2)$ **20.** $(X +15)(X - 4)$ **21.** $(X +60)(X + 1)$
22. $(X -20)(X - 3)$ **23.** $(X -20)(X + 3)$ **24.** $(X -60)(X + 1)$
25. $(X - 8)(X + 7)$ **26.** $(X - 9)(X + 8)$ **27.** $(X + 9)(X + 8)$
28. $(X + 3)(X +14)$ **29.** $(X -21)(X + 2)$ **30.** $(X -21)(X - 2)$
31. $3(X + 3)(X - 1)$ **32.** $5(X + 4)(X - 1)$
33. $8(X + 3)(X - 2)$ **34.** $6(X + 2)(X - 1)$
35. $12(X + 5)(X - 2)$ **36.** $10(X + 5)(X - 2)$
37. $X(X - 5)(X + 1)$ **38.** $X(X + 2)(X + 3)$
39. $2X(X - 5)(X - 2)$ **40.** $5X(X + 2)(X - 1)$
41. $7X(X + 6)(X + 1)$ **42.** $8X(X - 4)(X - 1)$
43. $X^2(X + 5)(X - 4)$ **44.** $X^2(X + 7)(X - 5)$
45. $15X^2(X - 4)(X + 1)$ **46.** $20X^2(X - 3)(X + 2)$
47. $30X^2(X + 5)(X - 2)$ **48.** $18X^2(X + 2)(X + 1)$

ANSWERS 2.01 Part I (Continued)

p.121:

1. (3X+1)(X+1) 2. (4X+1)(X+1) 3. (7X-1)(X-1)
4. (3X-1)(X+1) 5. (4X-1)(X+1) 6. (7X-1)(X+1)
7. (5X-1)(2X-1) 8. (10X+1)(X-1) 9. (5X-1)(2X+1)
10. (3X+5)(X+1) 11. (3X+1)(X+5) 12. (3X+5)(X-1)
13. (3X+1)(X-5) 14. (3X-1)(X-11) 15. (3X+11)(X-1)
16. (5X+8)(X-1) 17. (5X-1)(X+8) 18. (5X-4)(X+2)
19. (3X-8)(2X-1) 20. (3X-8)(2X+1) 21. (3X+8)(2X-1)
22. (3X+2)(2X+5) 23. (3X-2)(2X+5) 24. (6X+5)(X+2)

p.122-125:

1. (X - 8)(X + 8) 2. (X -10)(X +10) 3. (X - 9)(X + 9)
4. (X -13)(X +13) 5. (X - a)(X + a) 6. (X - b)(X + b)
7. (2X- 3)(2X+ 3) 8. (8X-11)(8X+11) 9. (4X- 7)(4X+ 7)
10. (9X-5Y)(9X+5Y) 11. (7X-6Y)(7X+6Y) 12. (5X-12a)(5X+12a)
13. 9(X-1)(X+1) 14. 3(X-2)(X+2) 15. 5(X - 3)(X + 3)
16. 4(X-4)(X+4) 17. 4(X-5)(X+5) 18. 8(X - 3)(X + 3)
19. 3X(X-5)(X+5) 20. 5X(X-4)(X+4) 21. 2X(X - 5)(X + 5)
22. $4X^2(4X-Y)(4X+Y)$ 23. $12Y^2(Y-X)(Y+X)$ 24. $79Y^2(Y-X)(Y+X)$
25. $(X + 2)^2$ 26. $(X + 7)^2$ 27. $(X + 10)^2$
28. $(X - 6)^2$ 29. $(X - 9)^2$ 30. $(X - 12)^2$
31. $5(X - 2)^2$ 32. $2(X - 5)^2$ 33. $3(X + 1)^2$
34. $3(X - 10)^2$ 35. $6(X - 3)^2$ 36. $4(X + 4)^2$
37. $X(X + 2)^2$ 38. $X(X - 6)^2$ 39. $9X(X - 1)^2$
40. $6X^2(X - 3)^2$ 41. $12Y^2(Y - 2)^2$ 42. $2Y^2(Y - 7)^2$
43. $(X - 2)(X + 2)(X^2 + 4)$ 44. $(X - 1)(X + 1)(X^2 + 1)$
45. $(X - 3)(X + 3)(X^2 + 9)$ 46. $(X - Y)(X + Y)(X^2 + Y^2)$
47. $(3X-2Y)(3X+2Y)(9X^2 +4Y^2)$ 48. $(2X - 3)(2X + 3)(4X^2 + 9)$
49. $(X^2 + 9)(X^2 + 1)$ 50. $(X^2 + 9)(X^2 + 4)$
51. (X-3)(X+3)(X-2)(X+2) 52. (X-5)(X+5)(X-2)(X+2)
53. $(Y^2+9)(Y-2)(Y+2)$ 54. $(Y - 3)^2(Y + 3)^2$
55. $X^2(5X - 8)(5X + 8)$ 56. $9X^2(X - 3)(X + 3)$
57. $9X^2(X - 2Y)(X + 2Y)$ 58. $9X(X - 3)^2$
59. $8X(X + 5)^2$ 60. $8Y^2 (Y + 1)^2$

p.127-130:

1. $(X-2)(X^2+2X+4)$; **2.** $(X-5)(X^2+5X+25)$;
3. $(X-4)(X^2+4X+16)$; **4.** $(X-3)(X^2+3X+9)$;
5. $(X+2)(X^2-2X+4)$; **6.** $(X+4)(X^2-4X+16)$;
7. $(X+5)(X^2-5X+25)$; **8.** $(X+3)(X^2-3X+9)$;
9. $(2X-5)(4X^2+10X+25)$; **10.** $(3X-2Y)(9X^2+6XY+4Y^2)$;
11. $(4X+5)(16X^2-20X+25)$; **12.** $(3X+2Y)(9X^2-6XY+4Y^2)$;
13. $(2X-3Y)(4X^2+6XY+9Y^2)$; **14.** $(5Y-2X)(25Y^2+10XY+4X^2)$;
15. $(2X+1)(4X^2-2X+1)$; **16.** $(5Y-1)(25Y^2+5Y+1)$;
17. $2X(2X-3)(4X^2+6X+9)$; **18.** $3(X-2Y)(X^2+2XY+4Y^2)$;
19. $5X(X+2)(X^2-2X+4)$; **20.** $10X^2Y(X+2Y)(X^2-2XY+4Y^2)$;

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p.127-130: **21.** $3X^2Y^2(XY-3)(X^2Y^2+3XY+9)$; **22.** $2X^2Y^2(2+5Y)(4-10Y+25Y^2)$;
23. $(X^2-Y^3)(X^4+X^2Y^3+Y^6)$; **24.** $(X^2+Y^3)(X^4-X^2Y^3+Y^6)$;
25. $(2X^2+5Y^2)(4X^4-10X^2Y^2+25Y^4)$ **26.** $(2X^2-5Y^2)(4X^4+10X^2Y^2+25Y^4)$;
27. $5X(X^2+2Y^3)(X^4-2X^2Y^3+4Y^6)$; **28.** $5X(X^2-2Y^3)(X^4+2X^2Y^3+4Y^6)$;
29. $2X(2X-Y^2)(4X^2+2XY^2+Y^4)$; **30.** $25X^5(1+2XY^3)(1-2XY^3+4X^2Y^6)$;
31. $(X+1)^2(X^2-X+1)^2$; **32.** $(X-2)(X^2+2X+4)(X-1)(X^2+X+1)$;
33. $(X-2)(X^2+2X+4)(X+1)(X^2-X+1)$;
34. $(X-2)(X^2+2X+4)(X+2)(X^2-2X+4)$.